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ABOUT SOME SOLUTIONS OF THE SYSTEM OF EQUATIONS OF THE THERMOELASTIC MATERIALS WITH DIFFUSION MICROTEMPERATURES AND MICROCONCENTRATIONS

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Abstract. In the present paper the 2D equations of thermoelasticity with diffusion, microtemperatures and micro-concentrations are considered. The fundamental and singular matrices of solutions are constructed by means of the elementary (harmonic, bi-harmonic and meta-harmonic) functions.

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1. Introduction

To study basic thermoelastic problems involving diffusion, microtemperatures, and microconcentrations is of interest in many engineering applications such as satellite problems, aircraft landing on water, the oil extraction.

The basic problems have been considered by several authors, by Bazarra et al. in [1] the proposed dynamical problem for the thermoelastic body with diffusion whose microelements are assumed to possess microtemperatures and microconcentrations. In [2] a nonlinear theory of thermodinamics is considered for elastic materials whose particles are subjected to the classical displacement, temperature and mass diffusion fields and whose microelements possess microtemperatures and microconcentrations. It is shown that there exists coupling between temperature, chemical potential, microtemperatures and microconcentrations for isotropic bodies.

The diffusion theory was established by Nowacki [3] and developed later by Sherief et.al [4]. The linear theory of thermoelasticity for materials with the classical displacement and temperature fields, possess microtemperatures, was established by Grot [5]. Ieşan and Quintanilla in [6] have developed the linear theory of thermoelastic materials with microtemperatures, have formulated the boundary value problems and presented an uniqueness result and a solution of the Boussinesq-Somigliana-Galerkin type.

A thermoelastic problems involving diffusion effect(i.e.the coupling among the fields of strain, temperature, and mass diffusion which leads to a random walk of an ensemble of particles, from regions of high concentrations to regions of lower concentrations)has been considered by other authors. In [7-9] the thermoelastic diffusion theory with voids is considered.

Many problems are investigated for elastic materials with microtemperatures by several researchers (some of those articles can be seen in [10-21] and references therein).

In the present paper the 2D linear equilibrium theory equations of thermoelasticity with diffusion, microtemperatures and microconcentrations are considered and the fundamental and singular matrices of solutions are constructed by means of the elementary (harmonic, bi-harmonic and meta-harmonic) functions.

2. Basic equations

Let $\mathbf{x} = (x_1, x_2)$ be a point of the Euclidean two-dimensional space E^2. The system of equations of isotropic and homogeneous thermoelastic bodies with diffusion, microtemperatures and microconcentrations can be written as follows [1,2]

$$\mu\Delta\mathbf{u} + (\lambda+\mu)\mathrm{grad}\mathrm{div}\mathbf{u} - \gamma_1\mathrm{grad}\theta - \gamma_2\mathrm{grad}P = 0, \tag{1}$$

$$k^*\Delta\theta + k_1^*\mathrm{div}\mathbf{T} = 0, \tag{2}$$

$$h^*\Delta P + h_1\mathrm{div}\mathbf{C} = 0, \tag{3}$$

$$k_6\Delta\mathbf{T} + (k_4+k_5)\mathrm{grad}\mathrm{div}\mathbf{T} - k_3\mathrm{grad}\theta - k_2\mathbf{T} = 0, \tag{4}$$

$$h_6\Delta\mathbf{C} + (h_4+h_5)\mathrm{grad}\mathrm{div}\mathbf{C} - h_3\mathrm{grad}P - h_2\mathbf{C} = 0, \tag{5}$$

where $\mathbf{u} := (u_1, u_2)^\top$ is the displacement vector, $\lambda, \mu, k_j, h_j, k^*, k_1^*$, are the material constants, θ is the difference of the temperature between the current state and a reference temperature, $\mathbf{T} := (T_1, T_2)^\top$, $\mathbf{C} = (C_1, C_2)^\top$. T_i and C_i are called microtemperatures and microconcentrations, respectively, P is the particle chemical potential. Δ is the 2D Laplace operator.

We assume that the constitutive coefficients satisfy the following conditions

$$\mu > 0, \quad k^* > 0, \quad k_4 + k_5 > 0, \quad k_6 - k_5 > 0, \quad \frac{4kk_2}{T_0} - \left(\frac{k_1}{T_0} + k_3\right)^2 > 0,$$

$$h^* > 0, \quad k_6 > 0, \quad h_6 > 0, \quad k_2 > 0, \quad h_2 > 0, \quad 4hh_2 - (h_1+h_3)^2 > 0,$$

$$\lambda + \mu > 0, \quad 2k_4 + k_5 + k_6 > 0, \quad k_6 + k_5 > 0.$$

3. The basic fundamental matrix

In this section we will construct the fundamental solution of equations (1-5) explicitly, which consists of harmonic, bi-harmonic and meta-harmonic functions. For this we introduce the following matrix differential operator

$$\mathbf{A}(\partial_\mathbf{x}) = \| A_{lj}(\partial_x) \|_{8\times 8}, \quad l, j = 1, 2, ..., 8, \tag{6}$$

where

$$A_{lj} = \delta_{lj}\mu\Delta + (\lambda+\mu)\frac{\partial^2}{\partial x_l \partial x_j}, \quad l, j = 1, 2, \quad A_{1m} = 0, \quad m = 5, ..., 8,$$

$$A_{j3} = -\gamma_1\frac{\partial}{\partial x_j}, \quad A_{j4} = -\gamma_2\frac{\partial}{\partial x_j}, \quad j = 1, 2, \quad A_{2m} = 0, \quad m = 5, ..., 8,$$

$$A_{3j} = 0, \quad j = 1, 2, \quad A_{4j} = 0, \quad j = 1, 2, 3 \quad A_{5j} = 0, \quad j = 1, 2,$$

$$A_{6j} = 0, \quad j = 1, 2, \quad A_{7j} = 0, \quad j = 1, 2, 3, \quad A_{8j} = 0, \quad j = 1, 2, 3,$$

$$A_{33} = k * \Delta, \quad A_{34} = 0, \quad A_{35} = k_1^* \frac{\partial}{\partial x_1}, \quad A_{36} = h^* \frac{\partial}{\partial x_2}, \quad A_{37} = 0, \quad A_{38} = 0,$$

$$A_{44} = k * \Delta, \quad A_{45} = 0, \quad A_{46} = 0, \quad A_{47} = h_1 \frac{\partial}{\partial x_1}, \quad A_{48} = h_1 \frac{\partial}{\partial x_2},$$

$$A_{53} = -k_3 \frac{\partial}{\partial x_1}, \quad A_{54} = 0, \quad A_{55} = \left[k_6 \Delta - k_2 + (k_4 + k_5) \frac{\partial^2}{\partial x_l^2} \right],$$

$$A_{56} = (k_4 + k_5) \frac{\partial^2}{\partial x_1 \partial x_2}, \quad A_{57} = 0, \quad A_{58} = 0, \quad A_{63} = -k_3 \frac{\partial}{\partial x_2}, \quad A_{64} = 0,$$

$$A_{65} = (k_4 + k_5) \frac{\partial^2}{\partial x_1 \partial x_2}, \quad A_{66} = \left[k_6 \Delta - k_2 + (k_4 + k_5) \frac{\partial^2}{\partial x_2^2} \right],$$

$$A_{67} = 0, \quad A_{68} = 0, \quad A_{74} = -h_3 \frac{\partial}{\partial x_1}, \quad A_{75} = 0, \quad A_{76} = 0,$$

$$A_{77} = \left[h_6 \Delta - h_2 + (h_4 + h_5) \frac{\partial^2}{\partial x_1^2} \right], \quad A_{78} = (h_4 + h_5) \frac{\partial^2}{\partial x_1 \partial x_2},$$

$$A_{84} = -h_3 \frac{\partial}{\partial x_2}, \quad A_{85} = 0, \quad A_{86} = 0, \quad A_{87} = (h_4 + h_5) \frac{\partial^2}{\partial x_1 \partial x_2}$$

$$A_{88} = \left[h_6 \Delta - h_2 + (h_4 + h_5) \frac{\partial^2}{\partial x_2^2} \right],$$

$\delta_{\alpha\gamma}$ is the Kronecker delta. Then system (1-5) can be rewritten as

$$\mathbf{A}(\partial_x)\mathbf{U} = 0, \tag{7}$$

where

$$\mathbf{U} = (\mathbf{u}, \theta, P, \mathbf{T}, \mathbf{C}).$$

We also consider the system of the equation

$$\mathbf{A}^\top(\partial_x)U = 0. \tag{8}$$

where $\mathbf{A}^\top(\partial_x)$ is the transpose of matrix $\mathbf{A}(\partial_x)$.

The determinant of equation (7) is

$$\Delta\Delta\Delta\Delta(\Delta - s_1^2)(\Delta - s_2^2)(\Delta - s_3^2)(\Delta - s_4^2)\mathbf{\Psi} = 0. \tag{9}$$

where

$$s_1^2 = \frac{k^* k_2 - k_1^* k_3}{k^* k_7} > 0, \quad s_2^2 = \frac{k_2}{k_6} > 0, \quad s_3^2 = \frac{h^* h_2 - h_1 h_3}{h^* h_7} > 0,$$

$$s_4^2 = \frac{h_2}{h_6} > 0, \quad k_7 = k_4 + k_5 + k_6, \quad h_7 = h_4 + h_5 + h_6.$$

$\mathbf{\Psi}$ is an eight-component vector function. We assume that the values s_j^2 are distinct and different from zero.

From (9), after some calculations, we obtain

$$\Delta^4\mathbf{\Psi}=\sum_{k=1}^{4}d_k\varphi_k,\quad \Delta^3\mathbf{\Psi}=\frac{\varphi}{s_1^2s_2^2s_3^2s_4^2}+\sum_{k=1}^{4}\frac{d_k\varphi_k}{s_k^2},\quad \Delta^2\mathbf{\Psi}=\frac{\varphi_0}{s_1^2s_2^2s_3^2s_4^2}+\sum_{k=1}^{4}\frac{d_k\varphi_k}{s_k^4},$$

where

$$\Delta\varphi=0,\quad \Delta\varphi_0=\varphi,\quad (\Delta-s_k^2)\varphi_k=0,$$

$$d_1^{-1}=(s_1^2-s_2^2)(s_1^2-s_3^2)(s_1^2-s_4^2),\quad d_2^{-1}=(s_2^2-s_1^2)(s_2^2-s_3^2)(s_2^2-s_4^2),$$

$$d_3^{-1}=(s_3^2-s_1^2)(s_3^2-s_2^2)(s_3^2-s_4^2),\quad d_4^{-1}=(s_4^2-s_1^2)(s_4^2-s_2^2)(s_4^2-s_3^2),$$

$$\varphi=\ln r,\quad \varphi_0=\frac{r^2(\ln r-1)}{4},\quad \varphi_m=\frac{\pi}{2i}H_0^{(1)}(is_mr),$$

$H_0^{(1)}(is_mr)$ is Hankel's function of the first kind with the index 0,

$$H_0^{(1)}(is_mr)=\frac{2i}{\pi}J_0(is_mr)\ln r+\frac{2i}{\pi}\left(\ln\frac{is_m}{2}+C-\frac{i\pi}{2}\right)J_0(is_mr)$$

$$-\frac{2i}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{is_mr}{2}\right)^{2k}\left(\frac{1}{k}+\frac{1}{k-1}+...+1\right),\quad m=1,..,4,$$

$$J_0(is_mr)=\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{is_mr}{2}\right)^{2k},\quad r^2=(x_1-y_1)^2+(x_2-y_2)^2,$$

$$\sum_{j=1}^{4}d_j=0,\quad \sum_{j=1}^{4}d_js_j^2=0,\quad \sum_{j=1}^{4}d_js_j^4=0,\quad \sum_{j=1}^{4}d_js_j^6=1.$$

We introduce the matrix differential operator $\mathbf{B}(\partial\mathbf{x})$ consisting of cofactors of elements of the matrix $\mathbf{A}^\top$ divided on $h_6h_7k_6k_7\mu\mu_0k^*h^*$:

$$B_{ij}^*=\left[\frac{\delta_{ij}}{\mu}\Delta-\frac{\lambda+\mu}{\mu\mu_0}\frac{\partial^2}{\partial x_i\partial x_j}\right]\Delta^2(\Delta-s_1^2)(\Delta-s_2^2)(\Delta-s_3^2)(\Delta-s_4^2),\ \ i,j=1,2,$$

$$B_{31}^*=0,\quad B_{32}^*=0,\quad B_{41}^*=0,\quad B_{42}^*=0,\quad B_{43}^*=0,\quad B_{34}^*=0,$$

$$B_{15}^*=-\frac{\gamma_1k_1^*}{\mu_0k^*k_7}\Delta^2(\Delta-s_2^2)(\Delta-s_3^2)(\Delta-s_4^2)\frac{\partial^2}{\partial x_1^2},$$

$$B_{16}^*=-\frac{\gamma_1k_1^*}{\mu_0k^*k_7}\Delta^2(\Delta-s_2^2)(\Delta-s_3^2)(\Delta-s_4^2)\frac{\partial^2}{\partial x_1\partial x_2}=B_{25}^*,$$

$$B^*_{17} = -\frac{\gamma_2 h_1}{\mu_0 h^* h_7}\Delta^2(\Delta - s_2^2)(\Delta - s_1^2)(\Delta - s_4^2)\frac{\partial^2}{\partial x_1^2},$$

$$B^*_{18} = -\frac{\gamma_2 h_1}{\mu_0 h^* h_7}\Delta^2(\Delta - s_2^2)(\Delta - s_1^2)(\Delta - s_4^2)\frac{\partial^2}{\partial x_1 \partial x_2} = B^*_{27},$$

$$B^*_{26} = -\frac{\gamma_1 k_1^*}{\mu_0 k^* k_7}\Delta^2(\Delta - s_2^2)(\Delta - s_3^2)(\Delta - s_4^2)\frac{\partial^2}{\partial x_2^2},$$

$$B^*_{27} = -\frac{\gamma_2 h_1}{\mu_0 h^* h_7}\Delta^2(\Delta - s_2^2)(\Delta - s_1^2)(\Delta - s_4^2)\frac{\partial^2}{\partial x_1 \partial x_2},$$

$$B^*_{28} = -\frac{\gamma_2 h_1}{\mu_0 h^* h_7}\Delta^2(\Delta - s_2^2)(\Delta - s_1^2)(\Delta - s_4^2)\frac{\partial^2}{\partial x_2^2},$$

$$B^*_{35} = -\frac{k_1^*}{k^* k_7}\Delta^3(\Delta - s_2^2)(\Delta - s_3^2)(\Delta - s_4^2)\frac{\partial}{\partial x_1},$$

$$B^*_{36} = -\frac{k_1^*}{k^* k_7}\Delta^3(\Delta - s_2^2)(\Delta - s_3^2)(\Delta - s_4^2)\frac{\partial}{\partial x_2},$$

$$B^*_{37} = 0, \quad B^*_{38} = 0, \quad B^*_{45} = 0, \quad B^*_{46} = 0,$$

$$B^*_{47} = -\frac{h_1}{h^* h_7}\Delta^3(\Delta - s_2^2)(\Delta - s_1^2)(\Delta - s_4^2)\frac{\partial}{\partial x_1},$$

$$B^*_{48} = -\frac{h_1}{h^* h_7}\Delta^3(\Delta - s_2^2)(\Delta - s_1^2)(\Delta - s_4^2)\frac{\partial}{\partial x_2},$$

$$B^*_{51} = B^*_{52} = B^*_{61} = B^*_{54} = B^*_{62} = B^*_{64} = 0,$$

$$B^*_{55} = \frac{1}{k_6 k_7}\left[k_7(\Delta - s_1^2)\Delta - \left[(k_4 + k_5)\Delta + \frac{k_1^* k_3}{k^*}\right]\frac{\partial^2}{\partial x_1^2}\right]\Delta^3(\Delta - s_3^2)(\Delta - s_4^2),$$

$$B^*_{56} = -\frac{1}{k_6 k_7}\left[(k_4 + k_5)\Delta + \frac{k_1^* k_3}{k^*}\right]\frac{\partial^2}{\partial x_1 \partial x_2}\Delta^3(\Delta - s_3^2)(\Delta - s_4^2) = B^*_{65},$$

$$B^*_{66} = \frac{1}{k_6 k_7}\left[k_7(\Delta - s_1^2)\Delta - \left[(k_4 + k_5)\Delta + \frac{h_1 h_3}{h^*}\right]\frac{\partial^2}{\partial x_2^2}\right]\Delta^3(\Delta - s_3^2)(\Delta - s_4^2),$$

$$B^*_{77} = \frac{1}{h_6 h_7}\left[h_7(\Delta - s_3^2)\Delta - \left[(h_4 + h_5)\Delta + \frac{h_1 h_3}{h^*}\right]\frac{\partial^2}{\partial x_1^2}\right]\Delta^3(\Delta - s_1^2)(\Delta - s_2^2),$$

$$B^*_{78} = -\frac{1}{h_6 h_7}\left[(h_4 + h_5)\Delta + \frac{h_1 h_3}{h^*}\right]\frac{\partial^2}{\partial x_1 \partial x_2}\Delta^3(\Delta - s_1^2)(\Delta - s_2^2) = B^*_{87},$$

$$B^*_{88} = \frac{1}{h_6 h_7}\left[h_7(\Delta - s_3^2)\Delta - \left[(h_4+h_5)\Delta + \frac{h_1 h_3}{h^*}\right]\frac{\partial^2}{\partial x_2^2}\right]\Delta^3(\Delta - s_1^2)(\Delta - s_2^2),$$

$$B^*_{71} = B^*_{72} = B^*_{73} = B^*_{81} = B^*_{82} = B^*_{83} = 0,$$

$$B^*_{53} = \frac{k_3}{k^* k_7}\Delta^3(\Delta - s_1^2)(\Delta - s_3^2)(\Delta - s_4^2)\frac{\partial}{\partial x_1},$$

$$B^*_{63} = \frac{k_3}{k^* k_7}\Delta^3(\Delta - s_1^2)(\Delta - s_3^2)(\Delta - s_4^2)\frac{\partial}{\partial x_2},$$

$$B^*_{74} = \frac{h_3}{h^* h_7}\Delta^3(\Delta - s_1^2)(\Delta - s_2^2)(\Delta - s_4^2)\frac{\partial}{\partial x_1},$$

$$B^*_{84} = \frac{h_3}{h^* h_7}\Delta^3(\Delta - s_1^2)(\Delta - s_2^2)(\Delta - s_4^2)\frac{\partial}{\partial x_2},$$

$$B^*_{67} = B^*_{68} = B^*_{75} = B^*_{76} = B^*_{85} = B^*_{86} = 0,$$

$$B^*_{33} = \left[\Delta - \frac{k_2}{k_7}\right]\frac{\Delta^3}{k^*}(\Delta - s_2^2)(\Delta - s_3^2)(\Delta - s_4^2),$$

$$B^*_{44} = \left[\Delta - \frac{h_2}{h_7}\right]\frac{\Delta^3}{h^*}(\Delta - s_1^2)(\Delta - s_2^2)(\Delta - s_4^2),$$

$$B^*_{13} = \left[\Delta - \frac{k_2}{k_7}\right]\frac{\gamma_1}{k^*\mu_0}\Delta^2(\Delta - s_2^2)(\Delta - s_3^2)(\Delta - s_4^2)\frac{\partial}{\partial x_1},$$

$$B^*_{14} = \left[\Delta - \frac{h_2}{h_7}\right]\frac{\gamma_2}{h^*\mu_0}\Delta^2(\Delta - s_1^2)(\Delta - s_2^2)(\Delta - s_4^2),\frac{\partial}{\partial x_1},$$

$$B^*_{23} = \left[\Delta - \frac{k_2}{k_7}\right]\frac{\gamma_1}{k^*\mu_0}\Delta^2(\Delta - s_2^2)(\Delta - s_3^2)(\Delta - s_4^2)\frac{\partial}{\partial x_2},$$

$$B^*_{24} = \left[\Delta - \frac{h_2}{h_7}\right]\frac{\gamma_2}{h^*\mu_0}\Delta^2(\Delta - s_2^2)(\Delta - s_3^2)(\Delta - s_4^2)\frac{\partial}{\partial x_2}.$$

Substituting $\mathbf{\Psi}$ into $\mathbf{U} = \mathbf{B\Psi}$, we obtain the matrix of fundamental solutions for the equation (7), that we denote by $\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y})$

$$\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y}) = \| \, \Gamma_{kj}(\mathbf{x}\text{-}\mathbf{y})(\partial x) \, \|_{8\times 8}, \quad l,j = 1,2,...,8, \tag{10}$$

where

$$\Gamma_{ij} = \frac{\varphi}{\mu} - \frac{\lambda+\mu}{\mu\mu_0}\frac{\partial^2\varphi_0}{\partial x_i \partial x_j}, \qquad i,j = 1.2, \quad \Delta\varphi_0 = \varphi, \quad \Delta\varphi = 0,$$

$$\Gamma_{i3}=\frac{\gamma_1}{k^*\mu_0}\frac{\partial\alpha_{11}}{\partial x_i},\ \ \Gamma_{i4}=\frac{\gamma_2}{h^*\mu_0}\frac{\partial\alpha_{14}}{\partial x_i}\quad i=1.2,$$

$$\alpha_{11}=\frac{\varphi_1-\varphi}{s_1^2}-\frac{k_2}{k_7}\left[\frac{\varphi_1-s_1^2\varphi_0}{s_1^4}+\frac{(s_2^2s_3^2+s_2^2s_4^2+s_4^2s_3^2)}{s_1^2s_2^2s_3^2s_4^2}\varphi\right],$$

$$\alpha_{14}=\frac{\varphi_3-\varphi_0}{s_3^2}-\frac{h_2}{h_7}\left[\frac{\varphi_3-s_3^2\varphi_0}{s_3^4}+\frac{(s_2^2s_1^2+s_1^2s_4^2+s_4^2s_2^2)}{s_1^2s_2^2s_3^2s_4^2}\varphi\right],$$

$$\Gamma_{37}=\Gamma_{38}=\Gamma_{45}=\Gamma_{46}=\Gamma_{51}=\Gamma_{52}=\Gamma_{54}=\Gamma_{57}=\Gamma_{58}=0,$$

$$\Gamma_{61}=\Gamma_{62}=\Gamma_{64}=\Gamma_{67}=\Gamma_{58}=\Gamma_{71}=\Gamma_{72}=\Gamma_{73}=\Gamma_{75}=0,$$

$$\Gamma_{76}=\Gamma_{81}=\Gamma_{82}=\Gamma_{83}=\Gamma_{85}=\Gamma_{86}=0,$$

$$\Gamma_{33}=\frac{1}{k^*}\left[\varphi_1-\frac{k_2}{k_7}\left(\frac{\varphi_1-\varphi}{s_1^2}\right)\right],\quad \Gamma_{31}=\Gamma_{32}=\Gamma_{34}=0,$$

$$\Gamma_{44}=\frac{1}{h^*}\left[\varphi_3-\frac{h_2}{h_7}\left(-\frac{\varphi_3-\varphi}{s_3^2}\right)\right],\quad \Gamma_{41}=\Gamma_{42}=\Gamma_{43}=0,$$

$$\Gamma_{15}=-\frac{\gamma_1k_1^*}{\mu_0k^*k_7}\frac{\partial^2\alpha_{15}}{\partial^2x_1^2},\qquad \Gamma_{16}=-\frac{\gamma_1k_1^*}{\mu_0k^*k_7}\frac{\partial^2\alpha_{15}}{\partial x_1\partial x_2}=\Gamma_{25},$$

$$\Gamma_{26}=-\frac{\gamma_1k_1^*}{\mu_0k^*k_7}\frac{\partial^2\alpha_{15}}{\partial x_2^2},\qquad \Gamma_{17}=-\frac{\gamma_2h_1}{\mu_0h^*h_7}\frac{\partial^2\alpha_{17}}{\partial x_1^2},$$

$$\Gamma_{27}=-\frac{\gamma_2h_1}{\mu_0h^*h_7}\frac{\partial^2\alpha_{17}}{\partial x_1\partial x_2}=\Gamma_{18},\qquad \Gamma_{28}=-\frac{\gamma_2h_1}{\mu_0h^*h_7}\frac{\partial^2\alpha_{17}}{\partial x_2^2},$$

$$\Gamma_{35}=-\frac{k_1^*}{k^*k_7}\frac{\partial}{\partial x_1}\frac{\varphi_1-\varphi}{s_1^2},\qquad \Gamma_{36}=-\frac{k_1^*}{k^*k_7}\frac{\partial}{\partial x_2}\frac{\varphi_1-\varphi}{s_1^2},$$

$$\Gamma_{47}=-\frac{h_1}{h^*h_7}\frac{\partial}{\partial x_1}\frac{\varphi_3-\varphi}{s_3^2},\qquad \Gamma_{48}=-\frac{h_1}{h^*h_7}\frac{\partial}{\partial x_2}\frac{\varphi_3-\varphi}{s_3^2},$$

$$\Gamma_{55}=\frac{\varphi_2}{k_6}-\frac{\partial^2\alpha_{55}}{\partial x_1^2},\qquad \Gamma_{65}=\Gamma_{56}=-\frac{\partial^2\alpha_{55}}{\partial x_1\partial x_2},$$

$$\Gamma_{66}=\frac{\varphi_2}{k_6}-\frac{\partial^2\alpha_{55}}{\partial x_2^2},\qquad \Gamma_{77}=\frac{\varphi_4}{h_6}-\frac{\partial^2\alpha_{77}}{\partial x_1^2},$$

$$\Gamma_{78}=\Gamma_{87}=-\frac{\partial^2\alpha_{77}}{\partial x_1\partial x_2},\qquad \Gamma_{88}=\frac{\varphi_4}{h_6}-\frac{\partial^2\alpha_{77}}{\partial x_2^2},$$

$$\Gamma_{53}=\frac{k_3}{k^*k_7}\frac{\partial}{\partial x_1}\frac{\varphi_2-\varphi}{s_2^2},\qquad \Gamma_{63}=\frac{k_3}{k^*k_7}\frac{\partial}{\partial x_2}\frac{\varphi_2-\varphi}{s_2^2},$$

$$\Gamma_{74}=\frac{h_3}{h^*h_7}\frac{\partial}{\partial x_1}\frac{\varphi_3-\varphi}{s_3^2},\qquad \Gamma_{84}=\frac{h_3}{h^*h_7}\frac{\partial}{\partial x_2}\frac{\varphi_3-\varphi}{s_3^2},$$

$$\alpha_{55}=\frac{1}{k_6k_7(s_1^2-s_2^2)}\left[(k_4+k_5)(\varphi_1-\varphi_2)+\frac{k_1^*k_3}{k^*}\left(\frac{\varphi_1}{s_1^2}-\frac{\varphi_2}{s_2^2}\right)\right]+\frac{k_1^*k_3}{k_6k_7k^*}\frac{\varphi}{s_1^2s_2^2},$$

$$\alpha_{77}=\frac{1}{h_67_7(s_3^2-s_4^2)}\left[(h_4+h_5)(\varphi_3-\varphi_4)+\frac{h_1h_3}{h^*}\left(\frac{\varphi_5}{s_3^2}-\frac{\varphi_4}{s_4^2}\right)\right]+\frac{h_1h_3}{h_6h_7h^*}\frac{\varphi}{s_3^2s_4^2},$$

$$\alpha_{15}=\frac{\varphi_1-s_1^2\varphi_0}{s_1^4}+\frac{s_2^2s_3^2+s_3^2s_4^2+s_4^2s_2^2}{s_1^2s_2^2s_3^2s_4^2}\varphi,$$

$$\alpha_{17}=\frac{\varphi_3-s_3^2\varphi_0}{s_3^4}+\frac{s_2^2s_1^2+s_1^2s_4^2+s_4^2s_2^2}{s_1^2s_2^2s_3^2s_4^2}\varphi,$$

Clearly

$$\frac{\pi}{2i}H_0^{(1)}(\lambda r)=\ln|\mathbf{x}-\mathbf{y}|-\frac{\lambda^2}{4}|\mathbf{x}-\mathbf{y}|^2\ln|\mathbf{x}-\mathbf{y}|+const+O(|\mathbf{x}-\mathbf{y}|^2).$$

It is evident that all elements of $\mathbf{\Gamma}$(**x-y**) are single-valued functions on the whole plane and they have a logarithmic singularity at most. It can be shown that columns of the matrix $\mathbf{\Gamma}$(**x-y**) are solutions to the system (7) with respect to $\mathbf{x}$ for any $\mathbf{x}\neq\mathbf{y}$. By applying the methods, as in the classical theory of elasticity, we can directly prove the following;(for details see in [22])

Theorem 1. *The elements of the matrix* $\mathbf{\Gamma}$(**x-y**) *have a logarithmic singularity as* $\mathbf{x}\to\mathbf{y}$ *and each column of the matrix* $\mathbf{\Gamma}$(**x-y**), *considered as a vector, is a solution of system* (7) *at every point* $\mathbf{x}$ *if* $\mathbf{x}\neq\mathbf{y}$.

We consider the system of the equation $\widetilde{\boldsymbol{A}}(\partial_{\mathbf{x}})\mathbf{U}=\boldsymbol{A}^{\boldsymbol{T}}(-\partial_{\mathbf{x}})\mathbf{U}$. The fundamental matrix of solutions of system $\widetilde{\boldsymbol{A}}(\partial_{\mathbf{x}})\mathbf{U}=0$ is $\widetilde{\mathbf{\Gamma}}(\mathbf{x})=\mathbf{\Gamma}^\top(-\mathbf{x})$. The following basic properties of $\widetilde{\mathbf{\Gamma}}(\mathbf{x})$ may be easily verified:

Theorem 2. *Each column of the matrix* $\widetilde{\mathbf{\Gamma}}$(**x-y**), *considered as a vector, satisfies the associated system* $\widetilde{\boldsymbol{A}}(\partial\mathbf{x})\widetilde{\mathbf{\Gamma}}$(**x-y**) $=0$, *at every point* $\mathbf{x}$ if $\mathbf{x}\neq\mathbf{y}$ *and the elements of the matrix* $\widetilde{\mathbf{\Gamma}}$(**x-y**) *have a logarithmic singularity as* $\mathbf{x}\to\mathbf{y}$.

4. Singular matrix of solutions

Let $\boldsymbol{P}^{(1)}(\partial\mathbf{x},\mathbf{n})$ be the stress operator in the linear theory of thermoelasticity and $\boldsymbol{P}^{(1)}(\partial\mathbf{x},\mathbf{n})\mathbf{U}$ is the stress vector which acts on an element of the arc with the normal $\mathbf{n}=(n_1,n_2)$

$$\boldsymbol{P}^{(1)}(\partial\mathbf{x},\mathbf{n})\mathbf{U}=\boldsymbol{T}(\partial\mathbf{x},\mathbf{n})\mathbf{u}-\mathbf{n}(\gamma_1\theta+\gamma_2P),\tag{11}$$

where $\boldsymbol{T}(\partial\mathbf{x},\mathbf{n})$ is the stress operator of the classical theory of elasticity

$$\boldsymbol{T}(\partial_{\mathbf{x}},\mathbf{n})=\begin{pmatrix}\mu\dfrac{\partial}{\partial\mathbf{n}}+(\lambda+\mu)n_1\dfrac{\partial}{\partial x_1} & (\lambda+\mu)n_1\dfrac{\partial}{\partial x_2}+\mu\dfrac{\partial}{\partial s}\\ (\lambda+\mu)n_2\dfrac{\partial}{\partial x_1}-\mu\dfrac{\partial}{\partial s} & \mu\dfrac{\partial}{\partial\mathbf{n}}+(\lambda+\mu)n_2\dfrac{\partial}{\partial x_2}\end{pmatrix},$$

We introduce the following matrices of differential operators

$$\boldsymbol{T}^{(1)}(\partial_{\mathbf{x}},\mathbf{n})=\begin{pmatrix} k_6\dfrac{\partial}{\partial\mathbf{n}}+(k_4+k_5)n_1\dfrac{\partial}{\partial x_1} & (k_4+k_5)n_1\dfrac{\partial}{\partial x_2}+k_5\dfrac{\partial}{\partial s} \\ (k_4+k_5)n_2\dfrac{\partial}{\partial x_1}-k_5\dfrac{\partial}{\partial s} & k_6\dfrac{\partial}{\partial\mathbf{n}}+(k_4+k_5)n_2\dfrac{\partial}{\partial x_2} \end{pmatrix},$$

$$\boldsymbol{T}^{(2)}(\partial_{\mathbf{x}},\mathbf{n})=\begin{pmatrix} h_6\dfrac{\partial}{\partial\mathbf{n}}+(h_4+h_5)n_1\dfrac{\partial}{\partial x_1} & (h_4+h_5)n_1\dfrac{\partial}{\partial x_2}+h_5\dfrac{\partial}{\partial s} \\ (h_4+h_5)n_2\dfrac{\partial}{\partial x_1}-h_5\dfrac{\partial}{\partial s} & h_6\dfrac{\partial}{\partial\mathbf{n}}+(h_4+h_5)n_2\dfrac{\partial}{\partial x_2} \end{pmatrix},$$

$$\frac{\partial}{\partial\mathbf{n}}=n_1\frac{\partial}{\partial x_1}+n_2\frac{\partial}{\partial x_2},\quad \frac{\partial}{\partial s}=n_2\frac{\partial}{\partial x_1}-n_1\frac{\partial}{\partial x_2},$$

$$\boldsymbol{R}(\partial_{\mathbf{x}},\mathbf{n})=\begin{pmatrix} T_{11}(\partial\mathbf{x},\mathbf{n}) & T_{12}(\partial_{\mathbf{x}},\mathbf{n}) & -\gamma_1 n_1 & -\gamma_2 n_1 \\ T_{21}(\partial_{\mathbf{x}},\mathbf{n}) & T_{22}(\partial_{\mathbf{x}},\mathbf{n}) & -\gamma_1 n_2 & -\gamma_2 n_2 \\ 0 & 0 & k^*\dfrac{\partial}{\partial\mathbf{n}} & 0 \\ 0 & 0 & 0 & h^*\dfrac{\partial}{\partial\mathbf{n}} \end{pmatrix},$$

$$\boldsymbol{R}^{(1)}(\partial_{\mathbf{x}},\mathbf{n})=\begin{pmatrix} T^{(1)}_{11}(\partial\mathbf{x},\mathbf{n}) & T^{(1)}_{12}(\partial_{\mathbf{x}},\mathbf{n}) & 0 & 0 \\ T^{(1)}_{21}(\partial_{\mathbf{x}},\mathbf{n}) & T^{(1)}_{22}(\partial_{\mathbf{x}},\mathbf{n}) & 0 & 0 \\ 0 & 0 & T^{(2)}_{11}(\partial\mathbf{x},\mathbf{n}) & T^{(2)}_{12}(\partial_{\mathbf{x}},\mathbf{n}) \\ 0 & 0 & T^{(2)}_{11}(\partial\mathbf{x},\mathbf{n}) & T^{(2)}_{12}(\partial_{\mathbf{x}},\mathbf{n}) \end{pmatrix},$$

In the following we assume that

$$\boldsymbol{L}(\boldsymbol{\partial}_{\mathbf{x}},\mathbf{n})\boldsymbol{\Gamma}(\mathbf{x\text{-}y})=\begin{pmatrix} R & 0 \\ 0 & R^{(1)} \end{pmatrix}\boldsymbol{\Gamma}(\mathbf{x\text{-}y})\ =\parallel L_{lj}(\partial x)\parallel_{8\times 8},\ \ l,j=1,2,...,8,$$

where

$$\boldsymbol{\Gamma}(\mathbf{x\text{-}y})=\begin{pmatrix} \boldsymbol{\Gamma}^{(1)}(\mathbf{x\text{-}y}) & \boldsymbol{\Gamma}^{(2)}(\mathbf{x\text{-}y}) \\ \boldsymbol{\Gamma}^{(3)}(\mathbf{x\text{-}y}) & \boldsymbol{\Gamma}^{(4)}(\mathbf{x\text{-}y}) \end{pmatrix}$$

$$\boldsymbol{\Gamma}^{(1)}=\begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ 0 & 0 & \Gamma_{33} & 0 \\ 0 & 0 & 0 & \Gamma_{44} \end{pmatrix}$$

$$\mathbf{\Gamma}^{(2)} = \begin{pmatrix} \Gamma_{15} & \Gamma_{16} & \Gamma_{17} & \Gamma_{18} \\ \Gamma_{25} & \Gamma_{26} & \Gamma_{27} & \Gamma_{28} \\ \Gamma_{35} & \Gamma_{36} & 0 & 0 \\ 0 & 0 & \Gamma_{47} & \Gamma_{48} \end{pmatrix}$$

$$\mathbf{\Gamma}^{(3)} = \begin{pmatrix} 0 & 0 & \Gamma_{53} & 0 \\ 0 & 0 & \Gamma_{63} & 0 \\ 0 & 0 & 0 & \Gamma_{74} \\ 0 & 0 & 0 & \Gamma_{84} \end{pmatrix}$$

$$\mathbf{\Gamma}^{(4)} = \begin{pmatrix} \Gamma_{55} & \Gamma_{56} & 0 & 0 \\ \Gamma_{65} & \Gamma_{66} & 0 & 0 \\ 0 & 0 & \Gamma_{77} & \Gamma_{78} \\ 0 & 0 & \Gamma_{87} & \Gamma_{88} \end{pmatrix}$$

The elements L_{ik} are

$$L_{11} = \frac{\partial\varphi}{\partial\mathbf{n}} - 2\frac{\lambda+\mu}{\mu_0}\frac{\partial}{\partial s}\frac{\partial^2\varphi_0}{\partial x_1 x_2}, \quad L_{12} = \frac{\partial\varphi}{\partial\mathbf{s}} - 2\frac{\lambda+\mu}{\mu_0}\frac{\partial}{\partial s}\frac{\partial^2\varphi_0}{\partial x_2^2},$$

$$L_{21} = -\frac{\partial\varphi}{\partial\mathbf{s}} + 2\frac{\lambda+\mu}{\mu_0}\frac{\partial}{\partial s}\frac{\partial^2\varphi_0}{\partial x_1^2}, \quad L_{22} = \frac{\partial\varphi}{\partial\mathbf{n}} + 2\frac{\lambda+\mu}{\mu_0}\frac{\partial}{\partial s}\frac{\partial^2\varphi_0}{\partial x_1 x_2},$$

$$L_{13} = \frac{2\mu\gamma_1}{k^*\mu_0}\frac{\partial}{\partial\mathbf{s}}\frac{\partial\alpha_{11}}{\partial x_2}, \quad L_{14} = \frac{2\mu\gamma_2}{k^*\mu_0}\frac{\partial}{\partial\mathbf{s}}\frac{\partial\alpha_{14}}{\partial x_2},$$

$$L_{23} = -\frac{2\mu\gamma_1}{k^*\mu_0}\frac{\partial}{\partial\mathbf{s}}\frac{\partial\alpha_{11}}{\partial x_1}, \quad L_{24} = -\frac{2\mu\gamma_2}{k^*\mu_0}\frac{\partial}{\partial\mathbf{s}}\frac{\partial\alpha_{14}}{\partial x_1}, \quad L_{31} = L_{32} = 0,$$

$$L_{33} = k^*\frac{\partial\Gamma_{33}}{\partial\mathbf{n}}, \quad L_{34} = L_{41} = L_{42} = L_{43} = 0 \quad L_{44} = h^*\frac{\partial\Gamma_{44}}{\partial\mathbf{n}},$$

$$L_{15} = -\frac{2\mu\gamma_1 k_1^*}{\mu_0 k^* k_7}\frac{\partial}{\partial s}\frac{\partial^2\alpha_{15}}{\partial x_1 x_2}, \quad L_{16} = -\frac{2\mu\gamma_1 k_1^*}{\mu_0 k^* k_7}\frac{\partial}{\partial s}\frac{\partial^2\alpha_{15}}{\partial x_2^2},$$

$$L_{17} = -\frac{2\mu\gamma_2 h_1}{\mu_0 h^* h_7}\frac{\partial}{\partial s}\frac{\partial^2\alpha_{17}}{\partial x_1 x_2}, \quad L_{18} = -\frac{2\mu\gamma_2 h_1}{\mu_0 h^* h_7}\frac{\partial}{\partial s}\frac{\partial^2\alpha_{17}}{\partial x_2^2},$$

$$L_{25} = \frac{2\mu\gamma_1 k_1^*}{\mu_0 k^* k_7}\frac{\partial}{\partial s}\frac{\partial^2\alpha_{15}}{\partial x_1^2}, \quad L_{26} = \frac{2\mu\gamma_1 k_1^*}{\mu_0 k^* k_7}\frac{\partial}{\partial s}\frac{\partial^2\alpha_{15}}{\partial x_1 x_2},$$

$$L_{27} = \frac{2\mu\gamma_2 h_1}{\mu_0 h^* h_7}\frac{\partial}{\partial s}\frac{\partial^2\alpha_{17}}{\partial x_1^2}, \quad L_{28} = \frac{2\mu\gamma_2 h_1}{\mu_0 h^* h_7}\frac{\partial}{\partial s}\frac{\partial^2\alpha_{17}}{\partial x_1 x_2},$$

$$L_{35} = k^*\frac{\partial\Gamma_{35}}{\partial\mathbf{n}}, \quad L_{36} = k^*\frac{\partial\Gamma_{36}}{\partial\mathbf{n}}, \quad L_{37} = L_{38} = 0$$

$$L_{45} = L_{46} = 0, \quad L_{47} = h^*\frac{\partial\Gamma_{47}}{\partial\mathbf{n}}, \quad L_{48} = h^*\frac{\partial\Gamma_{48}}{\partial\mathbf{n}},$$

$$L_{51} = L_{52} = L_{54} = L_{61} = L_{62} = L_{63} = L_{64} = L_{71} = L_{72} = 0,$$

$$L_{73} = L_{81} = L_{82} = L_{83} = 0,$$

$$L_{53} = \frac{k_3}{k_7 k^*}[k_7 n_1\varphi_2 + (k_5+k_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial}{\partial x_2}\frac{\varphi_2-\varphi}{s_2^2},$$

$$L_{63} = \frac{k_3}{k_7 k^*}[k_7 n_2\varphi_2 - (k_5+k_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial}{\partial x_1}\frac{\varphi_2-\varphi}{s_2^2},$$

$$L_{74} = \frac{h_3}{h_7 h^*}[h_7 n_1\varphi_3 + (h_5+h_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial}{\partial x_2}\frac{\varphi_3-\varphi}{s_3^2},$$

$$L_{84} = \frac{h_3}{h_7 h^*}[h_7 n_2\varphi_3 - (h_5+h_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial}{\partial x_1}\frac{\varphi_3-\varphi}{s_3^2},$$

$$L_{55} = \frac{\partial\varphi_2}{\partial\mathbf{n}} - (k_5+k_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial^2\alpha_{55}}{\partial x_1 x_2} + n_1\frac{\partial(\varphi_1-\varphi_2)}{\partial x_1},$$

$$L_{56} = \frac{k_5}{k_6}\frac{\partial\varphi_2}{\partial\mathbf{s}} - (k_5+k_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial^2\alpha_{55}}{\partial x_2^2} + n_1\frac{\partial(\varphi_1-\varphi_2)}{\partial x_2},$$

$$L_{65} = -\frac{k_5}{k_6}\frac{\partial\varphi_2}{\partial\mathbf{s}} + (k_5+k_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial^2\alpha_{55}}{\partial x_1^2} + n_2\frac{\partial(\varphi_1-\varphi_2)}{\partial x_1},$$

$$L_{66} = \frac{\partial\varphi_2}{\partial\mathbf{n}} + (k_5+k_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial^2\alpha_{55}}{\partial x_1 x_2} + n_2\frac{\partial(\varphi_1-\varphi_2)}{\partial x_2},$$

$$L_{77} = \frac{\partial\varphi_4}{\partial\mathbf{n}} - (h_5+h_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial^2\alpha_{77}}{\partial x_1 x_2} + n_1\frac{\partial(\varphi_3-\varphi_4)}{\partial x_1},$$

$$L_{88} = \frac{\partial\varphi_4}{\partial\mathbf{n}} + (h_5+h_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial^2\alpha_{77}}{\partial x_1 x_2} + n_2\frac{\partial(\varphi_3-\varphi_4)}{\partial x_2},$$

$$L_{78} = \frac{h_5}{h_6}\frac{\partial\varphi_4}{\partial\mathbf{s}} - (h_5+h_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial^2\alpha_{77}}{\partial x_2^2} + n_1\frac{\partial(\varphi_3-\varphi_4)}{\partial x_2},$$

$$L_{87} = -\frac{h_5}{h_6}\frac{\partial\varphi_4}{\partial\mathbf{s}} + (h_5+h_6)\frac{\partial}{\partial\mathbf{s}}\frac{\partial^2\alpha_{77}}{\partial x_1^2} + n_2\frac{\partial(\varphi_3-\varphi_4)}{\partial x_1}.$$

Let $[\mathbf{L}(\partial_\mathbf{y},\mathbf{n})\mathbf{\Gamma}(\mathbf{y}\text{-}\mathbf{x})]^\top$, be the matrix which we get from $[\mathbf{L}(\partial_\mathbf{x},\mathbf{n})\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y})]$ by transposition of the columns and rows and the variables $\mathbf{x}$ and $\mathbf{y}$ (analogously $\left[\tilde{\boldsymbol{L}}(\partial\mathbf{y},\mathbf{n})\tilde{\boldsymbol{\Gamma}}(\mathbf{y}\text{-}\mathbf{x})\right]^{\top}$).

Let us introduce the following single-layer and double-layer potentials :

The vector-functions defined by the equalities

$$\mathbf{V}(\mathbf{x};\mathbf{g}) = \frac{1}{\pi}\int\limits_{S} \boldsymbol{\Gamma}(\mathbf{x}-\mathbf{y})\mathbf{g}(\mathbf{y})d_{\mathbf{y}}S,$$

$$\widetilde{\boldsymbol{V}}(\mathbf{x};\mathbf{g}) = \frac{1}{\pi}\int\limits_{S} \boldsymbol{\Gamma}^{\top}(\mathbf{y}-\mathbf{x})\mathbf{g}(\mathbf{y})d_{\mathbf{y}}S$$

will be called single- layer potentials, while the vector-functions defined by the equalities

$$\mathbf{W}(\mathbf{x};\mathbf{h}) = \frac{1}{\pi}\int\limits_{S} \left[\boldsymbol{L}(\partial_{\mathbf{y}},\mathbf{n})\boldsymbol{\Gamma}(\mathbf{y}-\mathbf{x})\right]^{\top}\mathbf{h}(\mathbf{y})d_{\mathbf{y}}S,$$

$$\widetilde{\boldsymbol{W}}(\mathbf{x};\mathbf{h}) = \frac{1}{\pi}\int\limits_{S} \left[\widetilde{\boldsymbol{L}}(\partial_{\mathbf{y}},\mathbf{n})\boldsymbol{\Gamma}^{\top}(\mathbf{y}-\mathbf{x})\right]^{\top}\mathbf{h}(\mathbf{y})d_{\mathbf{y}}S$$

will be called double layer potentials. Here $\mathbf{g}$ and $\mathbf{h}$ are the continuous (or Hölder continuous) vectors and S is a closed Lyapunov curve.

By applying the methods, as in the classical theory of elasticity, we can state the following:(for details see in [22])

Theorem 3. *The vectors* $\widetilde{\boldsymbol{V}}(\boldsymbol{x};\mathbf{g})$ *and* $\boldsymbol{W}(\boldsymbol{x};\mathbf{h})$ *are the solutions of the system* $\widetilde{\boldsymbol{A}}(\partial_{\boldsymbol{x}})\boldsymbol{U} = \mathbf{0}$ *at any point* $\boldsymbol{x}$ *and* $\boldsymbol{x} \neq \boldsymbol{y}$*.The vectors* $\boldsymbol{V}(\boldsymbol{x};\mathbf{g})$ *and* $\widetilde{\boldsymbol{W}}(\boldsymbol{x};\mathbf{h})$ *are the solutions of the system* $\boldsymbol{A}(\partial_{\boldsymbol{x}})\boldsymbol{U} = \mathbf{0}$ *at any point* $\boldsymbol{x}$ *and* $\boldsymbol{x} \neq \boldsymbol{y}$*. The elements of the matrices* $[\boldsymbol{L}(\partial_{\boldsymbol{y}},\boldsymbol{n})\boldsymbol{\Gamma}(\boldsymbol{y}-\mathbf{x})]^{\top}$ *and* $\left[\widetilde{\boldsymbol{L}}(\partial_{\boldsymbol{y}},\boldsymbol{n})\boldsymbol{\Gamma}^{\top}(\boldsymbol{x}-\mathbf{y})\right]^{\top}$ *contain a singular part, which is integrable in the sense of the Cauchy principal value.*

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