Seminar of I. Vekua Institute of Applied Mathematics
REPORTS, Vol. 43, 2017

SOLUTION OF THE BOUNDARY PROBLEMS OF THERMOPOROELASTOSTATICS

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Abstract. In the present work, using absolutely and uniformly convergent series, the boundary value problems of thermoelastostatics for an elastic circle with double porosity are solved explicitly. The question on the uniqueness of a solution of the problem is investigated.

Keywords and phrases: thermoelasticity, double porosity, explicit solution, elastic circle.

AMS subject classification (2010): 74F10, 74G30, 74G10, 74F05.

Introduction

Recently the linear theory of thermoelasticity for materials with double porosity has been expanding and developing in different directions. For example, the basic equations of the one-and two-temperature thermohydromechanical coupling theories for elastic materials with double porosity were presented in [1-4]. In [5] the linear theory of thermoelasticity for solids with double porosity is considered. The fundamental solutions for the systems of steady vibrations, quasi-static and equalibrium equations are constructed by means of elementary functions. The fundamental solutions in the theory of elasticity for materials with single porosity were constructed in [6,7]. Iesan and Quintanilla [8] presented the theory of thermoelastic materials with double porosity structure.
Along with theoretical investigations of thermoporoelasticity problems, the development of methods for their solution is of great interest. From the point of view of applications, actual construction of solutions of problems in explicit form, which makes it possible to perform a numerical analysis of the problem under study. Various methods for solving boundary-value problems of statics for elastic bodies with double pores, are considered in [9-11].
In the present work, using absolutely and uniformly convergent series, the boundary value problems of thermoelastostatics for an elastic circle with double porosity are solved explicitly. The question on the uniqueness of a solution of the problem is investigated.

Basic equations and boundary value problems

We consider an isotropic elastic material with dual porosity occupying a circle D in a radius R and the boundary S. Equilibrium system of equations of thermoelasticity theory for isotropic materials dual porosity can be written as follows [5]:

$$\mu\Delta\mathbf{u} + (\lambda+\mu) graddiv\mathbf{u} - grad(\beta_1 p_1 + \beta_2 p_2 + \gamma_0 u_3) = 0, \tag{1}$$

$$(k_1\Delta - \gamma)p_1 + \gamma p_2 = 0,$$

$$\gamma p_1 + (k_2\Delta - \gamma)p_2 = 0, \tag{2}$$

$$\Delta u_3 = 0,$$

where $\mathbf{u(x)} = (u_1(x), u_2(x))$ is the displacement vector in a solid; $p_1(x)$ and $p_2(x)$ are the pressures in cracks and in pores, respectively; $u_3(x)$ is the temperature measured from some

constant absolute temperature $T_0(T_0 > 0)$. $U(x) = (u_1, u_2, p_1, p_2, u_3)$ is regular solution systems (1),(2); $\mathbf{U(x)} \in C^2(D) \bigcap C^1(\overline{D})$.
$\lambda, \mu, k_1, k_2, \beta_1, \beta_2, \gamma$ and γ_0 are the well-known thermoelastic and physical coefficients, $\gamma_0 = \alpha(3\lambda + 2\mu)$, α is the coefficient of linear thermal expansion; Δ is the Laplacion operator, $\mathbf{x} \in D$.
We will suppose that the following assumptions on the constitutive coefficients hold true: $\lambda, \quad \mu, \quad k_j, \quad \gamma > 0, \quad j = 1, 2$.

The problems

Find a regular solution $\mathbf{U(x)}$ to system (1) and (2) for $\mathbf{x} \in D$ satisfying the following boundary conditions on the circumference S:

$$\mathbf{u(z)} = \mathbf{f}((\mathbf{z}), \quad p_1(\mathbf{z}) = f_3(\mathbf{z}), \quad p_2(\mathbf{z}) = f_4(\mathbf{z}), \quad u_3(\mathbf{z}) = f_5(\mathbf{z}) \quad - \text{in the BVP I;} \tag{3}$$

$$\mathbf{P}(\partial_{\mathbf{z}}, \mathbf{n})\mathbf{U}(\mathbf{z}) = \mathbf{f}(\mathbf{z}), \quad \partial_{\mathbf{n}} p_1(\mathbf{z}) = f_3(\mathbf{z}), \quad \partial_{\mathbf{n}} p_2(\mathbf{z}) = f_4(\mathbf{z}), \quad \partial_{\mathbf{n}} u_3(\mathbf{z}) = f_5(\mathbf{z}) \quad - \text{in the BVP II,} \tag{4}$$

where $f(\mathbf{z}) = (f_1(\mathbf{z}), f_2(\mathbf{z}))$; $f_l(\mathbf{z})$ are known functions, $l = 1, 2, 3, 4, 5$; $\mathbf{n(z)}$ is the external unit normal vector on S at $\mathbf{z}$; $\mathbf{P}(\partial_x, \mathbf{n})\mathbf{u}$ is the stress vector in the considered theory

$$\mathbf{P}(\partial_x, \mathbf{n})\mathbf{U}(\mathbf{x}) = \mathbf{T}(\partial_x, \mathbf{n})\mathbf{u}(\mathbf{x}) - \mathbf{n}(\mathbf{x})[\beta_1 p_1(\mathbf{x}) + \beta_2 p_2(\mathbf{x}) + \gamma_0 u_3(\mathbf{x})], \tag{5}$$

$\mathbf{T}(\partial_x, \mathbf{n})\mathbf{u}$ is the stress vector in the classical theory of elasticity [9]

$$\mathbf{T}(\partial_x, \mathbf{n})\mathbf{u}(\mathbf{x}) = \mu \partial_n \mathbf{u}(\mathbf{x}) + \lambda \mathbf{n} div \mathbf{u}(\mathbf{x}) + \mu \sum_{i=1}^{2} n_i(\mathbf{x}) grad u_i(\mathbf{x}).$$

Separately we will study the following problems:

1. Find in a circle D solution $\mathbf{u(x)}$ of equation (1), if on the circumference S there are the values: of the vector $\mathbf{u(z)}$ (problem A_1), those of the vector $\mathbf{P}(\partial_z, \mathbf{n})\mathbf{U(z)}$ (problem A_2);
2. Find in the circle D solutions $p_1(x), p_2(x)$ and u_3 of the system of equations (2), if on the circumference S there are the values of the functions p_1, p_2 and u_3 (problem B_1) or those of the derivatives $\partial_n p_1, \partial_n p_2$ and $\partial_n u_3$ (problem B_2).

Thus the above-formulated BVPs of poroelastostatics can be considered as a union of two problems: I– (A_1, B_1), II– (A_2, B_2).

The uniqueness theorems

For the regular solution $U(x) = (u_1, u_2, p_1, p_2, u_3)$ Green's formulas have the form [12,16]:

$$\int_D [E(\mathbf{u}, \mathbf{u}) - (\beta_1 p_1 + \beta_2 p_2 + \gamma_0 u_3) div \mathbf{u}] dx = \int_S \mathbf{u} \mathbf{P}(\partial_y, n) \mathbf{U} d_y S; \tag{6}$$

$$\int_D \left\{ k_1 (grad p_1)^2 + k_2 (grad p_2)^2 + (grad u_3)^2 + \gamma (p_1 - p_2)^2 \right\} dx = \int_S \{ k_1 p_1 \partial_n p_1 + k_2 p_2 \partial_n p_2 + u_3 \partial_n u_3 \} d_y S, \tag{7}$$

where

$$E(\mathbf{u},\mathbf{u}) = (\lambda+\mu)(divu)^2 + \mu(\partial_{x_1}u_1 - \partial_{x_2}u_2)^2 + \mu(\partial_{x_2}u_1 + \partial_{x_1}u_2)^2.$$

For positive definiteness of the potential energy the inequalities $\lambda > 0, \quad \mu > 0$ are necessary and sufficient [13].

Let p_1, p_2 and u_3 be difference of two arbitrary solutions of problem B_1 (or B_2). Then p_1, p_2 and u_3 satisfy the homogeneous boundary conditions, therefore the product $p_j\,\partial_{\mathbf{n}}\,p_j$ and $u_3\,\partial_{\mathbf{n}}\,u_3$ vanishes on S $(j = 1, 2)$. On the basis of Green's formula (7) and by virtue of conditions $k_j > 0$ and $\gamma > 0$, follows that $p_1(\mathbf{x}) = p_2(\mathbf{x}) = c_0 = const$ and $u_3(x) = c_1 = const$, where $\mathbf{x} \in D$. In addition, by homogeneous boundary conditions in the problem B_1 we have $c_0 = c_1 = 0$.

The following theorems are true

Theorem 1. *The problem B_1 admits at most one regular solution.*

Theorem 2. *The difference of two arbitrary solutions of problem B_2 may differ only by an arbitrary constant: $p_1(\mathbf{x}) = p_2(\mathbf{x}) = c_0 = const$, $u_3(\mathbf{x}) = c_1 = const$.*

The homogeneous problems A_1 and A_2 for the difference $u(x)$ have the form

$$\begin{array}{l} \mu\Delta\mathbf{v} + (\lambda+\mu)graddiv\mathbf{v} = 0 \\ \mathbf{u}(\mathbf{z}) = 0 - in \quad problem \quad A_1; \\ \mathbf{T}(\partial_z,\mathbf{n})\mathbf{u} = \mathbf{n}(\mathbf{z})[\beta_1 p_1 + \beta_2 p_2 + \gamma_0 u_3] - in \quad problem \quad A_2. \end{array} \tag{8}$$

For both problems from (6) we obtain

$$E(\mathbf{u},\mathbf{u}) - (\beta_1 p_1 + \beta_2 p_2 + \gamma_0 u_3)div\mathbf{u} = 0. \tag{9}$$

In the case of the problem (A_1, B_1), by theorem 1, we have $p_1(\mathbf{x}) = p_2(\mathbf{x}) = u_3(\mathbf{x}) = 0$. From (9) we obtain $E(\mathbf{u},\mathbf{u}) = 0$. The solution of this equation has the form [13]

$$u_1(\mathbf{x}) = -cx_2 + \alpha_1, \quad u_2(\mathbf{x}) = cx_1 + \alpha_2, \tag{10}$$

where c, α_1, α_2 are arbitrary constants.

For problem (A_2, B_2) from (9) we obtain

$$E(\mathbf{u},\mathbf{u}) - c_2 div\mathbf{u} = 0. \tag{11}$$

where $c_2 = (\beta_1 + \beta_2)c_0 + \gamma_0 c_1$. The solution of problem A_2 with boundary condition $(8)_3$ is

$$u(\mathbf{x}) = ex + q, \tag{12}$$

where $e = \dfrac{c_2}{2(\lambda+\mu)}$; $\quad q = (q_1, q_2), \quad q_1, \quad q_2, \quad c_0$ are arbitrary constant. (12) also satisfies (11).

The following theorems are true:

Theorem 3. *The problem (A_1, B_1) admit at most one regular solution.*

Theorem 4. *The difference of two arbitrary solutions of problem (A_2, B_2) is the vector $\mathbf{U} = (u_1, u_2, p_1, p_2, u_3)$, where u_1 and u_2 are expressed by formulas: (12) and $p_1 = p_2 = c_0, \quad u_3 = c_1$, where c_0, c_1 are arbitrary constants.*

Solution of the Problems B_1 and B_2

From system (2) we can write

$$(\Delta+\lambda_1^2)\Delta p_j=0,$$

whence we obtain the solution of system (2) in the form [14]

$$p_1=\varphi_1+k_2\varphi_2,\quad p_2=\varphi_1-k_1\varphi_2, \tag{13}$$

where

$$\Delta\varphi_1=0,\quad (\Delta+\lambda_1^2)\varphi_2=0,$$

$$\lambda_1=i\sqrt{\frac{\gamma(k_1+k_2)}{k_1k_2}}=i\lambda_0,\quad i=\sqrt{-1},\quad k_1>0,\quad k_2>0,\quad \gamma>0.$$

We have to find functions $\varphi_j(\mathbf{x})$ and $u_3(\mathbf{x})$, $j=1,2$.

Problem B_1

Using (13), from (3) we can write:

$$\varphi_1(\mathbf{z})=d_1(\mathbf{z}),\quad \varphi_2(\mathbf{z})=d_2(\mathbf{z}),\quad u_3(\mathbf{z})=f_5(\mathbf{z}), \tag{14}$$

where

$$\begin{aligned} d_1(\mathbf{z})&=\frac{1}{k_1+k_2}[k_1f_3(\mathbf{z})+k_2f_4(\mathbf{z})],\\ d_2(\mathbf{z})&=\frac{1}{k_1+k_2}[f_3(\mathbf{z})-f_4(\mathbf{z})], \end{aligned} \tag{15}$$

functions f_3, f_4 and f_5 are defined under the conditions (3), $z\in S$.

Suppose that d_1, d_2 and f_5 can be expanded in Fourier series.

φ_1 and u_3 are Harmonic functions in the circle D and they seem in the form of the following series:

$$\varphi_1(\mathbf{x})=\sum_{m=0}^{\infty}\left(\frac{\rho}{R}\right)^m(\mathbf{Y}_m\cdot\boldsymbol{\nu}_m(\psi)),\quad u_3(\mathbf{x})=\sum_{m=0}^{\infty}\left(\frac{\rho}{R}\right)^m(\mathbf{Z}_m\cdot\boldsymbol{\nu}_m(\psi)), \tag{16}$$

where

$$\mathbf{x}=(\rho,\psi),\quad \rho^2=x_1^2+x_2^2,\quad \mathbf{Y}_m=(A_m,B_m),\quad \boldsymbol{\nu}_m=(\cos m\psi,\sin m\psi),$$

$$A_0=\frac{1}{2\pi}\int_0^{2\pi}d_1(\theta)d\theta,\quad A_m=\frac{1}{\pi}\int_0^{2\pi}d_1(\theta)\cos m\theta d\theta,\quad B_m=\frac{1}{\pi}\int_0^{2\pi}d_1(\theta)\sin m\theta d\theta,$$

$$\mathbf{Z}_m=(C_m,D_m),\quad C_0=\frac{1}{2\pi}\int_0^{2\pi}f_5(\theta)d\theta,\quad C_m=\frac{1}{\pi}\int_0^{2\pi}f_5(\theta)\cos m\theta d\theta,$$

$$D_m=\frac{1}{\pi}\int_0^{2\pi}f_5(\theta)\sin m\theta d\theta.$$

The metaharmonic function $\varphi_2(\mathbf{x})$ in the circle D can be represented as follows [15]

$$\varphi_2(\mathbf{x})=I_0(\lambda_0\rho)E_0+\sum_{m=1}^{\infty}I_m(\lambda_0\rho)(\mathbf{E}_m\cdot\boldsymbol{\nu}_m(\psi)), \tag{17}$$

where $I_m(\lambda_0\rho)$ is the Bessel function of an imaginary argument, $\mathbf{E}_m = (L_m, K_m)$; L_0, L_m, K_m are the unknown quantities. Keeping in mind (12) and boundary condition (14) and (15), we obtain the values of L_m and K_m

$$L_0 = \frac{1}{2\pi\lambda_0 I_0(\lambda_0 R)} \int_0^{2\pi} d_2(\theta)d\theta, \qquad L_m = \frac{1}{\pi\lambda_0 I_m(\lambda_0 R)} \int_0^{2\pi} d_2(\theta)\cos(m\theta)d\theta,$$

$$K_m = \frac{1}{\pi\lambda_0 I_m(\lambda_0 R)} \int_0^{2\pi} d_2(\theta)\sin m\theta d\theta,$$

Using formulas (13), (16) and (17), we can find values of the functions $p_1(\mathbf{x}), p_2(\mathbf{x})$ and $u_3(\mathbf{x})$ for $x \in D$.

Problem B_2

Using (13), from (4) we can write:

$$\partial_R\varphi_1(\mathbf{z}) = h_1(\mathbf{z}), \quad \partial_R\varphi_2(\mathbf{z}) = h_2(\mathbf{z}), \quad \partial_R u_3(\mathbf{z}) = f_5(\mathbf{z}), \tag{18}$$

where h_1 and h_2 are defined by formulas (15); functions f_3, f_4 and f_5 are defined under the conditions (4), $z \in S$.

We come to the Neumann problem for the functions φ_1, φ_2 and u_3. Given the properties of harmonic functions φ_1 and u_3, for functions h_1 and f_5 have

$$\int_S h_1(y)d_yS = 0; \qquad \int_S f_5(y)d_yS = 0.$$

Neumann solution for φ_1 and u_3 represented by the series

$$\varphi_1(\mathbf{x}) = c_1 + \sum_{m=1}^{\infty} \frac{R}{m}\left(\frac{\rho}{R}\right)^m (\mathbf{Y1}_m \cdot \boldsymbol{\nu}_m(\psi)), \tag{19}$$

$$u_3(\mathbf{x}) = c_2 + \sum_{m=1}^{\infty} \frac{R}{m}\left(\frac{\rho}{R}\right)^m (\mathbf{Z1}_m \cdot \boldsymbol{\nu}_m(\psi)), \tag{20}$$

where c_1 and c_2 are arbitrary constants; $\mathbf{Y1}_m = (A1_m, B1_m)$ and $\mathbf{Z1}_m = (C1_m, D1_m)$; $A1_m, B1_m$ and $C1_m, D1_m$ are the Fourier coefficients of the functions h_1 and f_5, respectively.

The metaharmonic function $\varphi_2(\mathbf{x})$ in the circle D can be written as (17), where $\mathbf{E}_m = (L_m, K_m)$; L_0, L_m, K_m are the unknown quantities. Keeping in mind boundary conditions (18), we obtain the values of L_0, L_m and K_m

$$L_0 = \frac{1}{2\pi\lambda_0 I_0'(\lambda_0 R)} \int_0^{2\pi} h_2(\theta)d\theta, \qquad L_m = \frac{1}{\pi\lambda_0 I_m'(\lambda_0 R)} \int_0^{2\pi} h_2(\theta)\cos m\theta d\theta, \tag{21}$$

$$K_m = \frac{1}{\pi\lambda_0 I_m'(\lambda_0 R)} \int_0^{2\pi} h_2(\theta)\sin m\theta d\theta,$$

where

$$I'_m(\xi) = \frac{\partial I_m(\xi)}{\partial \xi}, \quad \frac{\partial I_m(\lambda_0 \rho)}{\partial \rho} = \lambda_0 I'_m(\lambda_0 \rho) \quad I'_m(\lambda_0 R) \neq 0, \quad m = 0, 1, 2, ...$$

Using formulas (13), (19),(20),(17) and (21), we can find values of the functions $p_1(\mathbf{x}), p_2(\mathbf{x})$ and $u_3(\mathbf{x})$ for $x \in D$.

Solution of the problem A

Substitute (13) in (1)

$$\mu\Delta\mathbf{u} + (\lambda+\mu) graddiv\mathbf{u} = grad[a\varphi_1 + b\varphi_2 + \gamma_0 u_3], \tag{22}$$

where $a = \beta_1 + \beta_2, \quad b = k_2\beta_1 - k_1\beta_2$.

Then a general solution of last equation is presented in the form

$$\mathbf{u}(\mathbf{x}) = \mathbf{v}(\mathbf{x}) + \mathbf{v}_0(\mathbf{x}), \tag{23}$$

where $\mathbf{v}(\mathbf{x})$ is a general solution of the equation

$$\mu\Delta\mathbf{v} + (\lambda+\mu) graddiv\mathbf{v} = 0 \tag{24}$$

and $\mathbf{v}_0(\mathbf{x})$ is a particular solution of the nonhomogeneous equation (22):

$$\mathbf{v}_0(\mathbf{x}) = \frac{1}{\lambda+2\mu} grad\left[a\varphi_0(\mathbf{x}) - \frac{b}{\lambda_1^2}\varphi_2(\mathbf{x}) + \gamma_0 u_{30}(\mathbf{x})\right], \tag{25}$$

where

$$\Delta\varphi_0(\mathbf{x}) = \varphi_1(\mathbf{x}), \quad \Delta u_{30}(\mathbf{x}) = u_3(\mathbf{x}); \tag{26}$$

$\Delta\Delta\varphi_0 = \Delta\varphi_1 = 0, \quad \Delta\Delta u_{30} = \Delta u_3 = 0.$

Using (16), for the solution of equations (26) in the case of problem B_1 we find in the form

$$\begin{aligned} \varphi_0(\mathbf{x}) &= \frac{R^2}{4}\sum_{m=0}^{\infty}\frac{1}{m+1}\left(\frac{\rho}{R}\right)^{m+2}(\mathbf{Y}_m \cdot \boldsymbol{\nu}_m(\psi)), \\ u_{30}(\mathbf{x}) &= \frac{R^2}{4}\sum_{m=0}^{\infty}\frac{1}{m+1}\left(\frac{\rho}{R}\right)^{m+2}(\mathbf{Z}_m \cdot \boldsymbol{\nu}_m(\psi)), \end{aligned} \tag{27}$$

where the values of $Y_m = (A_m, B_m)$ and $Z_m = (C_m, D_m)$ defined in (16).

Taking into account (19) and (20), in the case of B_2 task we get:

$$\begin{aligned} \varphi_0(\mathbf{x}) &= c_1 + \frac{R^3}{4}\sum_{m=1}^{\infty}\frac{1}{m(m+1)}\left(\frac{\rho}{R}\right)^{m}(\mathbf{Y1}_m \cdot \boldsymbol{\nu}_m(\psi)), \\ u_{30}(\mathbf{x}) &= c_2 + \frac{R^3}{4}\sum_{m=1}^{\infty}\frac{1}{m(m+1)}\left(\frac{\rho}{R}\right)^{m}(\mathbf{Z1}_m \cdot \boldsymbol{\nu}_m(\psi)), \end{aligned} \tag{28}$$

where c_1 and c_2 are arbitrary constants; the values of $\mathbf{Y1}_m = (A1_m, B1_m)$ and $\mathbf{Z1}_m = (C1_m, D1_m)$ defined in (19) and (20).

Now, define the vector $\mathbf{v}(\mathbf{x})$.

A solution $\mathbf{v}(\mathbf{x}) = (v_1, v_2)$ of homogeneous equation (24) is sought in the form

$$\begin{aligned} v_1(\mathbf{x}) &= \frac{\partial}{\partial x_1}(\Phi_1 + \Phi_2) - \frac{\partial \Phi_3}{\partial x_2}, \\ v_2(\mathbf{x}) &= \frac{\partial}{\partial x_2}(\Phi_1 + \Phi_2) + \frac{\partial \Phi_3}{\partial x_1}, \end{aligned} \tag{29}$$

where Φ_1, Φ_2 and Φ_3 are scalar functions, satisfying the following equations

$$\begin{aligned} &\Delta\Phi_1 = 0, \quad \Delta\Delta\Phi_2 = 0, \quad \Delta\Delta\Phi_3 = 0, \\ &(\lambda + 2\mu)\frac{\partial}{\partial x_1}\Delta\Phi_2 - \mu\frac{\partial}{\partial x_2}\Delta\Phi_3 = 0, \\ &(\lambda + 2\mu)\frac{\partial}{\partial x_2}\Delta\Phi_2 + \mu\frac{\partial}{\partial x_1}\Delta\Phi_3 = 0. \end{aligned} \tag{30}$$

In view of (30) we can represent the harmonic function Φ_1, biharmonic functions Φ_2 and Φ_3 in the form

$$\begin{aligned} \Phi_1 &= \sum_{m=0}^{\infty} \left(\frac{\rho}{R}\right)^m (\mathbf{X}_{m1} \cdot \boldsymbol{\nu}_m(\psi)), \\ \Phi_2 &= \sum_{m=0}^{\infty} \left(\frac{\rho}{R}\right)^{m+2} (\mathbf{X}_{m2} \cdot \boldsymbol{\nu}_m(\psi)), \\ \Phi_3 &= \frac{\lambda + 2\mu}{\mu} \sum_{m=0}^{\infty} \left(\frac{\rho}{R}\right)^{m+2} (\mathbf{X}_{m2} \cdot \mathbf{s}_m(\psi)), \end{aligned} \tag{31}$$

where $\mathbf{X}_{mi} = (X_{mi1}, X_{mi2})$, $i = 1, 2$ are the sought two-component vectors, $\boldsymbol{\nu}_k = (\cos m\psi, \sin m\psi)$, $\mathbf{s}_m = (-\sin m\psi, \cos m\psi)$.
First solve the problems A_1 and A_2.

Problem A_1

Taking into account (23) and relying on the condition (3), we can write

$$\mathbf{v}(\mathbf{z}) = \Psi(\mathbf{z}), \tag{32}$$

where $\Psi(\mathbf{z}) = f(\mathbf{z}) - v_0(\mathbf{z})$ is the known vector; v_0 is defined by formula (25), and $\varphi_0, u_{30}, \varphi_1$ and φ_2 by equalities (27), (16), (17), respectively.
We rewrite (32)and (29) in the form

$$v_n(\mathbf{z}) = \boldsymbol{\Psi}(\mathbf{z})_{\mathbf{n}}, \quad \mathbf{v_s}(\mathbf{z}) = \boldsymbol{\Psi}(\mathbf{z})_{\mathbf{s}}, \tag{33}$$

$$\begin{aligned} v_n &= \frac{\partial}{\partial \rho}(\Phi_1 + \Phi_2) - \frac{1}{\rho}\frac{\partial}{\partial \psi}\Phi_3, \\ v_s &= \frac{1}{\rho}\frac{\partial}{\partial \psi}(\Phi_1 + \Phi_2) + \frac{\partial}{\partial \rho}\Phi_3, \end{aligned} \tag{34}$$

where $v_n(\mathbf{z})$ and $v_s(\mathbf{z})$, as well as $\Psi(\mathbf{z})_\mathbf{n}$ and $\Psi(\mathbf{z})_\mathbf{s}$ are the normal and tangent components of the vectors $\boldsymbol{v} = (v_1, v_2)$ and $\Psi(\mathbf{z})$, respectively; $\mathbf{n} = (n_1, n_2)$, $\mathbf{s} = (-n_2, n_1)$, $n_1 = \dfrac{x_1}{\rho}$, $n_2 = \dfrac{x_2}{\rho}$.

Expand the functions $\Psi_n(\mathbf{z})$ and$\Psi_s(\mathbf{z})$ in Fourier series, whose Fourier coefficients are $\alpha_m = (\alpha_{m1}, \alpha_{m2})$ and $\beta_m = (\beta_{m1}, \beta_{m2})$, respectively.
In (34) we substitute (31), the results are substituted in (33), then passing to limit as $\rho \longrightarrow R$, for determining the unknown values we obtain the following system of algebraic equations, whose solution has the form

$$\begin{aligned} &X_{01i} = \frac{\alpha_{0i}R}{4}, \quad X_{02i} = \frac{\beta_{0i}R\mu}{4(\lambda+2\mu)}, \\ &X_{m1i} = \frac{\alpha_{mi}R}{m} - \frac{(\beta_{mi}-\alpha_{mi})R}{2(\lambda+\mu)m}\,[(\lambda+3\mu)m+2\mu], \\ &X_{m2i} = \mu\frac{(\beta_{mi}-\alpha_{mi})R}{2(\lambda+\mu)m}, \quad m = 1,2,\ldots; i = 1,2. \end{aligned} \tag{35}$$

We substitute (35) in (31), and then in (29), we obtain the value of the vector $\mathbf{v}(\mathbf{x})$.

Problem A_2

We seek the solution of equation (22) in the form (23). $v_0(\mathbf{x})$ is determined by the formulas (25) and (28). Now, we seek a solution $v(\mathbf{x})$ of equation (24) with the following boundary condition:

$$T(\partial_\mathbf{z}, \mathbf{n})\mathbf{v}(\mathbf{z}) = \boldsymbol{\Psi}(\mathbf{z}), \quad \mathbf{z} \in \mathbf{S},$$

where, taking into account (4) and (5), we write

$$\Psi(\mathbf{z}) = f(\mathbf{z}) + n(\mathbf{z})[a\varphi_2(\mathbf{z}) + b\varphi_1(\mathbf{z}) + \gamma_0 u_3(x)] - T(\partial_\mathbf{z}, \mathbf{n})v_0(\mathbf{z})$$

is the known vector, $\Psi = (\Psi_1, \Psi_2)$. We rewrite the equality in the form of normal and tangential components. We get:

$$\begin{aligned} &(\lambda+\mu)\left[\frac{\partial v_n(\mathbf{z})}{\partial\rho}\right]_{\rho=R} + \frac{\lambda}{R}\frac{\partial v_s(\mathbf{z})}{\partial\psi} = \Psi_n(\mathbf{z}), \\ &\mu\left[\frac{\partial v_s(\mathbf{z})}{\partial\rho}\right]_{\rho=R} + \frac{\mu}{R}\frac{\partial v_n(\mathbf{z})}{\partial\psi} = \Psi_s(\mathbf{z}), \end{aligned} \tag{36}$$

where

$$\begin{aligned} &\Psi_n(\mathbf{z}) = f_n(\mathbf{z}) + a\varphi_2(\mathbf{z}) + b\varphi_1(\mathbf{z}) + \gamma_0 u_3 - \left[\mathbf{T}\left(\frac{\partial}{\partial\mathbf{z}}, \mathbf{n}\right)\mathbf{v}_0(\mathbf{z})\right]_n, \\ &\Psi_s(\mathbf{z}) = f_s(\mathbf{z}) - \left[\mathbf{T}\left(\frac{\partial}{\partial\mathbf{z}}, \mathbf{n}\right)\mathbf{v}_0(\mathbf{z})\right]_s, \quad \mathbf{z} \in S. \end{aligned}$$

v_n and v_s are defined from (29), $\mathbf{v}_0$ is defined by means of formula (25), where functions $\varphi_0(x)$ and u_{30} are determined by the formulas (28).

Expand the functions $\Psi_n(\mathbf{z})$ and$\Psi_s(\mathbf{z})$ in Fourier series, whose Fourier coefficients are $\gamma_m = (\gamma_{m1}, \gamma_{m2})$ and $\delta_m = (\delta_{m1}, \delta_{m2})$, respectively.

Substituting in (31) the values of v_n and v_s from (29), we obtain a system of algebraic equations. The solution of this system has the form

$$\begin{aligned} X_{01i} &= \frac{\gamma_0 i R^2}{4(\lambda+2\mu)}, \quad X_{02i} = \frac{\delta_0 i R^2}{4(\lambda+2\mu)}, \\ X_{m1i} &= \frac{R^2}{a_3}\delta_m i - \frac{a_4 R^2}{a_2 a_3 - a_1 a_4}(\mu\gamma_m i - a_1\delta_m i), \\ X_{m2i} &= \frac{a_3 R^2}{a_2 a_3 - a_1 a_4}(\mu\gamma_m i - a_1\delta_m i), \end{aligned} \tag{37}$$

where
$a_1 = \mu[2(\lambda+\mu)m^2 - (\lambda+2\mu)m]$, $\quad a_2 = 2(\lambda+\mu)(\lambda+3\mu)m^2 + (\lambda+2\mu)[(3\lambda+5\mu)m + 2\mu]$,
$a_3 = \mu m(2m-1)$, $\quad a_4 = (\lambda+3\mu)m(2m+3) + 2(\lambda+2\mu)$, $\quad m = 1, 2, \ldots$.
We substitute (37) in (31), and then in (29), we obtain the value of the vector $\mathbf{v}(\mathbf{x})$.

Conditions: $\mathbf{f}_j \in C^3(S)$ - in problem A_1 and conditions: $\mathbf{f}_j \in C^2(S)$ in problem A_2, provide absolutely and uniformly convergence of series.

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Received 15.07.2017; revised 14.09.2017; accepted 20.11.2017.

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