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# SOLUTION OF THE SECOND BOUNDARY VALUE PROBLEM OF STATICS OF THE THEORY OF ELASTIC MIXTURE FOR A CIRCULAR RING

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**Abstract**. For the two-dimensional homogeneous equation of statics of the linear theory of elastic mixture in a circular ring we consider the second boundary value problem (when on the boundary are given a stress vectors). The solution is presented in the form of absolutely and uniformly convergent series.

**Keywords and phrases**: Elastic mixture, second boundary value problem, Kolosov- Muskhelishvili type formulas, Laurent series.

**AMS subject classification (2010):** 74E35, 74E20, 74C05.

## 1. Introduction

The basic two-dimensional boundary value problems statics of the linear theory of elastic mixtures are studied in [1], [2], [3],[5] and also by many other authors. The paper deals with the construction of explicit solution to the second boundary value problem of the linear theory of elastic mixture in the case of a circular ring.For the solution of the problem the use will be made of the generalized Kolosov-Muskhelishvili's formula [3] and the method developed in [4] and [5]. The solution is obtained in the form of absolutely and uniformly convergent series.

## 2. Some auxiliary formulas and operators

A homogeneous equation of static of the linear theory of elastic mixtures in a complex form is of type [3]

$$\frac{\partial^2 U}{\partial z \partial \overline{z}} + K \frac{\partial^2 \overline{U}}{\partial \overline{z}^2} = 0, \tag{2.1}$$

where $z = x_1 + ix_2$, $\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)$, $\quad U = \begin{pmatrix} u_1 + iu_2 \\ u_3 + iu_4 \end{pmatrix}$,

$u = (u', u'')^T$, $u' = (u_1, u_2)^\top$ and $u'' = (u_3, u_4)^\top$ are partial displacements,

$$K = -\frac{1}{2} l m^{-1}, \quad l = \begin{bmatrix} l_4 & l_5 \\ l_5 & l_6 \end{bmatrix}, \qquad m^{-1} = \begin{bmatrix} m_1 & m_2 \\ m_2 & m_3 \end{bmatrix}^{-1},$$

$$m_k = l_k + \frac{1}{2} l_{3+k}, \quad k = 1, 2, 3, \quad l_1 = \frac{a_2}{d_2}, \quad l_2 = -\frac{c}{d_2}, \quad l_3 = \frac{a_1}{d_2},$$

$$d_2 = a_1 a_2 - c^2 > 0, \quad a_1 = \mu_1 - \lambda_5, \quad a_2 = \mu_2 - \lambda_5, \quad c = \mu_3 + \lambda_5, \quad l_1 + l_4 = \frac{b}{d_1},$$

$$l_2 + l_5 = -\frac{c_0}{d_1}, \quad l_3 + l_6 = \frac{a}{d_1}, \quad d_1 = ab - c_0^2 > 0, \quad a = a_1 + b_1, \tag{2.2}$$

$$b = a_2 + b_2, \quad c_0 = c + d, \quad b_1 = \mu_1 + \lambda_1 + \lambda_5 - \alpha_2 \frac{\rho_2}{\rho},$$

$$b_2 = \mu_2 + \lambda_2 + \lambda_5 + \alpha_2 \frac{\rho_1}{\rho}, \quad \alpha_2 = \lambda_3 - \lambda_4, \quad \rho = \rho_1 + \rho_2,$$

$$d = \mu_2 - \lambda_3 - \lambda_5 - \alpha_2 \frac{\rho_1}{\rho} \equiv \mu_3 + \lambda_4 - \lambda_5 + \alpha_2 \frac{\rho_2}{\rho}.$$

Here $\mu_1, \mu_2, \mu_3, \quad \lambda_p, \quad p = \overline{1,5}$ are elastic modules characterizing mechanical properties of the mixture, $\rho_1$ and $\rho_2$ are partial densities of the mixture. It will be assumed that the elastic constants $\mu_1, \mu_2, \mu_3, \quad \lambda_p, \quad p = \overline{1,5}$ and partial rigid densities $\rho_1$ and $\rho_2$ satisfy the certain conditions given in [1].

In [3] M. Basheleishvili obtained the following representation (analogous to the Kolosov-Muskhelishvili formulas)

$$U = (u_1 + iu_2, u_3 + iu_4)^\top = m\varphi(z) + \frac{1}{2} l z \overline{\varphi'(z)} + \overline{\psi(z)}, \tag{2.3}$$

$$TU = [(Tu)_2 - i(Tu)_1, (Tu)_4 - i(Tu)_3]^\top$$
$$\frac{\partial}{\partial s(x)} \left[ (A - 2E)\varphi(z) + Bz\overline{\varphi'(z)} + 2\mu\overline{\psi}(z) \right], \tag{2.4}$$

where $\varphi = (\varphi_1, \varphi_2)^\top$ and $\psi = (\psi_1, \psi_2)^\top$ are arbitrary analytic vector functions,

$(Tu)_p, p = \overline{1,4}$ are components of the stress vector [1],

$$A = \begin{bmatrix} A_1, A_2 \\ A_3, A_4 \end{bmatrix} = \mu m, \quad B = \begin{bmatrix} B_1, B_2 \\ B_3, B_4 \end{bmatrix} = \mu l,$$

$$\mu = \begin{bmatrix} \mu_1, \mu_3 \\ \mu_3, \mu_2 \end{bmatrix}, \quad m = \begin{bmatrix} m_1, m_2 \\ m_2, m_3 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \tag{2.5}$$

$$\frac{\partial}{\partial s(x)} = n_1 \frac{\partial}{\partial x_2} - n_2 \frac{\partial}{\partial x_1},$$

$n = (n_1, n_2)$ is the unit normal vector.

We remark also that [2]

$$\Delta_0 = det\,|m| > 0, \quad \Delta_1 = det\,|\mu| > 0, \quad \Delta_2 = det\,|A - 2E| > 0,$$

$$\Delta_3 = det\,|2E - A - B| > 0, \quad \Delta^* = det\,|A - B - 2E| = 0, \tag{2.6}$$

To solve the problem we use also the formulas [2]

$$A_1 - 2 + A_3 = B_1 + B_3, \quad A_2 + A_4 - 2 = B_2 + B_4, \tag{2.7}$$

$$B_1 + H_0 B_3 = K_0(A_1 - 2 + H_o A_3), \quad B_2 + H_0 B_4 = K_0[A_2 + H_o(A_4 - 2)], \tag{2.8}$$

where $H_0$ is root of an equation [2]

$$\frac{B_2 + XB_4}{B_1 + XB_3} = \frac{A_2 + X(A_4 - 2)}{A_1 - 2 + XA_3},$$

$$X_1 = 1, \quad X_2 = H_0 = -\frac{A_2B_1 + B_2(2 - A_1)}{B_3(2 - A_4) + A_3B_4} \neq 1, \tag{2.9}$$

$$K_0 = \frac{\Delta_1(b_1b_2 - d^2)}{\Delta_2 d_1 d_2} = \frac{1}{\Delta_2} \det |B|, \quad |K_0| < 1, \quad K_0 \neq 0. \tag{2.10}$$

Here $b_1, b_2, d_1, d_2, d, \Delta_1$ and $\Delta_2$ are given by the (2.2) and (2.6).

**3. Statement of the second boundary value problem and scheme of its solution**

Let us assume that an elastic mixture occupies the circular ring $G = \{r < |\sigma| < 1\}$ bounded by the circumferences $\Gamma_0 = |\sigma| = 1$ and $\Gamma_1 = |\sigma| = r$, $\Gamma = \Gamma_0 \cup \Gamma_1$.

We consider the following problem. Find in the domain $G$,a vector

$U = (u_1 + iu_2, u_3 + iu_4)^\top$ which belongs to the class $C^2(G) \cap C^{1,\alpha}(G \cup \Gamma)$ is a solution of equation (2.1) and satisfies the following boundary conditions (see (2.4))

$$[TU(t)]_{\Gamma_0} = i\sigma \left[(A - 2E)\phi(\sigma) + B\overline{\phi(\sigma)} - B\sigma\overline{\phi'(\sigma)} - 2\mu\overline{\sigma}^2\overline{\Psi(\sigma)}\right]$$

$$= f^{(0)}(\theta_0), \quad t = \sigma = e^{i\theta_0}, \quad 0 \leq \theta_0 \leq 2\pi, \tag{3.1}$$

$$[TU(t)]_{\Gamma_1} = i\sigma \left[(A - 2E)\phi(r\sigma) + B\overline{\phi(r\sigma)} - Br\sigma\overline{\phi'(r\sigma)} - 2\mu\overline{\sigma}^2\overline{\Psi(r\sigma)}\right]$$

$$= f^{(1)}(\theta_0), \quad t = r\sigma = re^{i\theta_0}, \quad 0 \leq \theta_0 \leq 2\pi,$$

where $f^{(j)} = (f_1^{(j)}, f_2^{(j)})^\top$ $(j = 0, 1)$ are given complex vector-functions on the $\Gamma_j$, $(j = 0, 1)$, satisfying certain conditions,

$$\phi(t) = \varphi'(t), \quad \Psi(t) = \psi'(t). \tag{3.2}$$

The following assertion is true [6].

**Theorem 3.1** *The general solution of the second homogeneous boundary value problem in $G$ is represented by the formula* $U = a^0 + i\varepsilon^0 \begin{pmatrix} 1 \\ 1 \end{pmatrix} z$ , *where* $z = x_1 + ix_2$, $a^0 = (a_1^0, a_2^0)$*is an arbitrary complex constant vector, and* $\varepsilon^0$ *is an arbitrary real constant.*

Let us change the (3.1) boundary conditions by the equivalent conditions:

$$(A - 2E)\overline{\phi(\sigma)} + B\left[\phi(\sigma) - \sigma\phi'(\sigma)\right] - 2\mu\sigma^2\Psi(\sigma) = \overline{f(\theta_0)}, \tag{3.3}$$

$$(A - 2E)\overline{\phi(r\sigma)} + B\left[\phi(r\sigma) - r\sigma\phi'(r\sigma)\right] - 2\mu\sigma^2\Psi(r\sigma) = \overline{-F(\theta_0)}, \tag{3.4}$$

where

$$f(\theta_0) = ie^{i\theta_0}f^0(\theta_0), \quad \textit{and} \quad F(\theta_0) = -ie^{i\theta_0}f^{(1)}_{(\theta_0)}, \quad 0 \leq \theta_0 \leq 2\pi.$$

In the sequel we will assume that

$$\oint f(\theta_0)d\sigma + r\oint F(\theta_0)d\sigma = 0, \tag{3.5}$$

$$\oint \begin{pmatrix} 1 \\ 1 \end{pmatrix} \left[ f(\theta_0) - \overline{f(\theta_0)} \right] \frac{d\sigma}{\sigma} + r^2 \oint \begin{pmatrix} 1 \\ 1 \end{pmatrix} \left[ F(\theta_0) - \overline{F(\theta_0)} \right] \frac{d\sigma}{\sigma} = 0. \tag{3.6}$$

Note that (see [3.P.439] and [4]) conditions (3.5) and (3.6) express that the principal vector and the principal moment of external forces are equal to zero.

Let us assume that vector-functions $f(\theta_0)$ and $F(\theta_0)$satisfy the sufficient smoothness conditions that allow us to represent them into a Laurent series

$$f(\theta_0) = f_0 + \sum_{k=1}^{\infty} f_k \sigma^k + \sum_{k=1}^{\infty} f_{-k} \sigma^{-k}, \tag{3.7}$$

$$F(\theta_0) = F_0 + \sum_{k=1}^{\infty} F_k \sigma^k + \sum_{k=1}^{\infty} F_{-k} \sigma^{-k}, \tag{3.8}$$

where $\sigma = e^{i\theta_0}$, $f_q = (f_{q_1}, f_{q_2})^T$ and $F_q = (F_{q_1}, F_{q_2})^T$, $q = 0, \pm 1, \pm 2, \pm ...$ are of the Laurent coefficients.

Below we assume that

$$f_{-1} + rF_{-1} = 0, \tag{3.9}$$

$$f_0 + r^2 F_0 = \overline{f_0} + r^2 \overline{F_0}. \tag{3.10}$$

Obviously the conditions (3.5) and (3.6) are fulfilled if the coefficients $f_{-1}$, $F_{-1}$, $f_0$ and $F_0$ (see (3.7) and (3.8)) satisfy the conditions (3.9) and (3.10).

Let us now consider the vector-function

$$2\mu \Psi(t) = t^{-2} \left\{ (A - 2E) \overline{\Phi(\frac{1}{\overline{t}})} + B[\phi(t) - t\phi'(t)] - L(t) \right\}, \quad r \leq |t| \leq 1, \tag{3.11}$$

where $L(t)$ is a Laurent series

$$L(t) = \overline{f_0} + \sum_{k=1}^{\infty} \overline{f_k} t^{-k} + \sum_{k=1}^{\infty} \overline{f_{-k}} t^k. \tag{3.12}$$

It is obvious that (3.11) vector-function satisfies condition (3.3).

Keeping in mind (3.11) in (3.4) we obtain

$$(1 - r^2) B \left[ \phi(r\sigma) - r\sigma\phi'(r\sigma) \right] - (A - 2E) \left[ r^2 \overline{\phi(r\sigma)} - \overline{\phi(\frac{\sigma}{r})} \right] = r^2 \overline{F(\theta_0)}$$

$$+L(r\sigma) = r^2 \overline{F_0} + \overline{f_0} + \sum_{k=1}^{\infty} \left( r^2 \overline{F_k} + r^{-k} \overline{f_k} \right) \sigma^{-k} + \sum_{k=1}^{\infty} \left( r^2 \overline{F_{-k}} + r^k \overline{f_{-k}} \right) \sigma^k. \tag{3.13}$$

Let us consider the boundary condition conjugate to the condition (3.13)

$$(1 - r^2) B \left[ \overline{\phi(r\sigma)} - \frac{r}{\sigma} \overline{\phi'(r\sigma)} \right] - (A - 2E) \left[ r^2 \phi(r\sigma) - \phi(\frac{\sigma}{r}) \right] = r^2 F_0 + f_0$$

$$+\sum_{k=1}^{\infty}\left(r^2F_k+r^{-k}f_k\right)\sigma^k+\sum_{k=1}^{\infty}\left(r^2F_{-k}+r^kf_{-k}\right)\sigma^{-k}. \tag{3.14}$$

We look for the analytic vector-function $\phi(t)$ in the following form

$$\phi(t)=C_0+\sum_{k=1}^{\infty}C_kt^k+C_{-1}t^{-1}+\sum_{k=2}^{\infty}C_{-k}t^{-k},\quad r\le|t|\le 1, \tag{3.15}$$

where $C_k=(C_{k_1},C_{k_2})^T$, $k=0,\pm1,\pm2,\pm...$ are unknown constant vectors.

By substituting (3.15) into (3.11) we get

$$2\mu\Psi(t)=t^{-2}\ \{(A-2E)\overline{C_0}+B(C_0+2C_{-1}t^{-1})$$

$$(A-2E)\left[\sum_{k=1}^{\infty}\overline{C_k}t^{-k}+\overline{C_{-1}}t+\ sum_{k=2}^{\infty}\overline{C_{-k}}t^k\right]$$

$$B\left[\sum_{k=2}^{\infty}(1-k)C_kt^k+\sum_{k=2}^{\infty}(1+k)C_{-k}t^{-k}\right]-L(t)\}. \tag{3.16}$$

Keeping in mind (3.15) and (3.16) in (3.2) by integration we obtain

$$\varphi(t)=(C_{-1}lnt+...),\quad 2\mu\psi(t)=\left[(A-2E)\overline{C_{-1}}-\overline{f_{-1}}\right]lnt+.... \tag{3.17}$$

Now note that since displacement $u=(u',u'')^T=(u_1,u_2,u_3,u_4)^T$ is one-valued in the circular ring G, therefore owing to formulas (3.17) we can conclude that $2\mu U=(u_1+iu_2,u_3+iu_4)$ (see (2.3), (2.5) and (3.2)) represent one-valued vector in G when

$$AC_{-1}-[(A-2E)C_{-1}-f_{-1}]=0,\qquad C_{-1}=\frac{1}{2}f_{-1}. \tag{3.18}$$

Taking into account (3.15) and (3.16) in (3.13) and (3.14) after some calculations, for the determination of the coefficients $C_k(\overline{C_k})$ and $C_{-k}(\overline{C_{-k}})$ we obtain the following system of equations:

$$(1-r^2)\left[BC_0+(A-2E)\overline{C_0}\right]=r^2\overline{F_0}+\overline{f_0}, \tag{3.19}$$

$$(1-r^2)(1-k)r^kBC_k+(r^k-r^{2-k})(A-2E)\overline{C_{-k}}=r^2\overline{F_{-k}}+r^k\overline{f_{-k}},$$

$$(r^{-k}-r^{2+k})(A-2E)C_k+(1-r^2)(1+k)r^{-k}B\overline{C_{-k}}=r^2F_k+r^{-k}f_k. \tag{3.20}$$

Owing to (3.10) and (2.6) from (3.19) by exactness $ImC_0=0$ we obtain

$$C_0=(A+B-2E)^{-1}\frac{r^2\overline{F_0}+\overline{f_0}}{1-r^2}=(A+B-2E)^{-1}\frac{r^2F_0+f_0}{1-r^2}. \tag{3.21}$$

From (3.20) by virtue of (3.9) when $k=1$, $(k=-1)$ we have

$$(1-r^4)(A-2E)C_1+2(1-r^2)B\overline{C_{-1}}=r^3F_1+f_1,$$

whence owing to (2.6) and (3.18) we get

$$C_1=(A-2E)\frac{r^3F_1+f_1}{1-r^4}+(A-2E)^{-1}B\frac{\overline{f_{-1}}}{1+r^2}. \tag{3.22}$$

Bearing in mind the formulas (2.7), (2,8), (2,9) and (2.10), after some calculations, we can rewrite (3.20) in the form of two systems of equations:

$$(1-k)(1-r^2)r^k\begin{pmatrix} B_1+B_3\\ B_2+B_4\end{pmatrix}C_k+(r^k+r^{2-k})\begin{pmatrix} B_1+B_3\\ B_2+B_4\end{pmatrix}\overline{C_{-k}}$$

$$=\begin{pmatrix}1\\1\end{pmatrix}(r^2\overline{F_{-k}}+r^k\overline{f_{-k}})=\nu_k^{(1)}, \tag{3.23}$$

$$(r^{-k}-r^{2+k})\begin{pmatrix} B_1+B_3\\ B_2+B_4\end{pmatrix}C_k+(1+k)(1-r^2)r^{-k}\begin{pmatrix} B_1+B_3\\ B_2+B_4\end{pmatrix}\overline{C_{-k}}$$

$$=\begin{pmatrix}1\\1\end{pmatrix}(r^2F_k+r^{-k}f_k)=\nu_k^{(2)};$$

$$(1-k)(1-r^2)r^k\begin{pmatrix} B_1+H_0B_3\\ B_2+H_0B_4\end{pmatrix}C_k+K_0^{-1}(r^k-r^{2-k})\begin{pmatrix} B_1+H_0B_3\\ B_2+H_0B_4\end{pmatrix}\overline{C_{-k}}$$

$$=\begin{pmatrix}1\\H_0\end{pmatrix}(r^2\overline{F_{-k}}+r^k\overline{f_{-k}})=\delta_k^{(1)}, \tag{3.24}$$

$$K_0^{-1}(r^{-k}-r^{2+k})\begin{pmatrix} B_1+H_0B_3\\ B_2+H_0B_4\end{pmatrix}C_k+(1+k)(1-r^2)r^{-k}\begin{pmatrix} B_1+H_0B_3\\ B_2+H_0B_4\end{pmatrix}\overline{C_{-k}}$$

$$=\begin{pmatrix}1\\H_0\end{pmatrix}(r^2F_k+r^{-k}f_k)=\delta_k^{(2)}.$$

Denoting the determinant of the systems (3.23) and (3.24) by $D_k$ and $D_k^*$ respectively. Simple calculations yield

$$D_k=r^4\left[(1-r^{-2})^2(1-k^2)+r^{2k-2}+r^{-2k-2}-r^{-4}-1\right]$$

$$=\xi^{-2}\left[(1-k^2)(1-\xi)^2+\xi^{k+1}+\xi^{1-k}-\xi^2-1\right], \tag{3.25}$$

$$D_k^*=K_0^{-2}\left[D_k+(k^2-1)(1-K_0^2)(1-r^2)^2\right],$$

where

$0<r<1, \quad \xi=r^{-2}, \quad |k|\geq 2; \quad |K_0|<1. \quad K_0\neq 0$, (see (2.10)) (3.26)

Now note that since [see [4.p.210.scholie 2] $(1-k^2)(1-\xi)^2+\xi^{k+1}-\xi^{1-k}-\xi^2-1>0$, when $\xi>1$ and $|k|\geq 2$ therefore from (3.25) owing to (3.26) we obtain $D_k>0$ and $D_k^*>0$

Thus, it follows from (3.23) and (3.24) that

$$(B_1+B_3)C_{k1}+(B_2+B_4)C_{k2}=\gamma_K,$$

$$(B_1+H_0B_3)C_{k1}+(B_2+H_0B_4)C_{k2}=\beta_k, \tag{3.27}$$

$$(B_1+B_3)\overline{C_{-k1}}+(B_2+B_4)\overline{C_{-k2}}=P_k,$$

$$(B_1 + H_0 B_3)\overline{C_{-k1}} + (B_2 + H_0 B_4)\overline{C_{-k2}} = q_k, \tag{3.28}$$

Here $C_k = (C_{k1}, C_{k2})^T$, $\overline{C_{-k}} = (\overline{C_{-k1}}, \overline{C_{-k2}})^T$

$$\gamma_k = \frac{1}{D_k}\left[\nu_k^{(1)}(1-r^2)(1+k)r^{-k} - \nu_k^{(2)}(r^k - r^{2-k})\right],$$

$$\beta_k = \frac{1}{K_0 D_k^*}\left[K_0\delta_k^{(1)}(1-r^2)(1+k)r^{-k} - \delta_k^{(2)}(r^k - r^{2-k})\right],$$

$$P_k = \frac{1}{D_k}\left[\nu_k^{(2)}(1-r^2)(1-k)r^{k} - \nu_k^{(1)}(r^{-k} - r^{2+k})\right],$$

$$q_k = \frac{1}{K_0 D_k^*}\left[K_0\delta_k^{(2)}(1-r^2)(1-k)r^{k} - \delta_k^{(1)}(r^{-k} - r^{2+k})\right] \tag{3.29}$$

Now note that the determinant of the system (3.27) [(3.28)] is equal to $K_0\Delta_2(H_0 - 1)$ and different from zero (see (2.6), (2.9) and (2.10)).

Finally note that by easy calculations it follows from (3.27) and (3.28) that

$$C_k = \frac{1}{K_0\Delta_2(H_0-1)}\begin{pmatrix} \gamma_k(B_2 + H_0B_4) - \beta_k(B_2 + B_4) \\ \beta_k(B_1 + B_3) - \gamma_k(B_1 + H_0B_3) \end{pmatrix}, \tag{3.30}$$

$$C_{-k} = \frac{1}{K_0\Delta_2(H_0-1)}\begin{pmatrix} P_k(B_2 + H_0B_4) - \beta_k(B_2 + B_4) \\ q_k(B_1 + B_3) - P_k(B_1 + H_0B_3) \end{pmatrix}, \tag{3.31}$$

where $\gamma_k, \beta_k, P_k$ and $q_k$ are defined by (3.29)

Substituting in formulas (3.15) and (3.16) the values $C_k, \overline{C_k}, C_{-k}$a and $\overline{C_{-k}}$ $(k = 0, 1, 2, 3, ...)$ appearing in (3.18), (3.21), (3.22), (3.30) and (3.31) we find the vector-functions $\phi$ and $\Psi$ in the form of series. Having found $\phi$ and $\Psi$ using formulas (3.2) we can find $\varphi$ and $\psi$ vector-functions and after by (2.3) formula we obtain the expression for the displacement vector in the form of a series.

Finally note that the series will be absolutely and uniformly convergent in the domain $G$ if $f''(\theta_0)$ and $(F''(\theta_0)\ 0 \leq \theta_0 \leq 2\pi)$ belong to the Dirichlet class (see [4]).

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