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TO PROBLEM OF HYDRODYNAMICS OF COLLECTOR HEAT EXCHANGERS

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Abstract. The devices equipped with collectors for distribution (collection) of flow through a permeable (slotted, perforated packed) surface belong to heat exchangers or switch- gears of the collector type. Typical feature of this class of heat exchangers is that in the course of coolant's motion due to outflow or inflow of the mass, the mass consumption of the main flow changes.

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Statement of the problem and its solution

Collector systems are widely used in power engineering, chemical and metallurgical productions, agricultural technology, drilling, cooling of parallel fuel elements in reactors, ecology, purification systems in pipeline transportation of oil or gas, etc. [2].

The main problems arising in the development of collector systems is associated with the need to ensure the distribution of the flow of medium between parallel canals according to a certain law with a minimum pressure drop (most often uniformly). Mathematically, these problems are reduced to the study of patterns of flow motion with mass transfer (in canals with permeable walls), for which application of ordinary equations (for example of Navier-Stocks or Euler) of motion of fluid in pipelines (canals) is impossible [1].

The mass transfer processes in collector heat exchange devices and systems are very complex and they have not been solved yet by the exact methods of hydrodynamics. Therefore, in this paper we consider the solution of this (very important for practice) problem using general equations of motion of the flow with external mass transfer. The solution is given in a quasi one-dimensional formulation[3].

For deriving dynamic equations of one-dimensional flow of medium with continuous change in mass, we accept that the direction of fluid motion coincides with hydrodynamic axis (of a pipe, canal), and attachment or detachment of the mass takes place at some angel. Equation of dynamics of perfect fluid with external mass transfer [4] is the starting point for further calculations:

$$\frac{d\vec{v}}{dt} = \vec{F} - \frac{1}{\rho} gradP + (\vec{u} - \vec{v})\, q, \quad q = \frac{d}{dt}(\ln m) \tag{1}$$

where $\vec{v}, \vec{F}$ is a vector of velocity and mass forces; P, m, ρ is pressure, mass and density of fluid; $\vec{u}$ is a velocity vector of attached (or detached) mass; q is specific attached (or detached) mass of fluid (provided attachment of the mass $q > 0$, provided detached $q < 0$). Projecting the vector equation $\omega = B\cos(Z/2)\cos\omega t$, (where B is amplitude of velocity pulsations, $Z = 4\pi z/\lambda$, z is a coordinate; λ and ω are wave length and frequency of forced pulsations, respectively) on the corresponding coordinate axes, we write

$$\frac{d\upsilon_x}{dt} = F_x - \frac{1}{\rho}\frac{\partial P}{\partial x} + (u_x - \upsilon_x)q,$$

$$\frac{d\upsilon_y}{dt} = F_y - \frac{1}{\rho}\frac{\partial P}{\partial y} + (u_y - \upsilon_y)q,$$

$$\frac{d\upsilon_z}{dt} = F_z - \frac{1}{\rho}\frac{\partial P}{\partial z} + (u_z - \upsilon_z)q. \tag{2}$$

We multiply the first equation of system (2) by dx, the second one by dy and the third one by dz, put them together and get:

$$(\upsilon_x d\upsilon_x + \upsilon_y d\upsilon_y + \upsilon_z d\upsilon_z) = (F_x dx + F_y dy + F_z dz)$$
$$-\frac{1}{\rho}\left(\frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz\right)$$
$$+\left[(u_x - \upsilon_x)\upsilon_x + (u_y - \upsilon_y)\upsilon_y + (u_z - \upsilon_z)\upsilon_z\right]\frac{dm}{m} \tag{3}$$

where $\upsilon_x = dx/dt, \upsilon_y = dy/dt, \upsilon_z = dz/dt$.

Taking into account $\upsilon_x d\upsilon_x + \upsilon_y d\upsilon_y + \upsilon_z d\upsilon_z = d\left(\upsilon^2/2\right)$, $\upsilon^2 = \upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2$ and $\frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz = dP$, as $P = P(x, y, z)$, we represent expression (3) in the form

$$d\left(\frac{\upsilon^2}{2}\right) = (F_x dx + F_y dy + F_z dz) - \frac{dP}{\rho}$$
$$+\left[(u_x\upsilon_x + u_y\upsilon_y + u_z\upsilon_z) - \upsilon^2\right]\frac{dm}{m}. \tag{4}$$

Denote the average speed of the jet of the basic fluid flow by υ, the average jet of the attached (or detached) mass by u. Then we can write $\upsilon_x = \upsilon\cos(\upsilon, x)$, $\upsilon_y = \upsilon\cos(\upsilon, y)$, $\upsilon_z = \upsilon\cos(\upsilon, z)$ and $u_x = u\cos(u, x)$, $u_y = u\cos(u, y)$, $u_z = u\cos(u, z)$. Substituting them in equation (4) and taking into account $[\cos(\upsilon, x)\cos(u, x) + \cos(\upsilon, y)\cos(u, y + \cos(\upsilon, z)\cos(u, z)] = \cos(\upsilon, u)$, we get

$$d\left(\frac{\upsilon^2}{2}\right) = F_x dx + F_y dy + F_z dz - \frac{dP}{\rho} + (u_* - \upsilon)\,\upsilon\frac{dm}{m}, \tag{5}$$

where $u_* = u\cos(\upsilon, u)$ is the projection of the average speed of the jet of the attached (or detached) mass on the direction of the average speed of the jet of the basic fluid flow. Take into account that the mass forces acting on the fluid have a potential (we can make this assumption because the mass force is mostly the gravity, and this as known has a potential), i.e. there exists the function $\Phi(x, y, z)$ that satisfies the condition $F_x = \partial\Phi/\partial x$, $F_y = \partial\Phi/\partial y$, $F_z = \partial\Phi/\partial z$. Consequently, $F_x dx + F_y dy + F_z dz = d\Phi = -gdz$, (since $F_x = 0, F_y = 0$, $F_z = -g$ is acceleration of gravity). Then after the transformation equation (5) takes the form

$$d\left(\frac{\upsilon^2}{2g} + \frac{P}{\gamma} + z\right) = (k-1)\frac{\upsilon^2 dm}{gm}, \tag{6}$$

where $k = u_*/\upsilon$; $\gamma = \rho g$ is specific gravity of fluid.

This is just the equation of one-dimensional flow of jet of perfect fluid with a change in mass. When there is no attached (or detached) mass (i.e. if we ignore the term $(k-1)\,\upsilon^2 dm/mg$ taking into account mass variability), from this equation as a particular case, we get the known Bernoulli-Euler equation $d\left(\upsilon^2/2g + P/\gamma + z\right) = 0$, that is the basic equation of hydrodynamics of one-dimensional flow of medium (with a constant mass of fluid) [5].

From equation (6) it follows that this equation allows to solve a number of applied problems of hydrodynamics of one-dimensional flow of medium with continuous change in mass, covering the cases when resistance (friction) forces are insignificant. When the fluid flows in real conditions (at the expense of viscosity and turbulence) there arise friction forces that lead to loss of energy (of hydrodynamic head) along the length of the fluid flow. Moreover, into the equation of hydrodynamics of one-dimensional flow of fluid we must introduce the correction dh_f, that takes into account the frictional pressure loss. Then we can represent equation (6) in the form

$$d\left(\frac{\upsilon^2}{2g}+\frac{P}{\gamma}+z+h_f\right)=(k-1)\,\frac{\upsilon^2 dm}{gm}. \tag{7}$$

When establishing (6) or(7) it was accepted that velocity of individual fluid jets are the same. Passing to the whole flow of real fluid of finite sizes, it is necessary to take into account the nonuniformity of speed distribution over the section. Moreover, instead of real field of velocities in cross section of the flow some average velocity dependent on the accepted method of averaging, is considered [5].

Let us consider the motion of the whole flow of finite sizes as totality of elementary jets moving with different velocities. Denote the second mass flow of jets by $m=\rho\upsilon d\omega$ (where $d\omega$ is the area of section of the jet). Then after appropriate transformation and integration over the cross sectional area of the flow ω, equation (7) will take the form

$$d\left(\int_\omega \upsilon^2 d\omega\right)=d\int_\omega\left(\frac{P}{\gamma}+gz+gh_f\right)d\omega-u_* d\left(\int_\omega \upsilon d\omega\right)=0 \tag{8}$$

or

$$-dy=\frac{\alpha_0}{g}d\left(\frac{\bar\upsilon^2}{2}\right)+dh_f+\frac{(\alpha_0\bar\upsilon-\bar\upsilon_*)}{gQ}dQ, \tag{9}$$

where $y=z+P/\gamma; \alpha_0=\left(\int_\omega \upsilon^2 d\omega\right)/\bar\upsilon Q$ is the coefficient taking into account nonuniformity of velocity distribution in the cross section of the flow ($\alpha_0>1$); Q is the volumetric flow of liquid ; $\bar\upsilon$ is the average velocity of the basic flow of liquid; ω is the cross sectional area of the flow; $\bar\upsilon_*$ is the projection of the average speed of the attachable mass (consumption) on the direction of the velocity $\bar\upsilon$.

To equation (9) it is necessary to add a flow equation (continuity equation) in the following form [3]

$$dQ=\pm q_0 dx \tag{10}$$

where q_0 is the intensity of change of the volumetric flow of the liquid (sign “$+$“ corresponds to the attachment, and the sign “ $-$ “ to detachment). Thus, equations (9) and (10) represent mathematical formulation of one-dimensional model of flow of medium with continuous change in the mass (flow) along the length of the flow. By means of these equations the problems related to both pressure flows and non-pressure (with free surface) flows.

Let us consider one-dimensional flow of fluid in pressure collector systems with permeable (porous of perforated) walls. The typical feature of such systems is that in these systems along the path of fluid motion as a consequence of inflow (blowing) or outflow (suction), there happens change of the flow velocity of the main flow. Furthermore, attachment or detachment of mass leads to change of longitudinal and transversal velocity of flow in the collector and also to change of hydrodynamical resistances, degree of turbulence and thickness of the boundary layer [3]. As a consequence of change in the flow and frictional losses, there happens pressure change along the length of the collector with permeable walls, and inertial effects are most often decisive.

Under the attachment (blowing) of flow pressure drops along the length of the canal ($dP/dx < 0$). The most complicated character of pressure change holds in the canal with distribution (suction) of consumption. In them, because of oppositely acting effects (pressure drop because of hydrodynamic resistance and pressure recovery as a result of distribution of consumption) the pressure in the parth of motion may both drop and increase. And with increasing intensity of distribution, inertial effects become predominant and on the length of the canal, pressure recovery takes place ($dP/dx > 0$). Thus, consideration of peculiarities of hydrodynamics of canals with permeable walls shows very complicated character of the fluid motion.

Below we give the solution to the problems of hydrodynamics of collector systems of constant section and with permeable walls. We will consider that along the length of the canal, distribution or attachment of fluid's flow happens either through perforation or through slit arranged in its lateral surface. For solving the stated problem, we use equation (9) as the initial one, that allowing for $Q = \upsilon\omega \quad \left(\omega = \pi D^2/4,\ D = const\right)$ and Darcy- Weisbach formula, for determining pressure (energy) loss on the area of the given dx

$$dh_f = \frac{\lambda \upsilon^2}{2gD} dx = \frac{8\lambda Q^2}{g\pi^2 D^5} dx, \tag{11}$$

is representable in the form [3]

$$dy = (2\alpha_0 - k) \frac{16Q}{\pi^2 g D^4} dQ + \frac{8\lambda Q^2}{\pi^2 g D^5} dx, \tag{12}$$

where D is the diameter of the canal (pipe); Q is the flow rate of the medium; λ is the friction resistance coefficient.

Equation (12) is a mathematical formulation of the quasi-one dimensional model of the pressure flow motion of the medium with continuous change in consumption by means of which one can obtain corresponding calculation formulas for determining pressure along the length of the collector (canal, pipe) of a constant section and with permeable walls. This equation does not impose restrictions of the flow regime. Here the difference is made by the choice of the friction resistance coefficient λ. In the canals with permeable walls it can be represented by the sum of two members $\lambda = \lambda_{.T} + \lambda_n$ one of which λ_T depends on the character of the regime and friction resistance domain [6]

$$\lambda_T = A/Re^m \tag{13}$$

the another, λ_n on permeability degree (porosity, perforation) φ [1]

$$\lambda_n = a\varphi^b. \tag{14}$$

Therefore, for determining the flow coefficient λ we can use the expression [3]

$$\lambda = A/Re^m + a\varphi^b \tag{15}$$

where $a = 0,0106$; $b = 0,413$; $\varphi = nd_0^2/4Dl$; d_0, n is the diameter and the number of holes; l is the length of the perforated part of the pipe; $Re = \upsilon D/\nu = 4\ Q/\pi D\nu$ is the Reynolds number; ν is the kinematic viscosity coefficient. The coefficient A and the power index m

depending on the character of the regime (and resistance zone) have different values (see Table 1).

Dependence of λ for different values of Re and Δ/D

No	Flow regime and resistance zone	Boundary of the zone		
1	Turbulent, smooth-walled	$(3-4)\cdot 10^3 \leq \mathrm{Re} \leq 10\mathrm{D}/\Delta$	0,3164	0,25
2	Turbulent, pre-quad-ratic	$10\mathrm{D}/\Delta < \mathrm{Re} < 500\mathrm{D}/\Delta$	10^{χ}, where $\chi = 0,127\lg\bar{\Delta} - 0,627$	0,123
3	Turbulent, quadratic	$\mathrm{Re} \geq 500\mathrm{D}/\Delta$	$\lambda_{quad.} = 0,11(\Delta/D)^{0,25}$	0

Table 1

According to numerous studies, it was established that under turbulent conditions (which is often found in practice) of flow of Newtonian viscous fluids, three resistance zones are observed: 1) in the zone of smooth-walled resistance (where $\lambda_T = f\,(Re)$ and $(3-4)\cdot 10^3 \leq Re \leq 10D/\Delta$), the coefficient λ_T more ofter is determined by the Blasius formula

$$\lambda_T = 0,3164/Re^{0,25} \tag{16}$$

in the zone of quadratic resistance (completely rough pipes $\lambda_T = f\,(\Delta/D)$ and $Re \geq 500D/\Delta$) by the Shifrinson formula

$$\lambda_T = 0,11\,(\Delta/D)^{0,25}\,. \tag{17}$$

In the zone of pre-quadratic resistance (where $\lambda_T = f\,(Re, \Delta/D)$ and $10D/\Delta < Re < 500D/\Delta$) the following formula [3] is recommended

$$\lambda_T = 10^{\chi}/\mathrm{Re}^{0,123}, \tag{18}$$

where $\chi = 0,127\lg\bar{\Delta} - 0,627$; $\bar{\Delta} = \Delta/D$ is the relative roughness.

Comparing (13) and (16) ÷ (18) we can set up the values of the coefficient A and the exponent m. Their values are given in table 1.

Substituting expression (15) in equation (12), after the corresponding transformations we get

$$-dy = \ (2\alpha_0 - k)\,\frac{16QdQ}{\pi^2 gD^4} + \left(\lambda_n + \frac{A\,(\pi D\nu/4)^m}{Q^m}\right)\frac{8Q^2}{g\pi^2 D^5}dx. \tag{19}$$

For the conditions of flow of fluid with continuous distribution of consumption $dQ = -q_0 dx$, after integration in the interval $(0, x)$ equation (19) takes $(0, x)$ the form [3]

$$y_x = y_0 + \frac{8Q_0^2}{g\pi^2 D^4}\left[(2\alpha_0 - k)\left(1 - z_x^2\right)\right.$$

$$\left. - \frac{\lambda_n Q_0}{3Dq_0}\left(1 - z_x^3\right) - \frac{A\,(\pi D\nu/4)^m\,Q_0^{1-m}}{(3-m)\,Dq_0}\left(1 - z_x^{3-m}\right)\right], \tag{20}$$

where Q_0 is the flow rate of the medium in the initial section (before distribution); $q_0 = Q_n/l = (Q_0 - Q_T)/l$; Q_n, Q_T are travel and transit flow; $z_x = 1 - \alpha x$; $\alpha = (1 - Q_T/Q_0)/l$.

Formula (20) enables to determine pressure change along the canal (pipe) with continuous distribution of consumption for different resistance zones. For the corresponding zone we get calculated dependences:

1) in the zone of smooth-walled resistance (for $m = 0,25$ and $A = 0,3164$)

$$2g(y_x - y_0)/\upsilon_0^2 = \left[(2\alpha_0 - k)\left(1 - z_x^2\right)\right]$$
$$- \left[\lambda_n\left(1 - z_x^3\right) + 0,35\left(1 - z_x^{2,75}\right)\Big/ Re_0^{0,25}\right] Q_0/3Dq_0 \qquad (21)$$

2) in the zone of pre-quadratic resistance (for $m = 0,1,2,3$ and $A = 10^{\chi}$, where $\chi = 0,127 \lg \bar{\Delta} - 0,627$)

$$2g(y_x - y_0)/\upsilon_0^2 = \left[(2\alpha_0 - k)\left(1 - z_x^2\right)\right]$$
$$- \left[\lambda_n\left(1 - z_x^2\right) + 1,05\left(1 - z_x^{2,877}\right)10^{\chi}\Big/ Re_0^{0,123}\right] Q_0/3Dq_0 \qquad (22)$$

3) in the zone of quadratic resistance (for $m = 0$ and $A = \lambda_{quad.} = 0,11(\Delta/\mathrm{D})^{0,25}$)

$$2g(y_x - y_0)/\upsilon_0^2 = \left[(2\alpha_0 - k)\left(1 - z_x^2\right)\right] - \left[(\lambda_n + \lambda_{quad.})\left(1 - z_x^3\right)\right] Q_0/3Dq_0. \qquad (23)$$

In formulas (21)-(23): $\upsilon_0 = 4Q_0/\pi D^2$; $Re_0 = 4Q_0/\pi D\nu$. When executing practical calculations, the coefficient α_0 should be taken equal to $\alpha_0 = 1,03 \div 1,1$. As for the flow distribution coefficient k, then in the case of distribution (detachment) of consumption there are different opinions on it (for example, prof. G.A. Petrov recommends to take $k = 0$, prof. P.G. Kisel'ev $k = 0,25 \div 0,5$, prof. I.E. Idelchik-$k = 0,5 \div 1,0$, prof. I.M. Konovalov $k = 1,0$) [1]. Therefore, this coefficient should be experimentally determined. To this end, and for establishment of real regularities of fluid motion in canals (pipes) with permeable walls, special experimental studies were carried out. The experimental stand included a system for creating pressure, perforated pipe, measuring and regulatory means. The perforated pipe consisted of the following models:

1) a steel pipe line (made of stainless steel) with inner diameter $D = 5,2 \cdot 10^{-2}m$ and $D = 3,42 \cdot 10^{-2}m$ of the length of the work site $l = 2,2m$ and $l = 3,6m$, diameter of holes (perforations) $d_0 = 4 \cdot 10^{-3}m$ and $d_0 = 2 \cdot 10^{-3}m$;

2) duralumin pipeline with inner diameter $D = 2,075 \cdot 10^{-2}m$, length of the working site $l = 2 \div$4m, diameter of holes $d_0 = 10^{-3}m$ and $2 \cdot 10^{-3}m$. In all models, the length of the initial section was $l_m = 2,0m$, of the final section $l_m = 1,0m$. Inner diameters of the studied models (pipelines) were found by the ,mean area of the pipe section, determined as the ratio of the fluid volume (measured by fulfilling the pipe by fluid) to its length. Diameters of holes (perforations) were determined by means of instrumental microscope.

During the process of experiments, certain total flow at the beginning of the pipe (before distribution) was skipped, for $Re_0 = (2 \div 9)10^5$ and ratio of consumptions $\beta = Q_\pi/Q_0$ (where Q_π is travel flow distributed along the length of the perforated area, $Q_\pi = Q_0 - Q_1$, Q_T is transit flow of liquid in the pipe after distribution, Q_0 is total flow in the pipe before the flow distribution) changed from 0 to 1. After achieving stationary mode of operation of the installation were fixed: initial Q_0, travel Q_π and transit Q_T flows, and also change of piezometric height (of pressure) along the working length of the perforated pipeline. Measurements were carried out by the standard technique. The graphs of change of flow (velocity) and pressure along the length, and distribution coefficient k of β were constructed on the base of experimental data. It was established that fluid flow (velocity) along the length decreases almost by the linear law, but pressure either decreases ($dP/dx < 0$), or increases ($dP/dx > 0$) [3]. The observed regularity follows directly from equation (20) and is determined by the character of the changes in the pulse and pressure loss along the length of the pipe.

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