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STABILIZATION OF LINEAR AUTONOMOUS SYSTEMS WITH INCOMPLETE INFORMATION ON THE BASIS OF MATLAB

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Abstract. The problem of stabilization of linear autonomous systems with incomplete information in the class of linear controls is considered. A way of solution of this problem on the basis of Matlab and Symbolic Math Toolbox is described. For brevity, only the cases of four and more dimensions are considered.

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Consider linear autonomous system with incomplete information

$$\frac{dx}{dt} = Ax + Bu\,, \quad y = Hx \tag{1}$$

where $x \in R^n, u \in R^m, y \in R^l$ are, respectively, the state, control and output vectors of the system. A, B and H are $n \times n$, $n \times m$ and $l \times n$ dimensional constant matrices. It is well known that, without loss of generality, one can suppose that

$$rankB = m\,.$$

We say that, (1) is a system with incomplete information (about of systems state) if

$$rankH < n.$$

Suppose that systems equilibrium state $x = 0$ is not asymptotically stable in the case of $u = 0$. Then the stabilization problem of system (1) (in the class of linear controls) can be stated as follows: find a control as a linear function of output

$$u = Cy, \tag{2}$$

such that, the closed (1)–(2) systems

$$\frac{dx}{dt} = (A + BCH)x$$

equilibrium state $x = 0$ became asymptotically stable.

Below we consider a way of solution of this problem on the basis of Matlab and Symbolic Math Toolbox (see [1]-[2]). However, for brevity, we consider only cases of four and more dimensions. For the beginning, consider the stabilization problem with incomplete information (see [3]) for systems having dimension four. Particularly, consider the system (1), in the class of control (2), where $x \in R^n, u \in R^m, y \in R^l$, ($m \leq 4$ and $l \leq 4$), A, B and H are, respectively, $4 \times 4, 4 \times m$ and $4 \times l$ dimensional constant matrices and C is an unknown $m \times l$ dimensional matrix. As in the cases of $rankB = 4$ or $rankH = 4$ we have a system with complete information, one can suppose that $rankB \le 3, rankH \le 3$. So, more general and complex is the case when $rankB = rankH = 3$. The consider following example.

Example 1. Consider system (1) $(n = 4, m = l = 1)$, with characters

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$b = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix},$$

$$h^T = \begin{pmatrix} 1 & 0 & 2 & -1 \end{pmatrix}.$$

It is clear, that A is not Hurwitz's matrix, so a consideration of the stabilization problem has a sense. Input characters of this system in the Matlab window:

$A = [0\ 1\ 0\ 0;\ 0\ 0\ 0\ 0;\ 0\ 0\ 1\ 1;\ 0\ 0\ 0\ 1]; b = [1;\ 1;\ -1;\ 1] : h = [1;\ 0;\ 2;\ -1]$
$syms\ \ c\ A1$
$A1 = A + b^* c^* h.'$
$A1 = [c, 1, 2*c, -c]$
$[0, 0, 0, 0]$
$[-c, 0, 1-2*c, c+1]$
$[0, 0, 0, 1]$

$A1 =$
$[c, 1, 2*c, -c]$
$[c, 0, 2*c, -c]$
$[-c, 0, 1-2*c, c+1]$
$[c, 0, 2*c, 1-c]$

$p = poly(A1)$
$p =$
$(2*c*(c+1)-c^2)*(c-2*c^2+c*(2*c-1)) - x^2*(c-(c-1)^2+2*c*(c+1)-3*c^2 + c*(2*c-1)) - 2*c*((2*c-1)*((2*c-1)*(c+1)-c^2)+c*(c-2*c*(c+1)+c^2)) + x^3*(2*c-2) - (c-1)*(2*c^3-c*(2*c^2+2*c)+c*(2*c-1)) - x*((2*c*(c+1) - c^2)*(c-1) - c*(2*c^2+2*c) - 2*c*((2*c-1)*(c+1)-c^2)+2*c^3-c*(c-2*c*(c+1) + c^2)+c*(2*c-1)+(c-1)*(c-2*c^2+c*(2*c-1))) + c*(2*c*((2*c-1)*(c+1) - c^2) - 2*c*(c+1)+c^2+c*(c-2*c*(c+1)+c^2)) + (2*c*((2*c-1)*(c+1)-c^2) + c*(c-2*c*(c+1)+c^2))*(c-1)+x^4$

$p = simple(p)$
$p =$
$c*x-c-4*c*x^2+2*c*x^3+x^2-2*x^3+x^4$

$p = simple(p)$
$p =$
$x^4+(2*c-2)*x^3+(1-4*c)*x^2+c*x-c$

Thus, the close characteristic polynomial of the system , in this case will have the form
$p = s4 + 2 * (c - 1) * s3 + (1 - 4 * c) * s2 + c * s - c$,
for which the Routh-Hurwitz stability criterion has the form:

$$\begin{array}{l} 2 * (c - 1) > 0 \\ (1 - 4 * c) > 0 \\ c > 0 \\ -c > 0 \\ 2 * (c - 1) * (1 - 4 * c) - c > 0 \\ 2 * (c - 1) * (1 - 4 * c) * c + 4 * (c - 1)^2 - c^2 > 0. \end{array} \tag{3}$$

Although, it's clear that this system can't be stable, because of contradiction in the third and fourth conditions in (3).

Example 2. Consider system (1) $(n = 4, m = l = 1)$, which turns out from example 1 by adding one more equation of observation and which has the following characters:

$A = [0\ 1\ 0\ 0;\ 0\ 0\ 0\ 0;\ 0\ 0\ 1\ 1;\ 0\ 0\ 0\ 1]; b = [1;\ 1;\ -1;\ 1] : H = [1\ 0\ 2\ \ -1;\ 1\ \ -1\ 1\ \ -1]$

$syms\ \ c1\ c2\ A1$

$c = [c1; c2];$

$A1 = A + b * c.' * H$
$A1 =$
$[c1 + c2, 1 - c2, 2 * c1 + c2, -c1 - c2]$
$[c1 + c2, -c2, 2 * c1 + c2, -c1 - c2]$
$[-c1 - c2, c2, 1 - c2 - 2 * c1, c1 + c2 + 1]$
$[c1 + c2, -c2, 2 * c1 + c2, 1 - c2 - c1]$

$p = poly(A1);$
$p = simple(p)$
$p =$
$c1 * x - c2 - c1 + 2 * c2 * x - 4 * c1 * x^2 + 2 * c1 * x^3 - 4 * c2 * x^2 + 2 * c2 * x^3 + x^2 - 2 * x^3 + x^4$

$p = simple(p)$
$p =$
$x^4 + (2 * c1 + 2 * c2 - 2) * x^3 + (1 - 4 * c2 - 4 * c1) * x^2 + (c1 + 2 * c2) * x - c1 - c2$

(We don't recall here expression for $p = poly(A1)$, because it covers more than half a page!) Hence, the close systems characteristic polynomial in this case will have the form
$p = s4 + 2 * (c1 + c2 - 1) * s3 + (1 - 4 * c2 - 4 * c1) * s2 + (c1 + 2 * c2) * s - (c1 + c2)$,
The Routh-Hurwitz stability criterion gives:
$2 * (c1 + c2 - 1) > 0$
$(1 - 4 * c2 - 4 * c1) > 0$
$(c1 + 2 * c2) > 0$
$-(c1 + c2) > 0$
$2 * (c1 + c2 - 1) * (1 - 4 * c2 - 4 * c1) + (c1 + c2) > 0$
$2 * (c1 + c2 - 1) * (1 - 4 * c2 - 4 * c1) * (c1 + c2) + 4 * (c1 + c2 - 1)^2 * (c1 + c2) - (c1 + 2 * c2)^2 > 0,$

or in convenient for Matlab solver form,

$$
\begin{aligned}
&-2*(c1+c2-1)<0\\
&-(1-4*c2-4*c1)<0\\
&-(c1+2*c2)<0\\
&-(c1+c2)<0-2*(c1+c2-1)*(1-4*c2-4*c1)-(c1+c2)<0\\
&-2*(c1+c2-1)*(1-4*c2-4*c1)*(c1+c2)-4*(c1+c2-1)^2*(c1+c2)\\
&+(c1+2*c2)^2<0.
\end{aligned}
\tag{4}
$$

Construct now goal file-function for minimax problem (in the frame of optimization toolbox), for this example: function $F = ogst1(c)$
4-dimensional stabilization $F = [-2*(c(1)+c(2)-1); -(1-4*c(2)-4*c(1)); -(c(1)+2*c(2)); -(c(1)+c(2));$
$-2*(c(1)+c(2)-1)*(1-4*c(2)-4*c(1))-(c(1)+c(2)); -2*(c(1)+c(2)-1)*(1-4*c(2)-4*c(1))*(c(1)+c(2))-4*(c(1)+c(2)-1)^2*(c(1)+c(2))+(c(1)+2*c(2))^2];$
end

Solving this problem using optimtools fminimax solver, we can see that, goal-function can receive only positive values, that is the stabilization problem hasn't a solution.

Example 3. Consider now two link manipulator's stabilization known problem (see [3]). This problem equations, in the version of incomplete information, has the form: System's state equations
$\frac{dx_1}{dt} = x_2$
$\frac{dx_2}{dt} = u_1 - u_2$
$\frac{dx_3}{dt} = x_4$
$\frac{dx_4}{dt} = u_2$
and the observations equations $y_1 = x_1$
$y_2 = x_2$
$y_3 = x_3$

In this case system's matrix,

$$
A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

is not a Hurwitz's matrix, so the stabilization problems consideration has a sense. Input this systems characters in the Matlab window:

$A = [0\ 1\ 0\ 0; 0\ 000; 0\ 0\ 0\ 1; 0\ 0\ 0\ 0]; B = [0\ 0; 1\ -1; 0\ 0; 0\ 1]; H = [1\ 0\ 0\ 0; 0\ 1\ 0\ 0; 0\ 0\ 1\ 0]$
$syms\ \ c1\ c2\ c3\ c4\ c5\ c6\ A1$
$c = [c1\ c2\ c3; c4\ c5\ c6]\ c =$
$[c1, c2, c3]$
$[c4, c5, c6]$
Then, as usual,

$A1 = A + B * c * H$

$A1 =$
$[0, 1, 0, 0]$
$[c1 - c4, c2 - c5, c3 - c6, 0]$
$[0, 0, 0, 1]$
$[c4, c5, c6, 0]$

$p = poly(A1);$
$p = simple(p)$

$p =$
$x^4+(c5-c2)*x^3+(c4-c1-c6)*x^2+(c6*(c2-c5)-c5*(c3-c6))*x+c6*(c1-c4)-c4*(c3-c6)$

Hence, the close systems characteristic polynomial in this case will have the form

$p = s4 + (c5 - c2) * s3 + (c4 - c1 - c6) * s2 + (c6 * (c2 - c5) - c5 * (c3 - c6)) * s + (c6 * (c1 - c4) - c4 * (c3 - c6))$,
for which the Routh-Hurwitz stability criterion gives:

$(c5 - c2) > 0$
$(c4 - c1 - c6) > 0$
$(c6 * (c2 - c5) - c5 * (c3 - c6)) > 0$
$(c6 * (c1 - c4) - c4 * (c3 - c6)) > 0$
$(c5 - c2) * (c4 - c1 - c6) - (c6 * (c2 - c5) - c5 * (c3 - c6)) > 0$
$(c5 - c2) * (c4 - c1 - c6) * (c6 * (c2 - c5) - c5 * (c3 - c6)) - (c5 - c2)^2 * (c6 * (c1 - c4) - c4 * (c3 - c6)) - (c6 * (c2 - c5) - c5 * (c3 - c6))^2 > 0$,

or, in convenient for Matlab solver form, $-(c5 - c2) < 0$
$-(c4 - c1 - c6) < 0$
$-(c6 * (c2 - c5) - c5 * (c3 - c6)) < 0$
$-(c6 * (c1 - c4) - c4 * (c3 - c6)) < 0$
$-(c5 - c2) * (c4 - c1 - c6) - (c6 * (c2 - c5) - c5 * (c3 - c6)) < 0$
$-(c5 - c2) * (c4 - c1 - c6) * (c6 * (c2 - c5) - c5 * (c3 - c6)) + (c5 - c2)^2 * (c6 * (c1 - c4) - c4 * (c3 - c6)) + (c6 * (c2 - c5) - c5 * (c3 - c6))^2 < 0$.

Construct now goal file-function for minimax problem (in the frame of optimization toolbox), for this example:
$functionF = ogst2(c)$
4-dimensional stabilization
$F = [-(c(5) - c(2)); -(c(4) - c(1) - c(6)); -(c(6) * (c(2) - c(5)) - c(5) * (c(3) - c(6)));$
$-(c(6) * (c(1) - c(4)) - c(4) * (c(3) - c(6))); -(c(5) - c(2)) * (c(4) - c(1) - c(6)) - (c(6) * (c(2) - c(5)) - c(5) * (c(3) - c(6)));$
$-(c(5) - c(2)) * (c(4) - c(1) - c(6)) * (c(6) * (c(2) - c(5)) - c(5) * (c(3) - c(6)))$
$+(c(5) - c(2))^2 * (c(6) * (c(1) - c(4)) - c(4) * (c(3) - c(6))) + (c(6) * (c(2) - c(5))$
$- c(5) * (c(3) - c(6)))^2]; end$

Solving this problem using optimtools fminimax solver, for starting point

$$[-1\ 1\ -1; 1\ -1\ 1]$$

we received:
Optimization running.
Objective function value: -0.03381567319184724
Local minimum possible. Constraints satisfied.

fminimax stopped because the predicted change in the objective function is less than the default value of the function tolerance and constraints are satisfied to within the default value of the constraint tolerance.

At the same time the ultimate point was $[-1.266\ 0.841\ -0.051; -0.085\ -0.755\ 0.502]$. Pushing solver once more for this point we received the same results. Although solving this problem starting from the point $[-1.266\ 0.841\ -0.051; -0.085\ -0.755\ 0.502]$ we received:

Optimization running.
Objective function value: -161.00780393072233
Local minimum possible. Constraints satisfied.

fminimax stopped because the size of the current search direction is less than twice the default value of the step size tolerance and constraints are satisfied to within the default value of the constraint tolerance.

At the same time the ultimate point was $[-277.0920\ -23.1590\ -5.0890; 248.2140\ 137.8490\ -4.4970]$.
Checking this solution gives:

$c =$
$-277.0920\ -23.1590\ -5.0890$
$248.2140\ 137.8490\ -4.4970$

$>> eig(A + B * c * H)$

$ans =$
$1.0e + 002*$
-1.5768
-0.0328
$-0.0002 + 0.0220i$
$-0.0002 - 0.0220i$

Hence, the closed system we received is really asymptotically stable. Besides this we can see that $(x1, x2, x3)$ represents original systems field of regulation.

Remark. It's must be taken into account that the solver gives results in the order of

$$c = \begin{pmatrix} c1 & c3 & c5 \\ c2 & c4 & c6 \end{pmatrix}$$

At last, it is clear that this method can be used also in the case of dimensions more than four.

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