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EFFECTIVE SOLUTIONS OF BVPs OF THE LINEAR THEORY OF THERMOELASTICITY WITH MICROTEMPERATURES

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Abstract. In the present paper the linear equilibrium theory of thermoelasticity with microtemperatures is considered. The explicit solutions of the Neumann type boundary value problems in the theory of thermoelasticity with microtemperatures for the sphere and for the whole space with a spherical cavity are constructed. The obtained solutions are represented by means of absolutely and uniformly convergent series.

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Introduction

A thermodynamic theory for elastic materials with inner structure whose particles, in addition to microdeformations, possess microtemperatures was proposed by Grot [1]. The mathematical model of the linear theory of thermoelasticity with microtemperature for materials with inner structure whose particles,in addition to the classical displacement and temperature fields, possess microtemperatures was presented by Ieşan and Quintanilla [2].The same authors have proved an existence of week solutions, have formulated the boundary value problems, have established the continuous dependence of solutions on the date and body loads. They have presented an uniqueness result and a solution of the type Boussinesq-Somigliana-Galerkin.

In recent years several continuum theories with microtemperatures have been formulated. An extensive review and the basic results in the microcontinuum field theories are given in the books of Eringen [3] and Ieşan [4]. The fundamental solution of the equations of the theory of thermoelasticity with microtemperatures was constructed by Svanadze [5]. The representations of the Galerkin type and general solutions of the system in this theory were obtained by Scalia, Svanadze and Tracinà [6]. The linear theory of thermoelasticity for microstretch elastic materials with microtemperatures was constructed by Ieşan [7] where the uniqueness and existence theorems are proved in the dynamical case for isotropic materials.

Riha developed a theory of micromorphic fluids with microtemperatures [8]. The exponential stability of solution of equations of the theory of thermoelasticity with microtemperatures has been established by Casas and Quintanilla [9].

Many investigators have studied different types of problems for materials with microtemperatures, that are published in a large number of papers (some of these results can be see in [10-30] and the references cited therein).

In the present paper the linear equilibrium theory of thermoelasticity with microtemperatures is considered. The explicit solutions of the Neumann type boundary value problems in the theory of thermoelasticity with microtemperatures for the sphere and for the whole space with a spherical cavity are constructed. The obtained solutions are represented by means of absolutely and uniformly convergent series.

2. Basic equations

Let $\mathbf{x} = (x_1, x_2, x_3)$ be a point of the Euclidean three-dimensional space E^3. $\partial\mathbf{x} := \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$. Assume D^+ to be a ball, of radius R_1, centered at point $O(0,0,0)$. Let S denotes the spherical surface of radius R_1. Let D^- be the whole space with spherical cavity and with the boundary S. $\overline{D^+} := D^+ \bigcup S$, $D^- := E_3 \backslash \overline{D^+}$.

The basic homogeneous (i.e., body forces are neglected) system of equations of motion in the linear theory of thermoelasticity with microtemperatures for isotropic materials can be written as [2]

$$\mu\Delta\mathbf{u} + (\lambda+\mu)\mathrm{graddiv}\mathbf{u} - \beta\mathrm{grad}\theta = 0, \tag{1}$$

$$k_6\Delta\mathbf{w} + (k_4+k_5)\mathrm{graddiv}\mathbf{w} - k_3\mathrm{grad}\theta - k_2\mathbf{w} = 0, \tag{2}$$

$$k\Delta\theta + k_1\mathrm{div}\mathbf{w} = 0, \tag{3}$$

where $\mathbf{u} := (u_1, u_2, u_3)^T$ is the displacement vector, $\mathbf{w} := (w_1, w_2, w_3)^T$ is the microtemperature vector, θ is the temperature measured from the constant absolute temperature T_0 $(T_0 > 0)$, λ, μ, β, k, k_j, $j = 1, ..., 6$, are constitutive coefficients, Δ is the 3D Laplace operator. The superscript "T" denotes transposition.

We assume that the constitutive coefficients satisfy the conditions

$$\mu > 0, \quad 3\lambda + 2\mu > 0, \quad k_6 \pm k_5 > 0, \quad k > 0,$$

$$3k_4 + k_5 + k_6 > 0, \quad (k_1 + T_0k_3)^2 > 4T_0kk_2.$$

Definition 1. A vector-function $\mathbf{U}$ defined in the domain $D^+(D^-)$ is called regular if it has integrable continuous second order derivatives in $D^+(D^-)$, and $\mathbf{U}$ itself and its first order derivatives are continuously extendible at every point of the boundary of $D^+(D^-)$, i.e., $\mathbf{U} \in C^2(D^+) \cap C^1(\overline{D}^+)$ $(\mathbf{U} \in C^2(D^-) \cap C^1(\overline{D}^-)$). In the case of the domain D^- the vector $\mathbf{U}$ additionally should satisfy the following conditions at infinity:

$$\mathbf{U}(\mathbf{x}) = O(|\mathbf{x}|^{-1}) \qquad \frac{\partial\mathbf{U}}{\partial x_j} = O(|\mathbf{x}|^{-2}) \qquad |\mathbf{x}|^2 = x_1^2 + x_2^2 + x_3^2 >> 1, \qquad j = 1, 2, 3.$$

The basic boundary value problems (BVPs) of the theory of thermoelasticity with microtemperatures are formulated as follows:

Problem 1. Find in the domain D^+ a regular solution $\mathbf{U}(\mathbf{u}, \mathbf{w}, \theta)$ of equations (1-3) satisfying the following boundary conditions

$$(\mathbf{u})^+ = \mathbf{F}^+(\mathbf{y}), \quad \left(\mathbf{P}^{(2)}(\partial\mathbf{x}, \mathbf{n})\mathbf{w}\right)^+ = \mathbf{f}^+(\mathbf{y}), \quad \left(k\frac{\partial\theta}{\partial\mathbf{n}} + \frac{k_1}{R_1}\mathbf{x}\mathbf{w}\right)^+ = f_4^+(\mathbf{y}), \quad \mathbf{y} \in S, \tag{4}$$

Problem 2. Find in the domain D^- a regular solution $\mathbf{U}(\mathbf{u}, p_1, p_2)$ of equations (1-3), satisfying the following boundary conditions

$$(\mathbf{u})^- = \mathbf{F}^-(\mathbf{y}), \quad \left(\mathbf{P}^{(2)}(\partial\mathbf{x}, \mathbf{n})\mathbf{w}\right)^- = \mathbf{f}^-(\mathbf{y}), \quad \left(k\frac{\partial\theta}{\partial\mathbf{n}} + \frac{k_1}{R_1}\mathbf{x}\mathbf{w}\right)^- = f_4^-(\mathbf{y}), \quad \mathbf{y} \in S,$$

where $\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w}$ is the stress vector in the theory of thermoelasticity with microtemperatures and has the form [2]

$$\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w}=(k_5+k_6)\frac{\partial\mathbf{w}}{\partial\mathbf{n}}+k_4\mathbf{n}\mathrm{div}\mathbf{w}+k_5[\mathbf{n}\cdot\mathrm{rot}\mathbf{w}],\quad \mathbf{n}(\mathbf{x})=\frac{\mathbf{x}}{\rho}. \tag{5}$$

$\mathbf{F},\mathbf{f},f_4$ are a given functions on S, $\mathbf{n}$ is the external unit normal vector to S at $\mathbf{y}$, $\frac{\partial}{\partial\mathbf{n}}$ is the derivative along the vector $\mathbf{n}$, the symbol $[\cdot]^-$ denotes the limit on S from D

$$[\mathbf{U}(\mathbf{z})]^{\pm}=\lim_{D^{\pm}\ni\mathbf{x}\to\mathbf{z}\in S}\mathbf{U}(\mathbf{x}),\quad \left[\mathbf{P}^{(2)}(\partial_{\mathbf{z}},\mathbf{n})\mathbf{w}(\mathbf{z})\right]^{\pm}=\lim_{D^{\pm}\ni\mathbf{x}\to\mathbf{z}\in S}\mathbf{P}^{(2)}(\partial_{\mathbf{x}},\mathbf{n})\,\mathbf{w}(\mathbf{x}),$$

$$\left[k\frac{\partial\theta}{\partial\mathbf{n}}+\frac{k_1}{R_1}\mathbf{x}\mathbf{w}(\mathbf{z})\right]^{\pm}=\lim_{D^{\pm}\ni\mathbf{x}\to\mathbf{z}\in S}\left(k\frac{\partial\theta}{\partial\mathbf{n}}+\frac{k_1}{R_1}\mathbf{x}\mathbf{w}\right).$$

The following theorems are valid [6]

Theorem 1. *Any two regular solution of the BVP 1 may differ only to within the additive vector* $\boldsymbol{U}=(\boldsymbol{u}_0,\boldsymbol{w}_0,\theta_0)$, *where*

$$\boldsymbol{u}_0=0,\quad \boldsymbol{w}_0=0,\quad \theta_0=constants.$$

Theorem 2. *The BVP 2 admit at most one regular solution.*

3. Auxiliary formulas

Let us introduce the spherical coordinates equalities

$$\begin{aligned}&x_1=\rho\sin\vartheta\cos\varphi,\quad x_2=\rho\sin\vartheta\sin\varphi,\quad x_3=\rho\cos\vartheta,\quad x\in D^+,\\&y_1=R_1\sin\vartheta_0\cos\varphi_0,\quad y_2=R_1\sin\vartheta_0\sin\varphi_0,\quad y_3=R_1\cos\vartheta_0,\quad y\in S,\\&|x|^2=\rho^2=x_1^2+x_2^2+x_3^2,\quad 0\le\vartheta\le\pi,\quad 0\le\varphi\le 2\pi\quad 0\le\rho\le R_1.\end{aligned}$$

In the sequel we use the following notation: If $\mathbf{g}(\mathbf{x})=\mathbf{g}(g_1,g_2,g_3)$ and $\mathbf{q}(\mathbf{x})=\mathbf{q}(q_1,q_2,q_3)$ then by symbols $(\mathbf{g.q})$ and $[\mathbf{g.q}]$ we will denote the scalar product and the vector product respectively

$$(\mathbf{g.q})=\sum_{k=1}^{3}g_kq_k,\quad [\mathbf{g.q}]=(g_2q_3-g_3q_2,g_3q_1-g_1q_3,g_1q_2-g_2q_1),$$

We introduce the following notations:

$$[\mathbf{x}\cdot\nabla]_k=\frac{\partial}{\partial S_k(x)},\quad k=1,2,3,\quad \nabla=\left(\frac{\partial}{\partial x_1},\frac{\partial}{\partial x_2},\frac{\partial}{\partial x_{3,}}\right),$$

$$\frac{\partial}{\partial S_1(x)}=x_2\frac{\partial}{\partial x_3}-x_3\frac{\partial}{\partial x_2}=-cos\varphi ctg\vartheta\frac{\partial}{\partial\varphi}-sin\varphi\frac{\partial}{\partial\vartheta},$$

$$\frac{\partial}{\partial S_2(x)}=x_3\frac{\partial}{\partial x_1}-x_1\frac{\partial}{\partial x_3}=-sin\varphi ctg\vartheta\frac{\partial}{\partial\varphi}+cos\varphi\frac{\partial}{\partial\vartheta},$$

$$\frac{\partial}{\partial S_3(x)}=x_1\frac{\partial}{\partial x_2}-x_2\frac{\partial}{\partial x_1}=\frac{\partial}{\partial\varphi},$$

It is easy to show that the following identities are valid [27],[33]:

$$(\mathbf{x}\cdot \mathrm{rot}\mathbf{g})=\sum_{k=1}^{3}\frac{\partial g_k(\mathbf{x})}{\partial S_k(\mathbf{x})},\quad \sum_{k=1}^{3}\frac{\partial}{\partial S_k(\mathbf{x})}(\mathrm{rot}[\mathbf{x}\cdot\nabla]h)_k=0,$$

$$\sum_{k=1}^{3}\frac{\partial}{\partial S_k(\mathbf{x})}(\mathrm{rot}\mathbf{g}(\mathbf{x}))_k=\rho\frac{\partial}{\partial\rho}\mathrm{div}\mathbf{g}(\mathbf{x})-\sum_{k=1}^{3}x_k\Delta\mathbf{g}_k(\mathbf{x}),$$

$$\sum_{k=1}^{3}\frac{\partial}{\partial S_k(x)}[\mathbf{x}\cdot\mathbf{g}]_k=\rho^2\mathrm{div}\mathbf{g}(\mathbf{x})-\left(\rho\frac{\partial}{\partial\rho}+1\right)(\mathbf{x}\cdot\mathbf{g}(\mathbf{x})),$$

$$\sum_{k=1}^{3}\frac{\partial}{\partial S_k(\mathbf{x})}[\mathbf{x}\cdot \mathrm{rot}\mathbf{g}(\mathbf{x})]_k=-\left(\rho\frac{\partial}{\partial\rho}+1\right)\sum_{k=1}^{3}\frac{\partial g_k(\mathbf{x})}{\partial S_k(x)},$$

$$\sum_{k=1}^{3}x_k\frac{\partial}{\partial S_k(x)}=0,\quad \frac{\partial}{\partial S_k(x)}\frac{\partial}{\partial x_k}=\frac{\partial}{\partial x_k}\frac{\partial}{\partial S_k(x)},$$

$$\sum_{k=1}^{3}\frac{\partial^2}{\partial S_k^2(x)}=\frac{\partial^2}{\partial\vartheta^2}+ctg\vartheta\frac{\partial}{\partial\vartheta}+\frac{1}{sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2},\quad \frac{\partial x_k}{\partial S_k}=0,$$

$$\sum_{k=1}^{3}\frac{\partial}{\partial S_k(x)}\frac{\partial}{\partial x_k}=0,\quad \frac{\partial g(\rho)Y(\vartheta,\varphi)}{\partial S_k(x)}=g(\rho)\frac{\partial Y(\vartheta,\varphi)}{\partial S_k(x)},$$

$$\frac{\partial}{\partial S_k(x)}\frac{\partial}{\partial\rho}=\frac{\partial}{\partial\rho}\frac{\partial}{\partial S_k(x)},\quad \Delta\frac{\partial g(x)}{\partial S_k(x)}=\frac{\partial}{\partial S_k}\Delta g(x),$$

$$\sum_{k=1}^{3}\frac{\partial^2 g(x)}{\partial S_k^2(x)}=\rho^2\Delta g(x)-\rho^2\frac{\partial^2 g(x)}{\partial\rho^2}-2\rho\frac{\partial g(x)}{\partial\rho}.$$

Below we frequently use the formulas

$$[\mathbf{x}\cdot \mathrm{rot}\mathbf{g}(x)]=\mathbf{M}(x,\partial x)\mathbf{g}+\mathbf{x}\mathrm{div}\mathbf{g}-\rho\frac{\partial}{\partial\rho}\mathbf{g},$$

$$\sum_{kj=1}^{3}\frac{\partial}{\partial S_k(x)}\mathbf{M}(x,\partial x)_{kj}g_j(x)=-\sum_{j=1}^{3}\frac{\partial g_j(x)}{\partial S_j(x)},$$

$$\mathbf{M}(x,\partial x)=\begin{pmatrix}0 & -\frac{\partial}{\partial S_3(x)} & \frac{\partial}{\partial S_2(x)}\\ \frac{\partial}{\partial S_3(x)} & 0 & -\frac{\partial}{\partial S_1(x)}\\ -\frac{\partial}{\partial S_2(x)} & \frac{\partial}{\partial S_1(x)} & 0\end{pmatrix},$$

$$[\mathbf{x}[\mathbf{x}\cdot\mathbf{g}]]=\mathbf{x}(\mathbf{x}\cdot\mathbf{g})-|\mathbf{x}|^2\mathbf{g}(x),$$

$$\sum_{k=1}^{3}\frac{\partial}{\partial S_k(x)}[\mathbf{x}[\mathbf{x}\cdot\mathbf{g}]]_k=-|\mathbf{x}|^2\sum_{k=1}^{3}\frac{\partial}{\partial S_k(x)}g_k(x).$$

In the sequel we use the following notation

$$(\mathbf{x}\cdot\mathbf{P}^{(2)}\mathbf{w}) = (\mathbf{x}\cdot\mathbf{f}) = h_1(\mathbf{z}), \quad \sum_{k=0}^{3}\frac{\partial}{\partial S_k(z)}[\mathbf{x}\cdot\mathbf{P}^{(2)}\mathbf{w}]_k = \sum_{k=0}^{3}\frac{\partial}{\partial S_k(z)}[\mathbf{x}\cdot\mathbf{f}] = h_2(\mathbf{z}),$$

$$\sum_{k=0}^{3}\frac{\partial}{\partial S_k(z)}[\mathbf{P}^{(2)}\mathbf{w}]_k = \sum_{k=0}^{3}\frac{\partial f_k}{\partial S_k(z)} = h_3(\mathbf{z}), \quad f_4 = h_4(\mathbf{z}), \quad \mathbf{z}\in S.$$

We assume that the functions $h_k(\mathbf{x})$ are representable by the series form.

$$h_k(\mathbf{x}) = \sum_{m=0}^{\infty} h_{km}(\vartheta,\varphi),$$

where h_{km} is the spherical harmonic of order m :

$$h_{km} = \frac{2m+1}{4\pi R_1^2}\int\limits_S P_m(\cos\gamma)h_m(y)dS_y,$$

P_m is Legender polynomial of the m-th order, γ is an angle formed by the radius-vectors Ox and Oy.

From this formulas it follows that, if g_m is the spherical harmonic, the operator $\dfrac{\partial}{\partial S_k}$, $k = 1,2,3$, does not affect the order of the spherical function:

$$\sum_{k=0}^{3}\frac{\partial^2 g_m(\mathbf{x})}{\partial S_k^2(x)} = -m(m+1)g_m(\mathbf{x}).$$

4. Expansion of regular solutions

Theorem 3. *In the domain of regularity the regular solution $\boldsymbol{W} = (\boldsymbol{w},\theta)$ of system (2),(3) can be represented as*

$$\boldsymbol{W}(\boldsymbol{x}) = (\overset{\mathbf{1}}{\mathbf{w}} + \overset{\mathbf{2}}{\mathbf{w}}, \theta), \tag{6}$$

where

$$\Delta(\Delta - s_1^2)\overset{\mathbf{1}}{\mathbf{w}} = 0, \quad \mathrm{rot}\overset{\mathbf{1}}{\mathbf{w}} = 0, \quad (\Delta - s_1^2)\mathrm{div}\overset{\mathbf{1}}{\mathbf{w}} = 0, \quad (\Delta - s_2^2)\overset{\mathbf{2}}{\mathbf{w}} = 0,$$
$$\mathrm{div}\overset{\mathbf{2}}{\mathbf{w}} = 0, \quad \Delta(\Delta - s_1^2)\theta = 0, \quad s_1^2 = \frac{kk_2 - k_1k_3}{kk_7} > 0, \quad s_2^2 = \frac{k_2}{k_6} > 0. \tag{7}$$

Theorem 4. *In the domain of regularity the regular solution of equations (2),(3) can be represented in the form*

$$\boldsymbol{w}(\boldsymbol{x}) = a\,\mathrm{grad}\varphi_1(\boldsymbol{x}) + b\,\mathrm{grad}\varphi_2(\boldsymbol{x}) + \overset{\mathbf{2}}{\mathbf{w}}(\boldsymbol{x}),$$
$$\theta(\boldsymbol{x}) = \varphi_1(\boldsymbol{x}) + \varphi_2(\boldsymbol{x}) \quad a = -\frac{k_3}{k_2} \qquad b = -\frac{k}{k_1}, \tag{8}$$

where

$$\Delta\varphi_1 = 0, \quad (\Delta - s_1^2)\varphi_2 = 0, \quad (\Delta - s_2^2)\overset{\mathbf{2}}{\mathbf{w}} = 0, \quad \mathrm{div}\overset{\mathbf{2}}{\mathbf{w}} = 0. \tag{9}$$

Theorem 5. *The vector* $\boldsymbol{W}=(\boldsymbol{w},\theta)$, *where* $\boldsymbol{w}=(w_1,w_2,w_3)$, *is a regular solution of the homogeneous equations (2),(3), in* D^+, *if and only if when it is represented in the form*

$$\begin{cases} \boldsymbol{w}(\boldsymbol{x}) = a\,\mathrm{grad}\varphi_1(\boldsymbol{x}) + b\,\mathrm{grad}\varphi_2(\boldsymbol{x}) + c\,\mathrm{rot}\boldsymbol{\varphi^3}(\boldsymbol{x}), \\ \theta(\boldsymbol{x}) = \varphi_1(\boldsymbol{x}) + \varphi_2(\boldsymbol{x}), \end{cases} \tag{10}$$

where

$$\begin{cases} (\Delta - s_2^2)\boldsymbol{\varphi^3} = 0, \quad \mathrm{div}\boldsymbol{\varphi^3} = 0, \quad c = -\dfrac{k_6}{k_2}, \\ \boldsymbol{\varphi^3}(\boldsymbol{x}) = [\boldsymbol{x}\cdot\nabla]\varphi_3(\boldsymbol{x}) + \mathrm{rot}[\boldsymbol{x}\cdot\nabla]\varphi_4(\boldsymbol{x}), \\ \int\limits_{S(0,a_1)} \varphi_j ds = 0, \quad (\Delta - s_2^2)\varphi_j = 0, \quad j = 3,4, \end{cases} \tag{11}$$

$S(0,a_1)\subset D^+$ *is an arbitrary spherical surface of radius* a_1. *Between the vector* $\boldsymbol{W}(\boldsymbol{x}) = (\boldsymbol{w},\theta)$ *and the functions* φ_j, $j=1,..,4$, *there exist one-to-one correspondence.*

Theorems 3,4 and Theorem 5 are proved in [27].

Remark 1. By virtue of the equality

$$\mathrm{rotrot}[\mathbf{x}\cdot\nabla]\varphi_4 = -\Delta[\mathbf{x}\cdot\nabla]\varphi_4,$$

formula (10) can be written as

$$\begin{cases} \mathbf{w}(\mathbf{x}) = a\,\mathrm{grad}\varphi_1(\mathbf{x}) + b\,\mathrm{grad}\varphi_2(\mathbf{x}) + [\mathbf{x}\cdot\nabla]\varphi_4(\mathbf{x}) + c\,\mathrm{rot}[\mathbf{x}\cdot\nabla]\varphi_3(\mathbf{x}), \\ \theta(\mathbf{x}) = \varphi_1(\mathbf{x}) + \varphi_2(\mathbf{x}). \end{cases} \tag{12}$$

Remark 2. Note that BVPs for the system (2),(3), which contain only $\mathbf{w}$ and θ, can be investigated separately. Then supposing θ, as known we can study BVPs for the system (1) with respect to $\mathbf{u}$. Combining the results obtained we arrive at an explicit solution for BVPs for the system (1)-(3). Let us assume that $\theta(\mathbf{x})$ is known, when $\mathbf{x}\in D^+(D^-)$, then for $\mathbf{u}$ we get the following nonhomogeneous equation

$$\mu\Delta\mathbf{u} + (\lambda+\mu)\mathrm{graddiv}\mathbf{u} = \beta\mathrm{grad}(\varphi_1+\varphi_2). \tag{13}$$

It is easily to check that the general solution of equation (13) is representable in the form

$$\mathbf{u} = \mathbf{V} + \mathbf{v}_0,$$

where $\mathbf{v}_0$ is a particular solution of (13)

$$\mathbf{v}_0 = \frac{\beta}{\lambda+2\mu}\mathrm{grad}\left(\varphi_{10} + \frac{\varphi_2}{s_1^2}\right), \quad \Delta\varphi_{10} = \varphi_1$$

and

$$\mu\Delta\mathbf{V} + (\lambda+\mu)\mathrm{graddiv}\mathbf{V} = 0.$$

The last equation is the equation of an isotropic elastic body. The solutions of the BVPs under BCs $\mathbf{V}^{\pm} = \mathbf{F}^{\pm} - \mathbf{v}_0^{\pm}$, in the domains D^+ and D^-, are given in [32].

So, it remains to solve BVPs for the system (2),(3).

5. Solution of the BVP 1

The thermostress vector $\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w}$ has the form

$$\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w}=(k_5+k_6)\frac{\partial\mathbf{w}}{\partial\mathbf{n}}+k_4\mathbf{n}\mathrm{div}\mathbf{w}+k_5[\mathbf{n}\cdot\mathrm{rot}\mathbf{w}],\quad \mathbf{n}(\mathbf{x})=\frac{\mathbf{x}}{\rho}. \tag{14}$$

Taking into account (12) and the identities

$$\begin{aligned}&\frac{\partial}{\partial\mathbf{n}}\mathrm{grad}h(\mathbf{x})=\frac{1}{\rho}\mathrm{grad}\left(\rho\frac{\partial}{\partial\rho}-1)h(\mathbf{x})\right),\\ &\frac{\partial}{\partial\mathbf{n}}\mathrm{rot}h(\mathbf{x})=\frac{1}{\rho}\mathrm{rot}\left(\rho\frac{\partial}{\partial\rho}-1)h(\mathbf{x})\right),\end{aligned} \tag{15}$$

the vector $\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w}$ takes the form

$$\begin{aligned}&\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w}=\\ &\frac{k_5+k_6}{\rho}\left\{\mathrm{grad}\left(\rho\frac{\partial}{\partial\rho}-1)(a\varphi_1(\mathbf{x})+b\varphi_2(\mathbf{x}))\right)+c\,\mathrm{rot}\left(\rho\frac{\partial}{\partial\rho}-1)\boldsymbol{\varphi}^3(\mathbf{x})\right)\right\}\\ &+k_4bs_1^2\frac{1}{\rho}\mathbf{x}\varphi_2+k_5\frac{1}{\rho}[\mathbf{x}\cdot\boldsymbol{\varphi}^{(3)}].\end{aligned} \tag{16}$$

By direct calculation from (16) we obtain

$$\begin{cases}(\mathbf{x}\cdot\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w})=(k_5+k_6)a\rho\dfrac{\partial^2\varphi_1}{\partial\rho^2}+[(k_5+k_6)b\rho\dfrac{\partial^2}{\partial\rho^2}+k_4bs_1^2\rho]\varphi_2\\ +c(k_5+k_6)\left(\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\sum\limits_{k=0}^{3}\dfrac{\partial^2\varphi_3}{\partial S_k^2(x)},\\ \sum\limits_{k=0}^{3}\dfrac{\partial}{\partial S_k(x)}\left[\mathbf{x}\cdot\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w})\right]_k=(k_5+k_6)\left(\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\sum\limits_{k=0}^{3}\dfrac{\partial^2(a\varphi_1+b\varphi_2)}{\partial S_k^2(x)}\\ -\left\{c(k_5+k_6)\left[\rho\dfrac{\partial^2}{\partial\rho^2}+\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right]+k_5\rho\right\}\sum\limits_{k=0}^{3}\dfrac{\partial^2\varphi_3}{\partial S_k^2(x)},\\ \sum\limits_{k=0}^{3}\dfrac{\partial}{\partial S_k(x)}\left[\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w})\right]_k=k_5\left(\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\sum\limits_{k=0}^{3}\dfrac{\partial^2\varphi_4}{\partial S_k^2(x)},\\ \dfrac{\partial\theta}{\partial n}=\dfrac{\partial\varphi_1}{\partial\rho}+\dfrac{\partial\varphi_2}{\partial\rho},\quad(\mathbf{x}\cdot\mathbf{w})=a\rho\dfrac{\partial\varphi_1}{\partial\rho}+b\rho\dfrac{\partial\varphi_2}{\partial\rho}+c\sum\limits_{k=1}^{3}\dfrac{\partial^2\varphi_3}{\partial S_k^2(x)}.\end{cases} \tag{17}$$

It is well known that the general solutions of the equation $(\Delta-s_k^2)\psi=0,\quad k=1,2$, in the domains $D^+(D^-)$ have the form ([31])

$$\begin{cases} \psi(\mathbf{x}) = \sum\limits_{m=0}^{\infty} \phi_m^{(1)}(is_k\rho) Y_m(\vartheta,\varphi), \quad \phi_m^{(1)}(is_k\rho) = \dfrac{\sqrt{R_1} J_{m+\frac{1}{2}}(is_k\rho)}{\sqrt{\rho} J_{m+\frac{1}{2}}(is_k R_1)}, \quad \rho < R_1, \\ \\ \psi(\mathbf{x}) = \sum\limits_{m=0}^{\infty} \Psi_m^{(1)}(is_k\rho) Y_n(\vartheta,\varphi), \quad \Psi_m^{(1)}(is_k\rho) = \dfrac{\sqrt{R_1} H^{(1)}_{m+\frac{1}{2}}(is_k\rho)}{\sqrt{\rho} H^{(1)}_{m+\frac{1}{2}}(is_k R_1)}, \quad \rho > R_1, \end{cases} \tag{18}$$

and the general solutions of the equation $\Delta\phi = 0$ in the domains $D^+(D^-)$ have the form ([31])

$$\begin{cases} \phi(\mathbf{x}) = \sum\limits_{m=0}^{\infty} \dfrac{\rho^m}{(2m+1)R_1^{m-1}} Z_m(\vartheta,\varphi), \quad \rho < R_1, \\ \\ \phi(\mathbf{x}) = \sum\limits_{m=0}^{\infty} \dfrac{R^{m+2}}{(2m+1)\rho^{m+1}} Z_m(\vartheta,\varphi), \quad \rho > R_1, \end{cases} \tag{19}$$

where Y_m and $Z_m(\theta,\phi)$ are the spherical harmonics.

We seek solutions to equations (2),(3) with the boundary conditions

$$\left(\mathbf{P}^{(2)}(\partial\mathbf{x},\mathbf{n})\mathbf{w}\right)^+ = \mathbf{f}(\mathbf{y}), \quad \left(k\frac{\partial\theta}{\partial\mathbf{n}} + \frac{k_1}{R_1}\mathbf{x}\mathbf{w}\right)^+ = f_4(\mathbf{y}), \quad \mathbf{y} \in S,$$

in the form (12), where

$$\begin{cases} \varphi_1(\mathbf{x}) = \sum\limits_{m=0}^{\infty} \dfrac{\rho^m}{(2m+1)R_1^{m-1}} Y_{1m}(\vartheta,\varphi), \\ \\ \varphi_2(\mathbf{x}) = \sum\limits_{m=0}^{\infty} \phi_m^{(1)}(is_1\rho) Y_{2m}(\vartheta,\varphi), \\ \\ \varphi_j(\mathbf{x}) = \sum\limits_{m=0}^{\infty} \phi_m^{(1)}(is_2\rho) Y_{jm}(\vartheta,\varphi), \quad j = 3,4, \quad \rho < R_1, \end{cases} \tag{20}$$

$$\phi_m^{(1)}(is_k\rho) = \frac{\sqrt{R_1} J_{m+\frac{1}{2}}(is_k\rho)}{\sqrt{\rho} J_{m+\frac{1}{2}}(is_k R_1)}, \quad k = 1,2.$$

Substituting the expressions of $\varphi_m(x), \quad m=1,2,3,4$ into (17), we obtain

$$\begin{cases} (\mathbf{x}\cdot\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w}) = a(k_5+k_6)\sum\limits_{m=0}^{\infty}\dfrac{m(m-1)\rho^{m-1}}{(2m+1)R_1^{m-1}}Y_{1m}(\vartheta,\varphi) \\ +b\left[(k_5+k_6)\rho\dfrac{\partial^2}{\partial\rho^2}\phi_m^{(1)}(is_1\rho)+k_4s_1^2\rho\phi_m^{(1)}(is_1\rho)\right]Y_{2m}(\vartheta,\varphi) \\ -c(k_5+k_6)\left(\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\sum\limits_{m=0}^{\infty}m(m+1)\phi_m^{(1)}(is_2\rho)Y_{3m}, \\ \sum\limits_{k=0}^{3}\dfrac{\partial}{\partial S_k(x)}\left[\mathbf{x}.\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w})\right]_k = \\ -(k_5+k_6)\sum\limits_{m=0}^{\infty}m(m+1)\left\{\dfrac{a(m-1)\rho^{m-1}}{(2m+1)R_1^{m-1}}Y_{1m}+b\left(\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\phi_m^{(1)}(is_1\rho)Y_{2m}\right\} \\ +\sum\limits_{m=0}^{\infty}m(m+1)\left\{c(k_5+k_6)\left(\rho\dfrac{\partial^2}{\partial\rho^2}+\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)+k_5\rho\right\}\phi_m^{(1)}(is_2\rho)Y_{3m}, \\ \sum\limits_{k=0}^{3}\dfrac{\partial}{\partial S_k(x)}\left[\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w})\right]_k = -k_5\sum\limits_{m=0}^{\infty}m(m+1)\left(\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\phi_m^{(1)}(is_2\rho)Y_{4m}, \\ \dfrac{\partial\theta}{\partial n} = \sum\limits_{m=0}^{\infty}\dfrac{m\rho^{m-1}}{(2m+1)R_1^{m-1}}Y_{1m}(\vartheta,\varphi)+\sum\limits_{m=0}^{\infty}\dfrac{\partial}{\partial\rho}[\phi_m^{(1)}(is_1\rho)]Y_{2m}(\vartheta,\varphi), \\ (\mathbf{x}\cdot\mathbf{w}) = a\rho\sum\limits_{m=0}^{\infty}\dfrac{m\rho^{m-1}}{(2m+1)R_1^{m-1}}Y_{1m}(\vartheta,\varphi)+b\rho\sum\limits_{m=0}^{\infty}\dfrac{\partial}{\partial\rho}[\phi_m^{(1)}(is_1\rho)]Y_{2m}(\vartheta,\varphi) \\ -cm(m+1)\sum\limits_{m=0}^{\infty}\phi_m^{(1)}(is_2\rho)Y_{3m}. \end{cases} \tag{21}$$

Remark 3. The conditions $\int\limits_{S(0,a_1)}\varphi_j ds=0 \quad j=3,4$ in fact mean that

$$Y_{30}=Y_{40}=0.$$

Passing in (21) to the limit as $\rho\to R_1$ and taking into account boundary conditions we obtain for the determination of Y_{mj} the system of algebraic equations

$$\begin{cases} a(k_5+k_6)\dfrac{m(m-1)}{2m+1}Y_{1m}(\vartheta,\varphi) \\ +b\left\{(k_5+k_6)\left[\rho\dfrac{\partial^2}{\partial\rho^2}\phi_m^{(1)}(is_1\rho)\right]_{\rho=R_1}+k_4s_1^2R_1\right\}Y_{2m}(\vartheta,\varphi) \\ -c(k_5+k_6)m(m+1)\left[\left(\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\phi_m^{(1)}(is_2\rho)\right]_{\rho=R_1}Y_{3m}=h_{1m}^+, \\ -(k_5+k_6)m(m+1)\left\{\dfrac{a(m-1)}{2m+1}Y_{1m}+b\left[\left(\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\phi_m^{(1)}(is_2\rho)\right]_{\rho=R_1}Y_{2m}\right\} \\ +m(m+1)\left\{c(k_5+k_6)\left[\left(\rho\dfrac{\partial^2}{\partial\rho^2}+\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\phi_m^{(1)}(is_2\rho)\right]_{\rho=R_1}+k_5R_1\right\}Y_{3m}=h_{2m}^+, \\ -k_5m(m+1)\left\{\left[\dfrac{\partial}{\partial\rho}\phi_m^{(1)}(is_2\rho)\right]_{\rho=R_1}-\dfrac{1}{R_1}\right\}Y_{4m}=h_{3m}^+, \\ \dfrac{kk_7s_1^2m}{k_2(2m+1)}Y_{1m}(\vartheta,\varphi)\dfrac{ck_1}{R_1}m(m+1)Y_{3m}=h_{4m}^+, \end{cases} \tag{22}$$

where

$$Y_{30}=Y_{40}=0,\quad h_{30}^+=h_{20}^+=0.$$

For $m=0$ we obtain Y_{10} is an arbitrary constant.

By virtue of Theorem 1 we conclude that the system (22) for $m\geq 1$ is uniquely solvable and the functions Y_{jm} can be expressed by the known functions h_{jm}^+.

Remark 4. Having defined φ_1, we find the solution of equation $\Delta\varphi_{10}=\varphi_1$

$$\varphi_{10}(\mathbf{x})=\frac{1}{2}\sum_{n=0}^{\infty}\frac{\rho^{2+n}Y_{1n}(\vartheta,\eta)}{(3+2n)(2n+1)R_1^{n-1}},\quad \rho<R_1. \tag{23}$$

6. Solution of the BVP 2

The solution of the problem

$$\left(\mathbf{P}^{(2)}(\partial\mathbf{x},\mathbf{n})\mathbf{w}\right)^-=\mathbf{f}(\mathbf{y}),\quad \left(k\frac{\partial\theta}{\partial\mathbf{n}}+\frac{k_1}{R_1}\mathbf{x}\mathbf{w}\right)^-=f_4(\mathbf{y}),\quad \mathbf{y}\in S,$$

in the domain D^- is sought in the form (12), where

$$\begin{cases} \varphi_1(\mathbf{x}) = \sum\limits_{m=0}^{\infty} \dfrac{R_1^{m+2}}{(2m+1)\rho^{m+1}} Y_{1m}(\vartheta,\varphi), \\ \varphi_2(\mathbf{x}) = \sum\limits_{m=0}^{\infty} \Psi_m^{(1)}(is_1\rho) Y_{2m}(\vartheta,\varphi), \\ \varphi_j(\mathbf{x}) = \sum\limits_{m=0}^{\infty} \Psi_m^{(1)}(is_2\rho) Y_{jm}(\vartheta,\varphi), \quad \rho > R_1, \quad j = 3,4, \end{cases} \tag{24}$$

$$\Psi_m^{(1)}(is_k\rho) = \frac{\sqrt{R_1} H^{(1)}_{m+\frac{1}{2}}(is_k\rho)}{\sqrt{\rho} H^{(1)}_{m+\frac{1}{2}}(is_k R_1)}, \quad k = 1,2.$$

Substituting the expressions of $\varphi_m(x), \quad m = 1,2,3,4$ into (17) we obtain

$$\begin{cases} (\mathbf{x}\cdot\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w}) = a(k_5+k_6) \sum\limits_{m=0}^{\infty} \dfrac{(m+1)(m+2)R_1^{m+2}}{(2m+1)\rho^{m+2}} Y_{1m}(\vartheta,\varphi) \\ +b\left[(k_5+k_6)\rho\dfrac{\partial^2}{\partial\rho^2}\Psi_m^{(1)}(is_1\rho) + k_4 s_1^2 \rho \Psi_m^{(1)}(is_1\rho)\right] Y_{2m}(\vartheta,\varphi) \\ -c(k_5+k_6)\left(\dfrac{\partial}{\partial\rho} - \dfrac{1}{\rho}\right) \sum\limits_{m=0}^{\infty} m(m+1)\Psi_m^{(1)}(is_2\rho) Y_{3m}, \\ \sum\limits_{k=0}^{3} \dfrac{\partial}{\partial S_k(x)} \left[\mathbf{x}\cdot\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w})\right]_k = \\ -(k_5+k_6)\sum\limits_{m=0}^{\infty} m(m+1)\left\{-\dfrac{a(m+2)R_1^{m+2}}{(2m+1)\rho^{m+2}} Y_{1m} + b\left(\dfrac{\partial}{\partial\rho} - \dfrac{1}{\rho}\right)\Psi_m^{(1)}(is_1\rho) Y_{2m}\right\} \\ +\sum\limits_{m=0}^{\infty} m(m+1)\left\{c(k_5+k_6)\left(\rho\dfrac{\partial^2}{\partial\rho^2} + \dfrac{\partial}{\partial\rho} - \dfrac{1}{\rho}\right) + k_5\rho\right\}\Psi_m^{(1)}(is_2\rho) Y_{3m}, \\ \sum\limits_{k=0}^{3} \dfrac{\partial}{\partial S_k(x)} \left[\mathbf{P}^{(2)}(\partial\mathbf{x},\partial\mathbf{n})\mathbf{w})\right]_k = -k_5 \sum\limits_{m=0}^{\infty} m(m+1)\left(\dfrac{\partial}{\partial\rho} - \dfrac{1}{\rho}\right)\Psi_m^{(1)}(is_2\rho) Y_{4m}, \\ \dfrac{\partial\theta}{\partial n} = -\sum\limits_{m=0}^{\infty} \dfrac{(m+1)R_1^{m+2}}{(2m+1)\rho^{m+2}} Y_{1m}(\vartheta,\varphi) + \sum\limits_{m=0}^{\infty} \dfrac{\partial}{\partial\rho}[\Psi_m^{(1)}(is_1\rho)] Y_{2m}(\vartheta,\varphi), \end{cases} \tag{25}$$

$$(\mathbf{x}\cdot\mathbf{w}) = -a\rho \sum_{m=0}^{\infty} \frac{(m+1)R_1^{m+2}}{(2m+1)\rho^{m+2}} Y_{1m}(\vartheta,\varphi) + b\rho \sum_{m=0}^{\infty} \frac{\partial}{\partial\rho}[\Psi_m^{(1)}(is_1\rho)] Y_{2m}(\vartheta,\varphi)$$

$$-cm(m+1)\sum_{m=0}^{\infty} \Psi_m^{(1)}(is_2\rho) Y_{3m},$$

Passing in (25) to the limit as $\rho \to R_1$ and taking into account boundary conditions we obtain for the determination of Y_{mj} the system of algebraic equations

$$\begin{cases} a(k_5+k_6)\dfrac{(m+1)(m+2)}{2m+1}Y_{1m}(\vartheta,\varphi) \\ +b\left\{(k_5+k_6)\left[\rho\dfrac{\partial^2}{\partial\rho^2}\Psi_m^{(1)}(is_1\rho)\right]_{\rho=R_1}+k_4s_1^2R_1\right\}Y_{2m}(\vartheta,\varphi) \\ -c(k_5+k_6)m(m+1)\left[\left(\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\Psi_m^{(1)}(is_2\rho)\right]_{\rho=R_1}Y_{3m}=h_{1m}^-, \\ -(k_5+k_6)m(m+1)\left\{-\dfrac{a(m+2)}{2m+1}Y_{1m}+b\left[\left(\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\Psi_m^{(1)}(is_2\rho)\right]_{\rho=R_1}Y_{2m}\right\} \\ +m(m+1)\left\{c(k_5+k_6)\left[\left(\rho\dfrac{\partial^2}{\partial\rho^2}+\dfrac{\partial}{\partial\rho}-\dfrac{1}{\rho}\right)\Psi_m^{(1)}(is_2\rho)\right]_{\rho=R_1}+k_5R_1\right\}Y_{3m}=h_{2m}^-, \\ k_5m(m+1)\left\{\left[\dfrac{\partial}{\partial\rho}\Psi_m^{(1)}(is_2\rho)\right]_{\rho=R_1}-\dfrac{1}{R_1}\right\}Y_{4m}=-h_{3m}^-, \\ \dfrac{kk_7s_1^2(m+1)}{k_2(2m+1)}Y_{1m}(\vartheta,\varphi)+\dfrac{ck_1}{R_1}m(m+1)Y_{3m}=-h_{4m}^-, \end{cases} \tag{26}$$

where

$$Y_{30}=Y_{40}=0, \quad h_{30}^-=h_{20}^-=0.$$

By virtue of Theorem 2 we conclude that the system (26) for $m\geq 0$ is uniquely solvable and the functions Y_{jm} can be expressed by the known functions h_{jm}^+.

Remark 5. Having defined φ_1, we find the solution of equation $\Delta\varphi_{10}=\varphi_1$

$$\varphi_{10}(\mathbf{x})=\frac{\rho^2}{2}\sum_{n=0}^{\infty}\frac{Y_{1n}(\vartheta,\eta)}{(1-4n^2)}\frac{R_1^{n+2}}{\rho^{n+1}}, \quad \rho>R_1.$$

Conclusions

By using the above-mentioned method, it is possible:

1. To construct explicitly the solutions of basic BVPs for systems (1)-(3) for simple cases of 2D domains (circle, plane with circular hole) in the form of absolutely and uniformly convergent series that are useful in the engineering practice;
2. To obtain numerical solutions of the boundary value problems;
3. To construct explicitly the solutions of basic BVPs of the systems of equations in the modern linear theories of elasticity, thermoelasticity and poroelasticity for materials with microstructures for a circle, for a sphere etc.
4. In practice, such BVPs are quite common in many areas of science. The potential users of the obtained results will be the scientists and engineers working on the problems of solid mechanics, micro and nanomechanics, mechanics of materials, engineering mechanics etc.

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