Seminar of I. Vekua Institute of Applied Mathematics REPORTS, Vol. 42, 2016

SOLUTION OF SOME BOUNDARY VALUE PROBLEMS OF STATICS OF THE THEORY OF ELASTIC MIXTURE IN AN INFINITE DOMAIN WITH AN ELLIPTICAL HOLE

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Abstract. For homogeneous equation of statics of the linear theory of elastic mixture in the case of an outside the elliptical domain we consider the two boundary value problems which are analogous to III and IV exterior boundary value problem of the classic theory of elasticity. Applying the representation of the stress vector by the so-called mutually adjoint vector functions we obtain effective solutions (Poisson type formulas) of the problems.

Keywords and phrases: Elastic mixture, singular integral equation with a Hilbert kernel, general representation of the displacement and stress vectors, analogues of the general Kolosov-Muskhelishvilis representations, adjount vector-function.

AMS subject classification (2010): 74E35, 74E20, 74C05.

1. Introduction

The basic two-dimensional boundary value problems statics of the linear theory of elastic mixtures are studied in [1], [3]-[7] and also by many other authors.

In the paper we consider two boundary value problems for homogeneous equation of statics of the linear theory of elastic mixtures in an infinite domain with an elliptical hole, which for the cases of simple connected finite and infinite domains has been studied by M. Basheleishvili in [5].

To solve the problems we use the method described in [2, §28] and [4]. Applying the representation of the stress vector by the so-called mutually adjoint vector-functions the problems are reduced to the singular integral equations with Hilbert kernels, and owing to the above result, the solution of the problems can be reduced to the first order linear differential equations.

The solutions of the problems are represented in the form of Poisson type formulas.

2. Some auxiliary formulas and operators

The homogeneous equation of statics of the theory of elastic mixtures in a complex form looks as follows [4]

$$\frac{\partial^2 U}{\partial z \partial \overline{z}} + K \frac{\partial^2 \overline{U}}{\partial \overline{z}^2} = 0,$$

where $z = x_1 + ix_2$, $\overline{z} = x_1 - ix_2$,

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right), \frac{\partial}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right),$$

$U=(u_1+iu_2,u_3+iu_4)^T$, $u^{'}=(u_1,u_2)^T$ and $u^{''}=(u_3,u_4)^T$ are partial displacements,

$$K=-\frac{1}{2}lm^{-1},\quad l=\begin{bmatrix} l_4 & l_5\\ l_5 & l_6\end{bmatrix},\quad m^{-1}=\frac{1}{\Delta_0}\begin{bmatrix} m_3 & -m_2\\ -m_2 & m_1\end{bmatrix},\quad \Delta_0=m_1m_3-m_2^2>0,$$

$$m_k=l_k+\frac{1}{2}l_{3+k},\quad k=1,2,3,\quad l_1=\frac{a_2}{d_2},\quad l_2=-\frac{c}{d_2},\quad l_3=\frac{a_1}{d_2},$$

$$a_1=\mu_1-\lambda_5,\quad a_2=\mu_2-\lambda_5,\quad c=\mu_3+\lambda_5,\quad d_2=a_1a_2-c^2,\quad l_1+l_4=\frac{a_2+b_2}{d_1},\tag{2.2}$$

$$l_2+l_5=-\frac{c+d}{d_1},\quad l_3+l_6=\frac{a_1+b_1}{d_1},\quad d_1=(a_1+b_1)(a_2+b_2)-(c+d)^2,$$

$$b_1=\mu_1+\lambda_1+\lambda_5-\alpha_2\frac{\rho_2}{\rho},\quad b_2=\mu_2+\lambda_2+\lambda_5+\alpha_2\frac{\rho_1}{\rho},\quad \rho=\rho_1+\rho_2,$$

$$\alpha_2=\lambda_3-\lambda_4,\quad d=\mu_3+\lambda_3-\lambda_5-\alpha_2\frac{\rho_1}{\rho}\equiv\mu_3+\lambda_4-\lambda_5+\alpha_2\frac{\rho_2}{\rho}.$$

Here μ_1,μ_2,μ_3 and $\lambda_p,\quad p=\overline{1,5}$ are elastic modules characterizing mechanical properties of a mixture, ρ_1 and ρ_2 are its particular densities. The elastic constants $\mu_1,\mu_2,\mu_3,\quad \lambda_p,\quad p=\overline{1,5}$ and particular densities ρ_1 and ρ_2 will be assumed to satisfy the conditions of inequality [1].

In [4] M. Basheleishvili obtained the following representations:

$$U=\begin{pmatrix} u_1+iu_2\\ u_3+iu_4\end{pmatrix}=m\varphi(z)+\frac{1}{2}\ lz\overline{\varphi'(z)}+\overline{\psi(z)},\tag{2.3}$$

$$TU=\begin{pmatrix} (Tu)_2-i(Tu)_1\\ (Tu)_4-i(Tu)_3\end{pmatrix}=\frac{\partial}{\partial S(x)}\left(-2\varphi(z)+2\mu U(x)\right),\tag{2.4}$$

where $\varphi=(\varphi_1,\varphi_2)^T$ and $\psi=(\psi_1,\psi_2)^T$ are arbitrary analytic vector-functions, $(TU)_p$ $(p=\overline{1,4})$ are components of the stress vector [1],

$$\mu=\begin{bmatrix} \mu_1 & \mu_3\\ \mu_3 & \mu_2\end{bmatrix},\quad m=\begin{bmatrix} m_1 & m_2\\ m_2 & m_3\end{bmatrix},\quad \det\mu=\Delta_1>0,$$

$\frac{\partial}{\partial S(x)}=n_1\frac{\partial}{\partial x_2}-n_2\frac{\partial}{\partial x_1},\quad n_1$ and n_2 are the projections of the unit vector of the normal onto the axes x_1 and x_2.

Formulas (2.3) and (2.4) are analogous to the Kolosov-Muskhelishvilis formulas for the linear theory of elastic mixture.

To investigate the problems we use the vector [4]

$$V=\begin{pmatrix} V_1+iV_2\\ V_3+iV_4\end{pmatrix}=i\left[-m\varphi(z)+\frac{1}{2}lz\overline{\varphi'(z)}+\overline{\psi(z)}\right].\tag{2.5}$$

As is known (see [4]) V is a vector adjoint to U.

From (2.3) , (2.4) and (2.5) we obtain

$$TU=\begin{pmatrix} (TU)_2-i(TU)_1\\ (TU)_4-i(TU)_3\end{pmatrix}=\frac{\partial}{\partial S(x)}\left[(2\mu-m^{-1})U-im^{-1}V\right].\tag{2.6}$$

3. Statement of the posed boundary value problems and the uniqueness theorems

Let an infinite isotropic plane be weakened by an elliptic hole with the semi-axis a and b ($a > b$). This unbound domain will be denoted by D^-. The symmetry axis of the ellipse is taken at the coordinate axis, and the major axis coincides with the real axis ox_1. By L we denote the elliptic curve $(a\cos\theta, b\sin\theta) \in L$.

We consider the following boundary value problems: Find in the domain D^- a vector $U = (u_1 + iu_2, u_3 + iu_4)^T$ which belongs to the class $C^2(D^-)\bigcap C^{1,\alpha}(D^-\bigcup L)$ is a solution of equation (2.1) and satisfies only one of the following conditions on the boundary L

$$(nU)^- = f^{(1)}, \qquad (STU)^- = f^{(2)}, \tag{3.1}$$

$$(SU)^- = F^{(1)}, \qquad (nTU)^- = F^{(2)}, \tag{3.2}$$

where $f^{(j)}$ and $F^{(j)}$, $\quad j = 1,2$ are the given scalar complex functions on the boundary L, note that

$$(f^{(1)}, F^{(1)}) \in C^{1,\alpha}(L), \quad (f^{(2)}, F^{(2)}) \in \sigma^{0,\alpha}(L), \quad \alpha > 0.$$

In the vicinity of infinity the vector $U = (u_1 + iu_2, u_3 + iu_4)^T$ satisfies the following conditions:

$$u_k = 0(1), \qquad |x|^2\frac{\partial u_k}{\partial x_j} = 0(1), \quad j = 1,2, \quad k = \overline{1,4}, \quad |x|^2 = x_1^2 + x_2^2.$$

It will be assumed that the stress and rotation components vanish at infinity; moreover, we suppose that the principal vector of external forces applied to the contour of the hole is equal to zero.

Let us denote by $(III_*)^-$ and $(IV_*)^-$ the problems (2.1), (3.1) and (2.1), (3.2) respectively.

The following assertion is true [5].

Theorem 3.1. *The problems $(III_*)^-$ and $(IV_*)^-$ are uniquely solvable.*

4. Solution of the $(III_*)^-$ and $(IV_*)^-$ problems

For the solution of the problems we use the method developed in [2]. Let us note that the solution of the first BVP of statics of the linear theory of elastic mixture for an infinite plane with an elliptic hole reads as ([7] or [3])

$$U(x) = \frac{1}{2\pi}\int_0^{2\pi} \frac{(1-\tau_1\overline{\tau_1})F(\theta)d\theta}{1-\tau_1 e^{i\theta} - \overline{\tau_1}e^{-i\theta} + \tau_1\overline{\tau_1}} - \frac{KA_0}{2\pi}\int_0^{2\pi}\frac{\overline{F(\theta)}\overline{\tau_1}e^{-i\theta}d\theta}{(1-\overline{\tau_1}e^{-i\theta})^2}, \tag{4.1}$$

where $U^- = F \in C^{1,\alpha}(L), \quad \alpha > 0, (a\cos\theta, \quad b\sin\theta) \in L; \quad K = -\frac{1}{2}lm^{-1}$ (see (2.2)),

$$A_0 = (1-\eta_1\overline{\eta_1})\left(\overline{\eta_1^{-1}} - \eta_2\right)(\overline{\eta_1} - \overline{\eta_2})^{-1}, \quad \tau_1 = \eta_1^{-1}, \quad |\tau_1| < 1,$$

$$\eta_1 = \frac{z + \sqrt{z^2 - a^2 + b^2}}{a+b}, \quad \eta_2 = \frac{z - \sqrt{z^2 - a^2 + b^2}}{a+b}, \quad z = x_1 + ix_2.$$

If $x = (x_1 x_2)$ belong to the boundary of the ellipse then $x_1 = a\cos\theta_0, x_2 = b\sin\theta_0$, and $\tau_1 = e^{-i\theta}$, $\overline{\tau_1} = e^{i\theta_0}$ and $A_0 = 0$.

Further, note that the adjoint vector of (4.1) has the form

$$V(x) = \begin{pmatrix} V_1 + iV_2 \\ V_3 + iV_4 \end{pmatrix} = \frac{1}{2\pi i}\int_0^{2\pi} \frac{(\tau_1 e^{i\theta} - \overline{\tau_1} e^{-i\theta})F(\theta)d\theta}{1 - \tau_1 e^{i\theta} - \overline{\tau_1} e^{-i\theta} + \tau_1\overline{\tau_1}} +$$

$$+ \frac{KA_0}{2\pi i}\int_0^{2\pi} \frac{\overline{F(\theta)}\overline{\tau_1} e^{-i\theta} d\theta}{(1 - \overline{\tau_1} e^{-i\theta})^2}. \tag{4.2}$$

1^0. A solution of the problem $(III)^-$ is sought in the form (see 4.1.)

$$U(x) = \frac{1}{2\pi}\int_0^{2\pi} \frac{(1 - \tau_1\overline{\tau_1})(nq + S\chi)d\theta}{1 - \tau_1 e^{i\theta} - \overline{\tau_1} e^{-i\theta} + \tau_1\overline{\tau_1}} - \frac{KA_0}{2\pi}\int_0^{2\pi} \frac{\overline{\tau_1} e^{-i\theta}(n\overline{q} + S\overline{\chi})d\theta}{(1 - \overline{\tau_1} e^{-i\theta})^2}, \tag{4.3}$$

where $(nU)^- = q = f^{(1)}$ is given by (3.1) and $(SU)^- = \chi$ is the unknown function

$$n = (n_1, n_2)^T = \frac{(b\cos\theta, a\sin\theta)^T}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}},$$

$$S = (-n_2, n_1)^T = \frac{(-a\sin\theta, b\cos\theta)^T}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}. \tag{4.4}$$

We remark also that, on $(a\cos\theta_0, b\sin\theta_0) \in L$

$$(U(\theta_0))^- = n(\theta_0)q(\theta_0) + S(\theta_0\chi(\theta_0)), \tag{4.5}$$

$$(V(\theta_0))^- = \int_0^{2\pi} ctg\frac{\theta - \theta_0}{2}\left[n(\theta)q(\theta) + S(\theta)\chi(\theta)\right]d\theta. \tag{4.6}$$

Using now (2.6) and taking into account (4.5) and (4.6) for the boundary value of the stress vector we obtain

$$\sqrt{a^2\sin^2\theta_0 + b^2 cos^2\theta_0}(TU)^- = (2\mu - m^{-1})\left(\frac{dU}{d\theta_0}\right)^-$$

$$+ \frac{m^{-1}}{2\pi i}\int_0^{2\pi} ctg\frac{\theta - \theta_0}{2}\left(\frac{dU}{d\theta}\right)^- d\theta. \tag{4.7}$$

If we take into account (4.4) and condition $(STU)^- = f^{(2)}$ (see(3.1)) then (4.7) can be rewritten in the form of one equation

$$\left[(2\mu - m^{-1})\left(\frac{dU}{d\theta_0}\right)^{-}\right]\begin{pmatrix} -a\sin\theta_0 \\ b\cos\theta_0 \end{pmatrix} + \left[\frac{m^{-1}}{2\pi i}\int_0^{2\pi} ctg\frac{\theta-\theta_0}{2}\left(\frac{dU}{d\theta}\right)^{-} d\theta\right]$$

$$\times\begin{pmatrix} -a\sin\theta_0 \\ b\cos\theta_0 \end{pmatrix} = (a^2\sin^2\theta_0 + b^2\cos^2\theta_0)f^{(2)}(\theta_0). \tag{4.8}$$

Represent U^- in the form (see (3.1) and (4.4))

$$(U(\theta_0))^{-} = \begin{pmatrix} b\cos\theta_0 \\ a\sin\theta_0 \end{pmatrix} f(\theta_0) + \begin{pmatrix} -a\sin\theta_0 \\ b\cos\theta_0 \end{pmatrix} h(\theta_0), \tag{4.9}$$

where

$$f(\theta_0) = \frac{f^{(1)}(\theta_0)}{\sqrt{a^2\sin^2\theta_0 + b^2\cos^2\theta_0}}, \qquad (4.10)_1$$

$$h(\theta_0) = \frac{(S(\theta_0)U(\theta_0))^{-}}{\sqrt{a^2\sin^2\theta_0 + b^2\cos^2\theta_0}} = \frac{\chi(\theta_0)}{\sqrt{a^2\sin^2\theta_0 + b^2\cos^2\theta_0}}. \qquad (4.10)_2$$

Substituting (4.9) in (4.8) after obvious transformations we get

$$\left[(2\mu - m^{-1})H'(\theta_0)\right]\begin{pmatrix} -a\sin\theta_0 \\ b\cos\theta_0 \end{pmatrix} + \left[\frac{m^{-1}}{2\pi i}\int_0^{2\pi} ctg\frac{\theta-\theta_0}{2}H'(\theta)d\theta\right]\begin{pmatrix} -a\sin\theta_0 \\ b\cos\theta_0 \end{pmatrix}$$

$$= \Phi(\theta_0), \tag{4.11}$$

where

$$H(\theta) = \begin{pmatrix} -a\sin\theta \\ b\cos\theta \end{pmatrix} h(\theta), \tag{4.12}$$

$$\Phi(\theta_0) = (a^2\sin^2\theta_0 + b^2\cos^2\theta_0)f^{(2)}(\theta_0) - (2\mu - m^{-1})\left[\begin{pmatrix} b\cos\theta_0 \\ a\sin\theta_0 \end{pmatrix} f(\theta_0)\right]'\begin{pmatrix} -a\sin\theta_0 \\ b\cos\theta_0 \end{pmatrix}$$

$$-\frac{m^{-1}}{2\pi i}\int_0^{2\pi} ctg\frac{\theta-\theta_0}{2}\left[\begin{pmatrix} b\cos\theta \\ a\sin\theta \end{pmatrix} f(\theta)\right]' d\theta \begin{pmatrix} -a\sin\theta_0 \\ b\cos\theta_0 \end{pmatrix}. \tag{4.13}$$

Bearing in mind the formulas

$$ctg\frac{\theta-\theta_0}{2}\begin{pmatrix} -a\sin\theta_0 \\ b\cos\theta_0 \end{pmatrix} = \begin{pmatrix} a\cos\theta + a\cos\theta_0 \\ b\sin\theta + b\sin\theta_0 \end{pmatrix} + ctg\frac{\theta-\theta_0}{2}\begin{pmatrix} -a\sin\theta \\ b\cos\theta \end{pmatrix},$$

after some calculations we can rewrite (4.11) in the form

$$\left[(2m\mu - E)H^{'}(\theta)\right] m^{-1} \begin{pmatrix} -a\sin\theta_0 \\ b\cos\theta_0 \end{pmatrix}$$

$$+\frac{1}{2\pi i}\int_0^{2\pi} ctg\frac{\theta-\theta_0}{2} H^{'}(\theta)m^{-1}\begin{pmatrix} -a\sin\theta \\ b\cos\theta \end{pmatrix} d\theta - iM = \Phi(\theta_0). \tag{4.14}$$

where

$$M = \frac{1}{2\pi}\int_0^{2\pi} H(\theta)m^{-1}\begin{pmatrix} -a\sin\theta \\ b\cos\theta \end{pmatrix} d\theta. \tag{4.15}$$

Applying the formula of composition of integrals with Hilbert kernels $(see[2], \S 28)$

$$\frac{1}{4\pi^2}\int_0^{2\pi} ctg\frac{\theta_0-\theta^*}{2}d\theta_0 \int_0^{2\pi} ctg\frac{\theta-\theta_0}{2}P(\theta)d\theta = -P(\theta^*) + \frac{1}{2\pi}\int_0^{2\pi} P(\theta)d\theta,$$

from (4.14) we find

$$H^{'}(\theta_0)m^{-1}\begin{pmatrix} -a\sin\theta_0 \\ b\cos\theta_0 \end{pmatrix} + \frac{1}{2\pi i}\int_0^{2\pi} ctg\frac{\theta-\theta_0}{2}\left[(2m\mu - E)H^{'}(\theta)\right] m^{-1}\begin{pmatrix} -a\sin\theta \\ b\cos\theta \end{pmatrix} d\theta$$

$$-N = \frac{1}{2\pi i}\int_0^{2\pi} ctg\frac{\theta-\theta_0}{2}\phi(\theta)d\theta, \tag{4.16}$$

where

$$N = \frac{1}{2\pi}\int_0^{2\pi} H^{'}(\theta)m^{-1}\begin{pmatrix} -a\sin\theta \\ b\cos\theta \end{pmatrix} d\theta. \tag{4.17}$$

The equalities (4.14) and (4.16) result in

$$\left[(2m\mu - 2E)H^{'}(\theta_0)\right] m^{-1}\begin{pmatrix} -a\sin\theta_0 \\ b\cos\theta_0 \end{pmatrix}$$

$$-\frac{1}{2\pi i}\int_0^{2\pi} ctg\frac{\theta_0-\theta^*}{2}\left[(2m\mu - 2E)H^{'}(\theta)\right] m^{-1}\begin{pmatrix} -a\sin\theta \\ b\cos\theta \end{pmatrix} d\theta + N - iM$$

$$= \phi(\theta_0) - \frac{1}{2\pi i})\int_0^{2\pi} ctg\frac{\theta-\theta_0}{2}\phi(\theta)d\theta \tag{4.18}$$

Thus, for determining $\left[(2m\mu - 2E)H^{'}(\theta)\right] m^{-1}\begin{pmatrix} -a\sin\theta \\ b\cos\theta \end{pmatrix}$ we have obtained a singular integral equation (4.18) with the Hilbert kernel.

Taking into account the fact that, when $f^{(1)} = f^{(2)} = 0$,then $U(x) = 0, x \in D^-$, (see theorem 3.1), also $\phi = 0, h = 0, H = 0$ and $M = N = 0$, (see $(4.10)_1$, $(4.10)_2$, (4.15) and (4.17)) we can conclude that solution of the equation (4.18) is

$$\left[(2m\mu - 2E)H^{'}(\theta)\right] m^{-1} \begin{pmatrix} -a\sin\theta \\ b\cos\theta \end{pmatrix} = \phi(\theta) - N + iM.$$

The last formula yields (see (4.12))

$$h^{'}(\theta) + \frac{1}{2}\frac{r^{'}(\theta)}{r(\theta)}h(\theta = \frac{\phi(\theta)}{r(\theta)} - \frac{N - iM}{r(\theta)}, \tag{4.19}$$

where

$$r(\theta) = 2\left[a^2\left(\mu_1 - \frac{m_3}{\Delta_0}\right)\sin^2\theta - ab\left(\mu_3 + \frac{m_2}{\Delta_0}\right)sin2\theta + b^2\left(\mu_2 - \frac{m_1}{\Delta_0}\right)\cos^2\theta\right] \neq 0,$$

$$0 \leq \theta \leq 2\pi. \tag{4.20}$$

Here (see [6])

$$\left(\mu_1 - \frac{m_3}{\Delta_0}\right)\left(\mu_2 - \frac{m_1}{\Delta_0}\right) - \left(\mu_3 + \frac{m_2}{\Delta_0}\right)^2 > 0, \quad \Delta_0 = m_1 m_3 - m_2^2 > 0. \tag{4.21}$$

From (4.19) by integration we obtain

$$h(\theta) = \frac{C}{\sqrt{r(\theta)}} + \frac{1}{\sqrt{r(\theta)}}\int_0^\theta \frac{\phi(\theta_0) - N + iM)}{\sqrt{r(\theta_0)}}d\theta_0, \tag{4.22}$$

where C is an arbitrary constant

As it is known conditions $f^{(1)} = f^{(2)} = 0$ imply that $U(x) = 0, x \in D^-$ and $\phi = H = h = M = N = 0$. Therefore from (4.22) we obtain $C = 0$ and finally

$$h(\theta) = \frac{1}{\sqrt{r(\theta)}}\int_0^\theta \frac{\phi(\theta_0) - N + iM}{\sqrt{r(\theta_0)}}d\theta. \tag{4.23}$$

Now let us find $N - iM$. Since $h(\theta)$ is periodic with the period 2π, i.e. $h(\theta + 2\pi) = h(\theta)$ (see (4.9) $(4.10)_1$ and $(4.10)_2$ and $r(2\pi) = r(0) \neq 0$ (see (4.20) and (4.21)) therefore from (4.23) we obtain

$$N - iM = \frac{\int_0^{2\pi}\phi(\theta)(r(\theta))^{-\frac{1}{2}}\ \ d\theta}{\int_0^{2\pi}(r(\theta))^{-\frac{1}{2}}\ \ d\theta}.$$

Having found $h(\theta)$ by formula $(4.10)_2$ we obtain value of $S(\theta)\chi(\theta)$ and after by (4.3) we obtain the solution of the problem $(III_*)^-$ represented in the form of Poisson type formula.

Thus the $(III_*)^-$ boundary value problem is solved. The BVP $(IV_*)^-$ is solved quite analogously.

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Received 16.03.2016; revised 09.07.2016; accepted 25.09.2016.

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