Seminar of I. Vekua Institute of Applied Mathematics
REPORTS, Vol. 42, 2016

ONE BOUNDARY VALUE PROBLEM FOR THE PLATES

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Abstract. In this work we consider equations of equilibrium of the isotropic elastic shell. By means of Vekua's method, the system of differential equations for thin and shallow shells is obtained, when on upper and lower face surfaces displacements are assumed to be known. The general solution for approximations $N = 1$ is constructed. The concrete problem is solved.

Keywords and phrases: Stress vectors, displacement vector, shallow shells.

AMS subject classification (2010): 74K25, 74B20.

1. Introduction

The refined theory of shells is constructed by reducing the three-dimensional problems of the theory of elasticity to the two-dimensional problems. I. Vekua had obtained the equations of shallow shells [1],[2]. It means that the interior geometry of the shell does not vary in thickness. This method for non-shallow shells in case of geometrical and physical nonlinear theory was generalized by T. Meunargia [3].

By means of Vekua's method, the system of differential equations for thin and shallow shells was obtained, when on upper and lower face surfaces displacements are assumed to be known [4].

The systems of equilibrium equations and stress-strain relations (Hooke's law) of the tow-dimensional shallow shells may be written in the following form [4]:

$$\begin{cases} \nabla_\alpha \overset{(m)}{\sigma}{}^{\alpha\beta} - b_\alpha^\beta \overset{(m)}{\sigma}{}^{\alpha 3} + \dfrac{2m+1}{h}\left(\overset{(m+1)}{\sigma}{}^{\beta 3} + \overset{(m+3)}{\sigma}{}^{\beta 3} + \ldots\right) + \overset{(m)}{\Phi}{}^{\beta} = 0, \\ \nabla_\alpha \overset{(m)}{\sigma}{}^{\alpha 3} + b_\alpha^\beta \overset{(m)}{\sigma}{}_\beta^\alpha + \dfrac{2m+1}{h}\left(\overset{(m+1)}{\sigma}{}^{33} + \overset{(m+3)}{\sigma}{}^{33} + \ldots\right) + \overset{(m)}{\Phi}{}^{3} = 0, \end{cases} \tag{1}$$

where

$$\begin{cases} \overset{(m)}{\sigma}{}^{\alpha\beta} = \lambda\left[\nabla_\gamma \overset{(m)}{u}{}^{\gamma} - 2H \overset{(m)}{u}{}^{3} - \dfrac{2m+1}{h}\left(\overset{(m-1)}{u}{}^{3} + \overset{(m-3)}{u}{}^{3} + \cdots\right)\right] a^{\alpha\beta} \\ +\mu\left(\nabla^\beta \overset{(m)}{u}{}^{\alpha} + \nabla^\alpha \overset{(m)}{u}{}^{\beta} - 2b^{\alpha\beta} \overset{(m)}{u}{}^{3}\right) + \lambda\dfrac{2m+1}{h}\left(\overset{(+)}{u}{}_3 - (-1)^m \overset{(-)}{u}{}_3\right) a^{\alpha\beta}, \\ \overset{(m)}{\sigma}{}^{\alpha 3} = \mu\left[\nabla^\alpha \overset{(m)}{u}{}^{3} + b_\beta^\alpha \overset{(m)}{u}{}^{\beta} - \dfrac{2m+1}{h}\left(\overset{(m-1)}{u}{}^{\alpha} + \overset{(m-3)}{u}{}^{\alpha} + \cdots\right)\right] \\ +\mu\dfrac{2m+1}{h}\left(\overset{(+)}{u}{}^{\alpha} - (-1)^m \overset{(-)}{u}{}^{\alpha}\right), \\ \overset{(m)}{\sigma}{}^{33} = \lambda\left(\nabla_\gamma \overset{(m)}{u}{}^{\gamma} - 2H \overset{(m)}{u}{}^{3}\right) - (\lambda+2\mu)\dfrac{2m+1}{h}\left(\overset{(m-1)}{u}{}^{3} + \overset{(m-3)}{u}{}^{3} + \cdots\right) \\ +(\lambda+2\mu)\dfrac{2m+1}{h}\left(\overset{(+)}{u}{}_3 - (-1)^m \overset{(-)}{u}{}_3\right). \end{cases} \tag{2}$$

Here λ and μ are Lame's constants, ∇_α are covariant derivatives on the midsurface, $a^{\alpha\beta}$ and $b^{\alpha\beta}$ are the contravariant components of the metric tensor and curvature tensor of the midsurface, H is middle curvature of the midsurface and

$$\left(\overset{(m)}{\sigma}{}^{ij}, \overset{(m)}{u}{}^{i}, \overset{(m)}{\Phi}{}^{i}\right) = \frac{2m+1}{2h}\int_{-h}^{h} \left(\sigma^{ij}, u^i, \Phi^i\right) P_m\left(\frac{x_3}{h}\right) dx_3,$$

$$(m = 0, 1, 2, \cdots)$$

$$\overset{(\pm)}{u}{}^{i} = u^i(x^1, x^2, \pm h),$$

where σ^{ij} are contravariant components of the stress vectors, u^i are contravariant components of the displacement vector, Φ^i are contravariant components of the volume force, $P_m\left(\frac{x^3}{h}\right)$ are Legendre polynomials, x^1, x^2 are the Gaussian parameters of the midsurfaces, $x^3 = x_3$ is the thickness coordinate and h is the semi-thickness. So, we have the infinite system.

An infinite system of equations (1) has the advantage that it contains two independent variables - Gaussian coordinates x^1, x^2 of the midsurface. But the decrease in the number of independent variables is achieved by increasing the number of equations to infinity, which, naturally, has an obvious practical inconvenience. Therefore it is necessary to make the next step for a further simplification of the problem.

2. $N = 1$ approximation for plates

we consider $N = 1$ approximation for plates. In other words, in the previous equations it is assumed that

$$\overset{(m)}{\sigma}{}^{ij} = 0, \quad \overset{(m)}{u}{}^{i} = 0, \quad if\ m > 1.$$

As a result we obtain a finite system of equilibrium equations

$$\begin{cases} \partial_\alpha \overset{(0)}{\sigma}_{\alpha\beta} + \dfrac{1}{h}\overset{(1)}{\sigma}_{\beta 3} + \overset{(0)}{\Phi}_{\beta} = 0, \\ \partial_\alpha \overset{(0)}{\sigma}_{\alpha 3} + \dfrac{1}{h}\overset{(1)}{\sigma}_{33} + \overset{(0)}{\Phi}_{3} = 0, \end{cases} \tag{3}$$

$$\begin{cases} \partial_\alpha \overset{(1)}{\sigma}_{\alpha\beta} + \overset{(1)}{\Phi}_{\beta} = 0, \\ \partial_\alpha \overset{(1)}{\sigma}_{\alpha 3} + \overset{(1)}{\Phi}_{3} = 0, \end{cases} \tag{4}$$

where

$$\begin{cases} \overset{(0)}{\sigma}_{\alpha\beta} = \lambda\left(\partial_\gamma \overset{(0)}{u}_{\gamma}\right)\delta_{\alpha\beta} + \mu\left(\partial_\beta \overset{(0)}{u}_{\alpha} + \partial_\alpha \overset{(0)}{u}_{\beta}\right) + \dfrac{\lambda}{h}\left(\overset{(+)}{u}_{3} - \overset{(-)}{u}_{3}\right)\delta_{\alpha\beta}, \\ \overset{(0)}{\sigma}_{\alpha 3} = \mu\left(\partial_\alpha \overset{(0)}{u}_{3}\right) + \dfrac{\mu}{h}\left(\overset{(+)}{u}_{\alpha} - \overset{(-)}{u}_{\alpha}\right), \\ \overset{(0)}{\sigma}_{33} = \lambda\left(\partial_\gamma \overset{(0)}{u}_{\gamma}\right) + \dfrac{\lambda + 2\mu}{h}\left(\overset{(+)}{u}_{3} - \overset{(-)}{u}_{3}\right), \end{cases} \tag{5}$$

$$\begin{cases} \overset{(1)}{\sigma}_{\alpha\beta} = \lambda\left(\partial_\gamma \overset{(1)}{u}_\gamma - \dfrac{3}{h}\overset{(0)}{u}_3\right)\delta_{\alpha\beta} + \mu\left(\partial_\beta \overset{(1)}{u}_\alpha + \partial_\alpha \overset{(1)}{u}_\beta\right) + \dfrac{3\lambda}{h}\left(\overset{(+)}{u}_3 + \overset{(-)}{u}_3\right)\delta_{\alpha\beta}, \\ \overset{(1)}{\sigma}_{\alpha 3} = \mu\left(\partial_\alpha \overset{(1)}{u}_3 - \dfrac{3}{h}\overset{(0)}{u}_\alpha\right) + \dfrac{3\mu}{h}\left(\overset{(+)}{u}_\alpha + \overset{(-)}{u}_\alpha\right), \\ \overset{(1)}{\sigma}_{33} = \lambda\left(\partial_\gamma \overset{(1)}{u}_\gamma\right) - \dfrac{3(\lambda+2\mu)}{h}\overset{(0)}{u}_3 + \dfrac{3(\lambda+2\mu)}{h}\left(\overset{(+)}{u}_3 + \overset{(-)}{u}_3\right). \end{cases} \tag{6}$$

Substituting these expressions (5) and (6) into equation (3) and (4), we obtain the system of second-order partial differential equations:

$$\begin{cases} \mu\Delta \overset{(0)}{u}_1 + (\lambda+\mu)\partial_1 \overset{(0)}{\theta} + \dfrac{1}{h}\left(\mu\partial_1 \overset{(1)}{u}_3 - \dfrac{3\mu}{h}\overset{(0)}{u}_1\right) = \overset{(0)}{\Psi}_1, \\ \mu\Delta \overset{(0)}{u}_2 + (\lambda+\mu)\partial_2 \overset{(0)}{\theta} + \dfrac{1}{h}\left(\mu\partial_2 \overset{(1)}{u}_3 - \dfrac{3\mu}{h}\overset{(0)}{u}_2\right) = \overset{(0)}{\Psi}_2, \\ \mu\Delta \overset{(0)}{u}_3 + \dfrac{1}{h}\left(\lambda \overset{(1)}{\theta} - \dfrac{3(\lambda+2\mu)}{h}\overset{(0)}{u}_3\right) = \overset{(0)}{\Psi}_3, \end{cases} \tag{7}$$

$$\begin{cases} \mu\Delta \overset{(1)}{u}_1 + (\lambda+\mu)\partial_1 \overset{(1)}{\theta} - \dfrac{3\lambda}{h}\partial_1 \overset{(0)}{u}_3 = \overset{(1)}{\Psi}_1, \\ \mu\Delta \overset{(1)}{u}_2 + (\lambda+\mu)\partial_2 \overset{(1)}{\theta} - \dfrac{3\lambda}{h}\partial_2 \overset{(0)}{u}_3 = \overset{(1)}{\Psi}_2, \\ \mu\Delta \overset{(1)}{u}_3 - \dfrac{3\mu}{h}\overset{(0)}{\theta} = \overset{(1)}{\Psi}_3, \end{cases} \tag{8}$$

where $\overset{(m)}{\Psi}_i$ are the known values and

$$\overset{(m)}{\theta} = \partial_1 \overset{(m)}{u}_1 + \partial_2 \overset{(m)}{u}_2, \quad m = 0, 1.$$

Introducing the well-known differential operators

$$\partial_z = \frac{1}{2}(\partial_1 - i\partial_2), \quad \partial_{\bar z} = \frac{1}{2}(\partial_1 + i\partial_2),$$

where $z = x_1 + ix_2$.

System (7) and (8) can be written in the complex form:

a) for the tension-pressure of plates

$$\begin{cases} \mu\Delta \overset{(0)}{u}_+ + 2(\lambda+\mu)\partial_{\bar z} \overset{(0)}{\theta} + \dfrac{1}{h}\left(2\mu\partial_{\bar z} \overset{(1)}{u}_3 - \dfrac{3\mu}{h}\overset{(0)}{u}_+\right) = \overset{(0)}{\Psi}_+, \\ \mu\Delta \overset{(1)}{u}_3 - \dfrac{3\mu}{h}\overset{(0)}{\theta} = \overset{(1)}{\Psi}_3, \end{cases} \tag{9}$$

b) for the bending of plates

$$\begin{cases} \mu\Delta \overset{(1)}{u}_+ + 2(\lambda+\mu)\partial_{\bar z} \overset{(1)}{\theta} - \dfrac{6\lambda}{h}\partial_{\bar z} \overset{(0)}{u}_3 = \overset{(1)}{\Psi}_+, \\ \mu\Delta \overset{(0)}{u}_3 + \dfrac{1}{h}\left(\lambda \overset{(1)}{\theta} - \dfrac{3(\lambda+2\mu)}{h}\overset{(0)}{u}_3\right) = \overset{(0)}{\Psi}_3, \end{cases} \tag{10}$$

where $\Delta = 4\dfrac{\partial^2}{\partial z \partial \bar{z}}$ and

$$\overset{(m)}{u}_+ = \overset{(m)}{u}_1 + i\,\overset{(m)}{u}_2, \quad \overset{(m)}{\theta} = \partial_z \overset{(m)}{u}_z + \partial_{\bar{z}} \overset{(m)}{\bar{u}}_+, \quad \overset{(m)}{\Psi}_+ = \overset{(m)}{\Psi}_1 + i\,\overset{(m)}{\Psi}_2.$$

The complex representation of the general solutions of the homogenous systems (9) and (10) are written in the following form [2, 5]:

$$\begin{cases} \overset{(0)}{u}_+ = f(z) + z\overline{f'(z)} + \dfrac{4(\lambda+2\mu)h^2}{3\mu}\overline{f''(z)} + \overline{g'(z)} - \dfrac{ih}{3}\dfrac{\partial\omega(z,\bar{z})}{\partial\bar{z}}, \\ \overset{(1)}{u}_3 = \dfrac{3}{2h}\left(\bar{z}f(z) + z\overline{f(z)}\right) + \dfrac{3}{2h}\left(g(z) + \overline{g(z)}\right), \end{cases} \tag{11}$$

$$\begin{cases} \overset{(1)}{u}_+ = \dfrac{5\lambda+6\mu}{3\lambda+2\mu}\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)} + \dfrac{\lambda h}{2(\lambda+\mu)}\dfrac{\partial\chi(z,\bar{z})}{\partial\bar{z}}, \\ \overset{(0)}{u}_3 = \chi(z,\bar{z}) + \dfrac{2\lambda h}{3(3\lambda+2\mu)}\left(\varphi'(z) + \overline{\varphi'(z)}\right), \end{cases} \tag{12}$$

where $f(z)$, $g(z)$, $\varphi(z)$ and $\psi(z)$ are any analytic functions of z, $\omega(z,\bar{z})$ and $\chi(z,\bar{z})$ are the general solutions of the following Helmholtz's equations, respectively:

$$\Delta\omega - \gamma^2\omega = 0, \quad \left(\gamma^2 = \frac{3}{h^2}\right),$$

$$\Delta\chi - \nu^2\chi = 0, \quad \left(\chi^2 = \frac{12(\lambda+\mu)h^2}{\lambda+2\mu}\right).$$

From eqs. (5), (6) the following relations follow

$$\begin{cases} \overset{(0)}{\sigma}_{11} + \overset{(0)}{\sigma}_{22} = 2(\lambda+\mu)\,\overset{(0)}{\theta}, & \overset{(0)}{\sigma}_{11} - \overset{(0)}{\sigma}_{22} + 2i\,\overset{(0)}{\sigma}_{12} = 4\mu\partial_{\bar{z}}\,\overset{(0)}{u}_+, \\ \overset{(1)}{\sigma}_{11} + \overset{(1)}{\sigma}_{22} = 2(\lambda+\mu)\,\overset{(1)}{\theta} - \dfrac{6\lambda}{h}\,\overset{(0)}{u}_3, & \overset{(1)}{\sigma}_{11} - \overset{(1)}{\sigma}_{22} + 2i\,\overset{(1)}{\sigma}_{12} = 4\mu\partial_{\bar{z}}\,\overset{(1)}{u}_+, \\ \overset{(0)}{\sigma}_{13} + i\,\overset{(0)}{\sigma}_{23} = 2\mu\partial_{\bar{z}}\,\overset{(0)}{u}_3, & \overset{(1)}{\sigma}_{13} + i\,\overset{(1)}{\sigma}_{23} = 2\mu\partial_{\bar{z}}\,\overset{(1)}{u}_3 - \dfrac{3\mu}{h}\,\overset{(0)}{u}_+. \end{cases} \tag{13}$$

3. The solution of the boundary problem for the circle

Let us solve the problem when the midsurface of the body is the circle with the radius R.

The boundary problem (in stresses) takes the form [3]:

$$\begin{cases} \overset{(m)}{\sigma}_{rr} + i\,\overset{(m)}{\sigma}_{r\alpha} = \dfrac{1}{2}\left[\overset{(m)}{\sigma}_{11} + \overset{(m)}{\sigma}_{22} - \left(\overset{(m)}{\sigma}_{11} - \overset{(m)}{\sigma}_{22} + 2i\,\overset{(m)}{\sigma}_{12}\right)\left(\dfrac{d\bar{z}}{ds}\right)^2\right] = \overset{(m)}{F}_+, \\ \overset{(m)}{\sigma}_{rn} = -\mathrm{Im}\left(\overset{(m)}{\sigma}_{+3}\dfrac{d\bar{z}}{ds}\right) = \overset{(m)}{F}_3, \quad \left(\overset{(m)}{\sigma}_{+3} = \overset{(m)}{\sigma}_{13} + i\,\overset{(m)}{\sigma}_{23}\right). \end{cases} \tag{14}$$

Using eqs. (12) and (13) the boundary conditions are written as

$$\begin{cases} (\lambda+\mu)(f'(z)+\overline{f'(z)})+\Big(2\mu z\overline{f''(z)}+\dfrac{8(\lambda+2\mu)}{3}\overline{f'''(z)} \\ +2\mu\overline{g''(z)}-\dfrac{2\mu ih}{3}\dfrac{\partial^2\omega(z,\bar z)}{\partial\bar z^2}\Big)e^{-2i\alpha}=\displaystyle\sum_{-\infty}^{+\infty}A_{n1}e^{in\alpha},\quad r=R, \\ \dfrac{\mu}{2h}\Big(ih\dfrac{\partial\omega}{\partial\bar z}-\dfrac{4(\lambda+2\mu)h^2}{3\mu}\overline{f''(z)}\Big)e^{-i\alpha} \\ -\dfrac{\mu}{2h}\Big(ih\dfrac{\partial\omega(z,\bar z)}{\partial\bar z}+\dfrac{4(\lambda+2\mu)h^2}{3\mu}\overline{f''(z)}\Big)e^{i\alpha}=\displaystyle\sum_{-\infty}^{+\infty}B_{n1}e^{in\alpha},\quad r=R, \end{cases} \tag{15}$$

$$\begin{cases} 2\mu(\varphi'(z)+\overline{\varphi'(z)})-\dfrac{3\lambda\mu}{(\lambda+2\mu)h}\chi(z,\bar z) \\ +2\mu\Big(\dfrac{\lambda h}{2(\lambda+\mu)}\dfrac{\partial^2\chi(z,\bar z)}{\partial\bar z^2}-z\overline{\varphi''(z)}-\overline{\psi'(z)}\Big)e^{-2i\alpha}=\displaystyle\sum_{-\infty}^{+\infty}A_{n2}e^{in\alpha},\quad r=R, \\ \Big(\mu\dfrac{\partial\chi(z,\bar z)}{\partial\bar z}+\dfrac{2\lambda\mu h}{3(3\lambda+2\mu)}\overline{\varphi''(z)}\Big)e^{-i\alpha} \\ +\Big(\mu\dfrac{\partial\chi(z,\bar z)}{\partial z}+\dfrac{2\lambda\mu h}{3(3\lambda+2\mu)}\varphi''(z)\Big)e^{i\alpha}=\displaystyle\sum_{-\infty}^{+\infty}B_{n2}e^{in\alpha},\quad r=R. \end{cases} \tag{16}$$

Inside the domain the analytic functions $f(z)$, $g(z)$, $\varphi(z)$ and $\psi(z)$ will have the following form:

$$f(z)=\sum_{n=1}^{+\infty}a_ne^{in\alpha},\qquad g(z)=\sum_{n=0}^{+\infty}b_ne^{in\alpha}, \tag{17}$$

$$\varphi(z)=\sum_{n=1}^{+\infty}c_ne^{in\alpha},\qquad \psi(z)=\sum_{n=1}^{+\infty}d_ne^{in\alpha}. \tag{18}$$

Solutions of the Helmholtz equations $\omega(z,\bar z)$ and $\chi(z,\bar z)$ inside of the domain are represented as follows

$$\omega(z,\bar z)=\sum_{-\infty}^{+\infty}\alpha_nI_n(\gamma r)e^{in\alpha}, \tag{19}$$

$$\chi(z,\bar z)=\sum_{-\infty}^{+\infty}\beta_nI_n(\nu r)e^{in\alpha}, \tag{20}$$

where $I_n(\cdot)$ are Bessel's modified functions.

In the boundary conditions (15) we substitute the corresponding expressions (17), (19) and compare the coefficients at identical degrees. We obtain the following system of equations

$$\begin{cases} (\lambda+\mu)(n+1)R^na_{n+1}-\dfrac{\mu i}{2h}I_{n+2}(\gamma R)\alpha_n=A_{n1}, \\ \dfrac{i\mu\gamma}{4}\Big(I_{n+1}(\gamma R)-I_{n-1}(\gamma R)\Big)\alpha_n-2(\lambda+2\mu)n(n+1)R^{n-1}a_{n+1}=B_{n1} \\ \left[(\lambda+\mu)R^n+2\mu nR^{n-1}+\dfrac{8(\lambda+2\mu)h^2}{3}(n-1)nR^{n-2}\right](n+1)a_{n+1} \\ +2\mu(n-1)nb_n=\bar A_{-n1}. \end{cases} \tag{21}$$

The solutions of the system (21) have the following forms:

$$\mathrm{Re}a_1 = \frac{\mathrm{Re}A_{01}}{2(\lambda+\mu)}, \quad \alpha_0 = -\frac{2h\mathrm{Im}A_{01}}{\mu I_2(\gamma R)},$$

$$a_{n+1} = \frac{2I_{n+2}(\gamma R)B_{n1} + (I_{n+1}(\gamma R) - I_{n-1}(\gamma R))\gamma h A_{n1}}{(n+1)R^{n-1}((\lambda+\mu)(I_{n+1}(\gamma R) - I_{n-1}(\gamma R))\gamma h R - 4(\lambda+2\mu)nI_{n+2}(\gamma R))},$$

$$\alpha_n = \frac{2h[(n+1)(\lambda+\mu)R^n a_{n+1} - A_{n1}]}{\mu i I_{n+2}(\gamma R)},$$

$$b_n = \frac{\bar{A}_{-n1}}{2\mu n(n-1)} - \left[\frac{(\lambda+\mu)R^n}{2\mu n(n-1)} + \frac{R^{n-1}}{n-1} + \frac{4(\lambda+2\mu)h^2R^{n-2}}{3}\right](n+1)a_{n+1}.$$

Now by substituting (18), (20) into (16) we obtain the system of algebraic equations:

$$\begin{cases} \dfrac{3\lambda\mu}{(\lambda+2\mu)h}\Big(I_{n+2}(\nu R) - I_n(\nu R)\Big)\beta_n + 2\mu(n+1)R^n c_{n+1} = A_{n2}, \\ \dfrac{\mu\nu}{2}\Big(I_{n+1}(\nu R) + I_{n-1}(\nu R)\Big)\beta_n + \dfrac{2\lambda\mu h}{3(3\lambda+2\mu)}n(n+1)c_{n+1} = B_{n2}, \\ \dfrac{3\lambda\mu}{(\lambda+2\mu)h}\Big(I_{n-2}(\nu R) - I_n(\nu R)\Big)\beta_n + 2\mu(n+1)(1-n)R^n c_{n+1} \\ -2\mu(n-1)R^{n-2}d_{n-1} = \bar{A}_{-n2}. \end{cases} \tag{22}$$

For coefficients c_n, d_n and β_n we have:

$$c_{n+1} = \frac{(3\lambda+2\mu)[6\lambda I'_n(\nu R)B_{n2} - (\lambda+2\mu)h^2 I''_n(\nu R)A_{n2}]}{4\lambda^2\mu h n(n+1)R^{n-1}I'_n(\nu R) - 2(\lambda+2\mu)(3\lambda+2\mu)\mu h^2(n+1)R^n I''_n(\nu R)},$$

$$\beta_n = \frac{(\lambda+2\mu)h(A_{n2} - 2\mu(n+1)R^n c_{n+1})}{3\lambda\mu I'_n(\nu R)}$$

$$d_{n-1} = \frac{3\lambda\Big(I_{n-2}(\nu R) - I_n(\nu R)\Big)}{2(\lambda+2\mu)h(n-1)R^{n-2}}\beta_n - (n+1)R^2 c_{n+1} - \frac{\bar{A}_{-n2}}{2\mu(n-1)R^{n-2}},$$

$$\beta_0 = \frac{B_{02}}{\mu\nu I_1(\nu R)}, \quad \mathrm{Re}c_1 = \frac{\mathrm{Re}A_{02}}{4\mu} - \frac{3\lambda I'_0(\nu R)B_{02}}{(\lambda+2\mu)\nu h I_1(\nu R)},$$

where

$$I'_n(\nu R) = I_{n+2}(\nu R) - I_n(\nu R), \quad I''_n(\nu R) = I_{n+1}(\nu R) + I_{n-1}(\nu R).$$

It is easy to prove that the absolute and uniform convergence of the series obtained in the circle (including the contours) when the functions set on the boundaries have sufficient smoothness.

Acknowledgement. The designated project has been fulfilled by a financial support of Shota Rustaveli National Science Foundation (Grant SRNSF/FR /358/5-109/14). Any idea in this publication is possessed by the author and may not represent the opinion of Shota Rustaveli National Science Foundation itself.

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Received 10.05.2016; revised 15.10.2016; accepted 16.11.2016.

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