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THE PROBLEM OF STATICS OF THE THEORY OF ELASTIC MIXTURE OF FINDING FULL-STRENGTH CONTOUR INSIDE THE POLYGON

Svanadze K.

Abstract. In the present work we consider the problem of statics of the linear theory of elastic mixture of finding a full-strength contour for a finite doubly-connected domain whose outer boundary is a convex polygon, while the inner boundary is a smooth closed contour. It is assumed that absolutely smooth rigid punches are applied to every link of the polygon. The punches are under the action of external normal contractive forces. The goal of the problem is to find an unknown contour under the condition that tangential normal stress vector on it takes constant value.

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1. Introduction

The problems of the plane theory of elasticity for infinite domains weakened by equally strong holes have been studied by many authors, particularly in [1], [9] the same problem for simple and doubly-connected domains with partially unknown boundaries are investigated in [2], [10] etc. The mixed boundary value problems of the plane theory of elasticity for domain with partially unknown boundaries have been studied by R. Bantsuri [3]. Analogous problem in the case of the plane theory of elastic mixtures is considered in [15].

In [14] using the method suggested by R. Bantsury in [4], the author gives a solution of the mixed problem of the plane theory of elasticity for a finite multiply connected domain with a partially unknown boundary having the axis of symmetry. Analogous problem in the case of the plane theory of elastic mixtures has been studied in [16]. The problem of statics of the plane theory of elasticity of finding an equally strong contour for square which is weakened by a hole and by cuttings at vertices have been investigated in [5] by R. Bantsuri and G. Kapanadze. The analogous problem in the case of the plane theory of elastic mixtures has been studied in [17].

In the work of R. Bantsuri and G. Kapanadze [6] the problem of statics of the plane theory of elasticity of finding a full-strength contour inside the polygon are considered.

In the present paper in the case of the plane theory of elastic mixtures we study the problem analogous to that solved in [6]. For the solution of the problem the use will be made of the generalized Kolosov-Muskhelishvili's formula [17] and the method developed in [6].

2. Some auxiliary formulas and operators

The homogeneous equation of statics of the theory of elastic mixtures in a complex form looks as follows [8]

$$\frac{\partial^2 U}{\partial z \partial \overline{z}} + K \frac{\partial^2 \overline{U}}{\partial \overline{z}^2} = 0, \tag{2.1}$$

where $z = x_1 + ix_2$, $\overline{z} = x_1 - ix_2$,

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right), \quad \frac{\partial}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right),$$

$$U = (u_1 + iu_2, u_3 + iu_4)^T, \quad u^{'} = (u_1, u_2)^T \quad and \quad u^{''} = (u_3, u_4)^T \quad ,$$

are partial displacements,

$$K = -\frac{1}{2}em^{-1}, \quad e = \begin{bmatrix} e_4 & e_5 \\ e_5 & e_6 \end{bmatrix}, \quad m^{-1} = \frac{1}{\triangle_0}\begin{bmatrix} m_3 & -m_2 \\ -m_2 & m_1 \end{bmatrix}, \quad \triangle_0 = m_1 m_3 - m_2^2,$$

$$m_k = e_k + \frac{1}{2}e_{3+k}, \quad e_1 = a_2/d_2 \quad e_2 = -c/d_2, \quad e_3 = a_1/d_2, \quad d_2 = a_1 a_2 - c^2,$$

$$a_1 = \mu_1 - \lambda_5, \quad a_2 = \mu_2 - \lambda_5, \quad c = \mu_3 + \lambda_5, \quad e_1 + e_4 = b/d_1, \quad e_2 + e_5 = -c_0/d_1,$$

$$e_3 + e_6 = a/d_1, \quad d_1 = ab - c_0^2, \quad b_1 = \mu_1 + \lambda_1 + \lambda_5 - \alpha_2\rho_2/\rho,$$

$$b_2 = \mu_2 + \lambda_2 + \lambda_5 + \alpha_2\rho_1/\rho, \quad \alpha_2 = \lambda_3 - \lambda_4, \quad \rho = \rho_1 + \rho_2, \quad a = a_1 + b_1, \quad b = a_2 + b_2$$

$$c_0 = c + d, \quad d = \mu_2 + \lambda_3 - \lambda_5 - \alpha_2\rho_1/\rho \equiv \mu_3 + \lambda_4 - \lambda_5 + \alpha_2\rho_2/\rho.$$

Here μ_1, μ_2, μ_3, λ_p, $p = \overline{1,5}$ are elasticity modules characterizing mechanical properties of a mixture, ρ_1 and ρ_2 are its particular densities. The elastic constants μ_1, μ_2, μ_3, λ_p, $p = \overline{1,5}$ and particular densities ρ_1 and ρ_2 will be assumed to satisfy the conditions of the inequality [13].

In [7] M. Basheleishvili obtained the following representations:

$$U = \begin{pmatrix} u_1 + iu_2 \\ u_3 + iu_4 \end{pmatrix} = m\varphi(z) + \frac{1}{2}ze\overline{\varphi'(z)} + \overline{\psi(z)}, \tag{2.2}$$

$$TU = \begin{pmatrix} (TU)_2 - i(TU)_1 \\ (TU)_4 - i(TU)_3 \end{pmatrix} = \frac{\partial}{\partial S(x)}\left[(A - 2E)\varphi(z) + Bz\overline{\varphi'(z)} + 2\mu\overline{\psi(z)}\right], \tag{2.3}$$

where $\varphi = (\varphi_1, \varphi_2)^T$ and $\psi = (\psi_1, \psi_2)^T$ are arbitrary analytic vector-functions;

$$A = 2\mu m, \quad \mu = \begin{bmatrix} \mu_1 & \mu_3 \\ \mu_3 & \mu_2 \end{bmatrix} \quad B = \mu e, \quad m = \begin{bmatrix} m_1 & m_2 \\ m_2 & m_3 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\frac{\partial}{\partial(x)} = -n_2\frac{\partial}{\partial x_1} + n_1\frac{\partial}{\partial x_2}, \quad \frac{\partial}{\partial n(x)} = n_1\frac{\partial}{\partial x_1} + n_2\frac{\partial}{\partial x_2},$$

$n = (n_1, n_2)^T$ is the unit vector of the outer normal, $(TU)_p, p = \overline{1,4}$, the stress components [7]

$$(TU)_1 = r'_{11}n_1 + r'_{21}n_2, \quad (TU)_2 = r'_{12}n_1 + r'_{22}n_2,$$
$$(TU)_3 = r''_{11}n_1 + r''_{21}n_2, \quad (TU)_4 = r''_{12}n_1 + r''_{22}n_2,$$
$$r'_{11} = a\theta' + c_0\theta'' - 2\frac{\partial}{\partial x_2}(\mu_1 u_2 + \mu_3 u_4), \quad r'_{21} = -a_1\omega' - c\omega'' + 2\frac{\partial}{\partial x_1}(\mu_1 u_2 + \mu_3 u_4),$$
$$r'_{12} = a_1\omega' + c\omega'' + 2\frac{\partial}{\partial x_2}(\mu_1 u_1 + \mu_3 u_3), \quad r'_{22} = a\theta' + c_0\theta'' - 2\frac{\partial}{\partial x_1}(\mu_1 u_1 + \mu_3 u_3),$$
$$r''_{11} = c_0\theta' + b\theta'' - 2\frac{\partial}{\partial x_2}(\mu_3 u_2 + \mu_2 u_4), \quad r''_{21} = -c\omega' - a_2\omega'' + 2\frac{\partial}{\partial x_1}(\mu_3 u_2 + \mu_2 u_4),$$
$$r''_{12} = c\omega' + a_2\omega'' + 2\frac{\partial}{\partial x_2}(\mu_3 u_1 + \mu_2 \mu_3), \quad r''_{22} = c_0\theta' + b\theta'' - 2\frac{\partial}{\partial x_1}(\mu_3 u_1 + \mu_2 u_3),$$
$$\theta'' = duv\nu', \quad \theta'' = duv\nu'', \quad \omega' = rotu', \omega'' = rotu''.$$

Introduce the vectors:

$$\tau^{(1)} = (r'_{11}, r''_{11})^T, \tau^{(2)} = (r'_{22}, r''_{22})^T, \tau = \tau^{(1)} + \tau^{(2)}, \tag{2.4}$$

$$\eta^{(1)} = (r'_{21}, r''_{21})^T, \eta^{(2)} = (r'_{12}, r''_{12})^T, \eta = \eta^{(1)} + \eta^{(2)}, \quad \varepsilon^* = \eta^{(1)} - \eta^{(2)}. \tag{2.5}$$

Let (n, S) be the right rectangular system, where S and n are respectively, the tangent and the normal of the curve L at the point $t = t_1 + it_2$. Assume that $n = (n_1, n_2)^T = (cos\alpha, sin\alpha)^T$ and $S^0 = (-n_2, n_1)^T = (-sin\alpha, cos\alpha)^T$, where α is the angle of inclination of the normal n to the ox_1 axis.

Introduce the vectors

$$U_n = (u_1n_1 + u_2n_2, u_3n_1 + u_4n_2)^T, \quad U_S = (u_2n_1 - u_1n_2, u_4n_1 - u_3n_2)^T, \tag{2.6}$$

$$\sigma_n = \begin{pmatrix} (TU)_1n_1 + (TU)_2n_2 \\ (TU)_3n_1 + (TU)_4n_2 \end{pmatrix}, \quad \sigma_S = \begin{pmatrix} (TU)_2n_1 - (TU)_1n_2 \\ (TU)_4n_1 - (TU)_3n_2 \end{pmatrix}, \tag{2.7}$$

$$\sigma_t = \begin{pmatrix} \left[r'_{21}n_1 - r'_{11}n_2, r'_{22}n_1 - r'_{12}n_2\right]^T S^0 \\ \left[r''_{21}n_1 - r''_{11}n_2, r''_{22}n_1 - r''_{12}n_2\right]^T S^0 \end{pmatrix}. \tag{2.8}$$

Let us call the vector (2.8) the tangential normal stress vector in the linear theory of elastic mixture.

After elementary calculations we obtain

$$\sigma_n = \tau^{(1)}cos^2\alpha + \tau^{(2)}sin^2\alpha + \eta sin\alpha cos\alpha,$$
$$\sigma_t = \tau^{(1)}sin^2\alpha + \tau^{(2)}cos^2\alpha - \eta sin\alpha cos\alpha,$$
$$\sigma_s = \frac{1}{2}\left[(\tau^{(2)} - \tau^{(1)})sin2\alpha + \eta cos2\alpha - \varepsilon^*\right].$$

Direct calculations allow us to check that on L [15]

$$\sigma_n + \sigma_t = \tau = 2(2E - A - B)Re\varphi'(t), \tag{2.9}$$

$$\sigma_n + 2\mu\left(\frac{\partial U_s}{\partial S} + \frac{U_n}{\varrho_0}\right) + i\left[\sigma_S - 2\mu\left(\frac{\partial U_n}{\partial S} - \frac{U_s}{\varrho_0}\right)\right] = 2\varphi^{'}(t), \tag{2.10}$$

$$[(A-2E)\varphi(t) + Bt\overline{\varphi'(t)} + 2\mu\overline{\psi(t)}]_L = -i\int_L e^{i\alpha}(\sigma_n + i\sigma_s)ds, \tag{2.11}$$

where $\det(2E - A - B) > 0$, $\frac{1}{\varrho_0}$ is the curvature of L at the point $t = t_1 + it_2$. Everywhere in the sequel it will be assumed that the components U_n and U_s are bounded [8].

Formulas (2.2), (2.3), (2.9) and (2.10) are analogous in the linear theory of elastic mixtures to those of Kolosov-Muskhelishvili [12].

3. Statement of the problem and the method of its solving

In the present work we consider the problem of statics of the linear theory of elastic mixture of finding a full-strength contour for a finite doubly-connected domain whose outer boundary is a convex polygon, while the inner boundary is a smooth closed unknown contour. It is assumed that the unknown contour is free from external stresses and absolutely smooth rigid punches are applied to the polygon boundary; the punches are under action of normal contractive forces.

Our problem is to find strained state of the polygon (with a hole) and analytic form of the unknown contour under the condition that the tangential normal stress vector (2.8) on it takes constant value (the condition of the unknown contour full-strength).

Statement of the problem. Let smooth rigid punches be applied to the boundary of a convex polygon which is weakened by an internal hole, and let the punches be under the action of external normal contractive forces; the hole boundary is free from external forces.

We consider the problem: Find elastic equilibrium of the polygon and analytic form of an unknown contour under the condition that the tangential normal stress vector on it takes constant value $\sigma_t = K^0$, $\quad K^0 = (K_1^0, K_2^0)^T = const.$

By D we denote a doubly-connected domain whose internal boundary is a smooth closed curve L_1 (an unknown part of the boundary), and the external boundary is a polygon L_0. By $A_j^0 \quad (j = \overline{1,n})$ we denote vertices (and their affixes) or the polygon (G_0) and assume that the point $z = 0$ lies inside the contour L_1. The positive direction on $L = L_0 \bigcup L_1$ is taken that which leaves the domain D on the left.

It is not difficult to note that in the case under consideration the $\sigma_S = 0$ (see (2.7)) on the entire boundary of D, and the $U_n(t)$ (see (2.6)) is a piecewise constant (unknown) vector on L_0.

Relying on the analogous Kolosov-Muskhelishvilis formulas (2.9) - (2.11) the above posed problem is reduced to finding two analytic vector-functions $\varphi(z)$ and $\psi(z)$ in domain D, by the following boundary conditions on $L = L_0 \bigcup L_1$:

$$Re\varphi^{'}(t) = H, \quad t \in L_1, \quad H = \frac{1}{2}(2E - A - B)^{-1}K^0, \tag{3.1}$$

$$Im\varphi^{'}(t) = 0, \quad t \in L_0, \tag{3.2}$$

$$Ree^{-i\alpha(t)}\left[(A-2E)\varphi(t)+Bt\overline{\varphi'(t)}+2\mu\overline{\psi(t)}\right]=C(t), \qquad t\in L_0, \tag{3.3}$$

$$(A-2E)\varphi(t)+Bt\overline{\varphi'(t)}+2\mu\overline{\psi(t)}=0, \quad t\in L_1, \tag{3.4}$$

where $\alpha(t)$ is the angle lying between the ox_1-axis and external normal to the boundary at the point $t\in L_0$,

$$C(t)=Re\{-i\int_{A^0}^{t}\sigma(t_0)expi[\alpha(t_0)-\alpha(t)]dS_0+(\delta^{(1)}+i\delta^{(2)})exp(-i\alpha(t))\}, t\in L_0,$$

$\delta^{(j)}=(\delta_1^{(j)},\delta_2^{(j)})^T, \quad (j=1,2)$, are arbitrary real constant vectors.

Moreover if $t\in L_0$ then we can write

$$Ret\ e^{-i\alpha(t)}=Ree^{-i\alpha(t)}A^0(t),$$

where $A^0(t)=A_k^0$ for $t\in A_k^0A_{k+1}^0$.

Since $\alpha(t)$ is the piecewise constant function, we obtain for $C(t)$ the representation

$$C(t)=\sum_{j=1}^{k}P^{(j)}sin(\alpha_k-\alpha_j)+\delta^{(1)}cos\alpha_k+\delta^{(2)}sin\alpha_k=C_k,$$

for $t\in A_k^0A_{k+1}^0, \quad k=\overline{1,n}, \quad (A_{k+1}^0\equiv A_1^0)$ where α_k is the value of the function $\alpha(t)$ on $A_k^0A_{k+1}^0$,

$$P^{(j)}=-\int_{S_j}^{S_{j+1}}\sigma_n(S)ds, \quad j=\overline{1,n}, \quad \sum_{k=1}^{n}P^{(k)}\cos\alpha_k=\sum_{k=1}^{n}P^{(k)}sin\alpha_k=0,$$

$$P^{(j)}=(P_1^{(j)},P_2^{(j)})^T,$$

(the equilibrium conditions), Thus, $C(t)$ is the piecewise constant vector-function containing n arbitrary real constants to be defined in the sequel.

Now note that, the conditions (3.1) and (3.2) is the Keldysh-Sedov problem having a solution [11]

$$\varphi(z)=Hz=\frac{1}{2}(2E-A-B)^{-1}K^0z, \qquad z\in D \tag{3.5}$$

(an arbitrary constant is assumed to be equal to zero).

Let the function $z=\omega(\zeta)$ map conformally a circular ring $G(1<|\zeta|<R)$ onto the domain D. We assume that the contour $l_0(|\zeta|=R)$ turns into L_0 and the contaur $l_1(|\zeta|=1)$ into L_1.

By virtue of (3.3), (3.4) and (3.5) for the vector-functions $\psi_0(\zeta)=\psi[\omega(\zeta)]$ holomorphic in the ring G, we obtain the following boundary value problem:

$$Ree^{-i\alpha(\xi)}[\frac{1}{2}K^0\omega(\xi)-2\mu\psi_0(\xi)]=-C(\xi), \quad |\xi|=R, \tag{3,6}$$

$$\frac{1}{2}K^0\omega(\sigma)-2\mu\overline{\psi_0(\sigma)}=0 \quad |\sigma|=1. \tag{3.7}$$

Note that on l_0 there takes place the equality

$$\frac{1}{2}Ree^{-i\alpha(\sigma)}K^0\omega(\sigma)=\frac{1}{2}K^0f_0(\sigma)=F_0(\sigma), \tag{3.8}$$

where $f_0(\sigma) = Re[e^{-i\alpha(\sigma)}A^0(\sigma)]$, $A^0(\sigma) = A^0_k$, $\sigma \in l_0^{(k)}$ (l_0^k are the arcs of the circumference l_0 corresponding to the sides L_0^k) $k=\overline{1,n}$.

Let us consider a new unknown vector-function $W(\zeta)=(W_1,W_2)^T$ defined by the formula

$$W(\zeta)=\begin{cases} \frac{1}{2}K^0\overline{\omega(\zeta)}, & 1<|\zeta|<R, \\ 2\mu\psi_0\left(\frac{1}{\bar{\zeta}}\right), & \frac{1}{R}<|\zeta|<1. \end{cases} \tag{3.9}$$

By the conditions (3.7) and (3.8) we can conclude that $W(\zeta)$ is the vector-function, holomorphic in the ring $G^*(\frac{1}{R}<|\zeta|<R)$ and satisfying the boundary conditions

$$\begin{aligned} Ree^{-\alpha(\xi)}W(\xi)&=F_0(\xi), \quad \xi\in l_0,\\ Ree^{-\alpha(\sigma)}W(\sigma)&=F_0^*(\sigma), \quad \sigma\in l_0^*, \end{aligned} \tag{3.10}$$

where l_0^* the circumference $|\zeta|=\frac{1}{R}$, $F_0^*(\sigma)=C(\sigma)+F_0(\sigma)$.

Since $F_0(\xi)$ and $F_0^*(\sigma)$ are the piecewise constant vector-functions, from (3.10) by means of multiplication by the abscissa s, with respect to the vector-function $W'(\zeta)$ we obtain the boundary value problem

$$Re[i\sigma e^{-i\alpha(\sigma)}W'(\sigma)]=0, \quad \sigma\in l_0Ul_0^* \tag{3.11}$$

Consider now the polygon (G_1) lying completely inside the contour L_1 and similar to the polygon (G_0); the corresponding vertices lie on one and the same ray emanating from the point $z=0$ (the similarity coefficient q remains unfixed yet).

We denote by A_j^* (that is, $A_j^*=q^{-1}A_j^0$,), vertices of the polygon (G_1) and by L_0^* the boundary.

By D^* we denote the doubly-connected domain which is bounded by the polygons (G_1) and (G_0), and as the positive direction on the boundary of D^* $(L_0\bigcup L_0^*)$ we choose that which leaves the domain D^* on the left.

Let the function $z=\omega_0(\zeta)$ map conformally the circular ring $G^*(R^{-1}<|\zeta|<R)$ onto the domain D^* (this can be achieved by the choice of q). Assume that $(|\zeta|=R)$ corresponds to L_0 and l_0^* $(|\zeta|=R^{-1})$ corresponds to L_0^*.

Taking into account that on l_0 and l_0^* the equalities:

$$Re[e^{-i\alpha(\xi)}\frac{1}{2}K^0\omega_0(\xi)]=F_0(\xi), \quad \xi\in l_0,$$

$$Re[e^{-i\alpha(\sigma)}\frac{1}{2}K^0\omega_0(\sigma)]=\frac{1}{q}F_0(\sigma), \quad \sigma\in l_0^*, \tag{3.12}$$

take place, we obtain with respect to the vector-function $\frac{1}{2}K^0\omega_0'(\zeta)$ the boundary value problem (3.11). Thus the vector-functions $W'(\zeta)$ and $\frac{1}{2}K^0\omega_0'(\zeta)$ satisfy one and the same boundary conditions on $l_0Ul_0^*$

Taking into account the results cited in [6], we can conclude that the necessary and sufficient condition for solving the problem (3.11) is of the form

$$\prod_{k=1}^{n}\left(\frac{a_k}{R^2}\right)^{\gamma_k-1}\left(\frac{a_k}{q}\right)^{1-\gamma_k}=1, \tag{3.13}$$

and the solution itself is given by the formula

$$W^{'}(z)=\nu\prod_{k=1}^{n}\left(\frac{a_k}{R}\right)^{\frac{1}{2}(\gamma_k-1)}\left(1-\frac{\zeta}{a_k}\right)^{\gamma_k-1}\left(1-\frac{a_k}{\zeta R^2}\right)^{\gamma_k-1}T(\zeta)[\zeta^2T(R^2\zeta)]^{-1}, \tag{3.14}$$

where by a_k we denote the preimages of the points $A_k^0 \quad (a_k\in l_0), \quad k=\overline{1,n}, \quad \nu=(\nu_1,\nu_2)^T$ is an arbitrary real constant vector,$\pi\gamma_k$ is the innear angle at the vertex $A_k, \quad k=\overline{1,n}$ and

$$T(\zeta)=\prod_{j=1}^{\infty}\prod_{k=1}^{n}\left(1-\frac{a_k}{R^{4j}\zeta}\right)^{\gamma_k-1}\left(1-\frac{\zeta}{R^{4j}a_k}\right)^{\gamma_k-1}.$$

Since $\sum_{k=1}^{n}(\gamma_k-1)=-2$ form (3.13) we get the relation $q=R^2$.

On the basis of the above results we can conclude that the problem of finding a full-strength contour inside the polygon is closely connected with the problem of conformal mapping of a doubly-connected domain, bounded by polygons, onto the circular ring. In order that the above-mentioned problems (3.10) and (3.12) be identical, it is necessary that the equality (see [6])

$$\left(1-\frac{1}{R^2}\right)F_0(\sigma)=C(\sigma), \quad \sigma\in l_0^*, \tag{3.15}$$

hold, or what is the same thing,

$$\frac{1}{2}\left(1-\frac{1}{R^2}\right)K^0(A_m^{(1)}cos\alpha_m+A_m^{(2)}sin\alpha_m)=$$

$$=\sum_{j=1}^{m}P^{(j)}sin(\alpha_m-\alpha_j)+\delta^{(1)}cos\alpha_m+\delta^{(2)}sin\alpha_m, \tag{3.16}$$

where $A_m^0=A_m^{(1)}+iA_m^{(2)}. \quad m=\overline{1,n}$.

If we choose the constants $P^{(j)}=(P_1^{(j)},P_2^{(j)})^T, \quad j=\overline{1,n}$ and $\delta^{(1)},\delta^{(2)}$ (two of $P^{(j)}$ are expressed through the rest ones) in such a way that the equality (3.16) holds, we obtain $W(\zeta)=\frac{1}{2}K^0\omega_0(\zeta)$, and hence the equation of the unknown contour L_1 will be

$$t=\omega_0(\sigma)=\frac{2}{K_1^0}W_1(\sigma)=\frac{2}{K_2^0}W_2(\sigma), \quad \sigma\in l_1$$

and the vector-function $2\mu\psi_0(\zeta)$ will be represented in the form $2\mu\psi_0(\zeta)=\frac{1}{2}K^0\overline{\omega_0\left(\frac{1}{\zeta}\right)}$, $\zeta\in G$.

As an example, we consider the case with the rectilinear polygon (G_0). Assume that to every polygon side are applied punches whose middle is under the action of normal concentrated force $-P$, $\ (P=(P_1,P_2)^T)$.

The coordinate origin is at the center of the polygon (G_0) and the ox_1-axis is perpendicular to the side A_1^0, A_2^0. Owing to the symmetry in the case we may assume that

$$A_k^0 = exp\left[-\frac{\pi i}{n}+\frac{2\pi i}{n}(k-1)\right]; \qquad \alpha_k=\frac{2\pi}{n}(k-1) \quad a_k = Rexp\left[\frac{2\pi i}{n}(k-1)\right].$$

It can be shown that the function $f_0(\sigma)=Re\left[e^{-i\alpha(\sigma)}A^0(\sigma)\right]$ is constant: $f_0(\sigma)=rcos\frac{\pi}{n}$, and the vector-function $C(t)$ in this case has the form

$$C(t)=\frac{P}{2\sin\frac{\pi}{n}}\left[\cos\frac{\pi}{n}-cos\frac{\pi}{n}(2k-1)\right]+\nu^{(1)}\cos\frac{2\pi}{n}(k-1)+$$

$$\nu^{(2)}\sin\frac{2\pi}{n}(k-1)=\frac{1}{2}P\left[ctg\frac{\pi}{n}-\cos\frac{2\pi}{n}(k-1)ctg\frac{\pi}{n}+\sin\frac{2\pi}{n}(k-1)\right]+$$

$$+\nu^{(1)}\cos\frac{2\pi}{n}(k-1)+\nu^{(2)}\sin\frac{2\pi}{n}(k-1).$$

Taking $\nu^{(1)}=\frac{1}{2}Pctg\frac{\pi}{n}$; $\quad \nu^{(2)}=-\frac{1}{2}P$, we get $C(t)=-\frac{1}{2}Pctg\frac{\pi}{n}$ and hence (3.15) results in the relation

$$K^0=\frac{PR^2}{r(R^2-1)\sin\frac{\pi}{n}}. \tag{3.17}$$

In particular, if we assume that the polygon side is equal to unity, i.e. $a_n = 2r\sin\frac{\pi}{n}=1$, then from (3.17) we obtain

$$K^0=\frac{2PR^2}{R^2-1},$$

whence we conclude that $K_j^0>2P_j$; $\ (j=1,2)$ and also, when R increases (i.e. when the hole shrinks to the point) $K^0\to 2P$, while as $R\to 1$ i.e., when K^0 increases and does not exceed critical value, the hole contour approaches to that of the polygon.

REFERENCES

1. Banichuk N.V. Optimization of forms of elastic bodies. (Russian) *Nauka. Moskow*, 1980.
2. Bantsuri R.D., Isakhanov R.S., A problem of elasticity theory for a domain with an unknown part of the boundary. (Russian) *Soobsch. Akad. Nauk Gruzin SSR*, **116**, 1 (1984), 45-48.
3. Bantsuri R.D. On one mixed problem of the plane theory of elasticity with a partially unknown boundary. *Proc. A. Razmadze Math. Inst.*, **140** (2006), 9-16.
4. Bantsuri R.D. Solution of the mixed problem of plane bending for a multi-connected domain with a partially unknown boundary in the presence of cyclic symmetry. *Pros. A. Razmadze Math. Inst.*, **145** (2007), 9-22.

5. Bantsuri R. , Kapanadze G. On one problem of finding an aqually strong contour for a square which is weekened by a hole and by cuttings at vertices. *Proc A. Razmadze Math. Inst*, **155** (2011), 9-16.

6. Bantsuri R., Kapanadze G. The problem of finding a full-strength contour inside the polygon. *Proc. A. Razmadze Math. Inst.*, **163** (2013), 1-7.

7. Basheleishvili M. Analogous of the Kolosov-Muskhelishvili general representation formulas and Cauchy-Riemann conditions in the theory of elastic mixtures. *Georgian Math. j.*, **4**, 3 (1997), 223-242.

8. Basheleishvili M., Svanadze K. A new method of solving the basic plane boundary value problems of statics of the elastic mixture theory. *Georgian Math. j*, **8**, 3 (2001), 427-446.

9. Cherepanov G.P. Inverse problems of the plane theory of elasticity. (Russian) T*ranslated from: Prikl. Math. Mech.*, **38**, 5 (1974), 963-979. *j Appl. Math. Mech.*, **38**, 6 (1975), 915-931.

10. Kapanadze G. On one problem of the plane theory of elasticity with a partially unknown boundary. *Proc. A. Razmadze Math. Inst.*, **143** (2007), 61-71.

11. Keldysh M.V., Sedov I.I. The effective solution of some boundary value problem for harmonec functions. (Russian) *DAN SSSR*, **16**, 1 (1997), 7-10.

12. Muskhelishvili N.I. Some basic problems of the mathematical theory of elasticity. (Russian) *Nauka, Moscow,* 1966.

13. Natroshvili D.G., Dzhagmaidze A.Y, Svanadze M.Zh. Some problem in the linear theory for elastic mixtures. (Russian) *Tbilisi Gos. Univ. Tbilisi*, 1986.

14. Odishelidze N.T. Solution of the mixed problem of the plane theory of elasticity for a multiply connected domain with partially unknown boundary in the presence of axial symmetry. *Proc. A. Razmadze Math. Inst.*, **146** (2008), 97-112.

15. Svanadze K. On one mixed problem of the plane theory of elastic mixture with a partially unknown boundary. *Proc A. Razmadze Math. Inst.*, **150** (2009), 121-131.

16. Svanadze K. Solution of a mixed boundary value problem of the plane theory of elastic mixture for a multiply connected domain with a partially unknown boundary having the axis of symmetry. *Proc. A. Razmadze Math. Inst.*, **155** (2011), 73-90.

17. Svanadze K. On one problem of statics of the theory of elastic mixtures for a square which is weakened by a hole and by cuttings at vertices. *Seminar of I. Vekua Institute of Applied Mathematics, REPORTS*, **38** (2012), 41-51.

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Author's address:

K. Svanadze
A. Tsereteli Kutaisi State University
59, Tamar Mepe St., Kutaisi 4600
Georgia
E-mail: kostasvanadze@yahoo.com