OPTIMAL HEDGING IN THE FINANCIAL MODEL WITH DISORDER

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On the filtered probability space $(\Omega, F, (F_n)_{0\le n\le N}, P)$ consider stochastic process indiscrete time as evolution of risky asset price

$S_n = S_{n-1}\exp\{I(n<\theta)\Delta M_n^{(1)} + I(n\ge\theta)\Delta M_n^{(2)}\},\ n=1,\dots,N,$ (1)

where $S_0>0$ is a constant, $M_n^{(1)}$ and $M_n^{(2)}$ are independent Gaussian martingales starting from 0. θ is a arandom variable which takes values $1,2,\dots,N$ with known probabilities $p_i = P(\theta=i),\ i=\overline{1,N}$. The vector $(M_n^{(1)}, M_n^{(1)})$ is independent of θ and $I(A)$ is an indicator of A.

In this model for the European contingent claim $f(S_N)$ we have constructed the GF hedging strategy $\pi=(\gamma_n,\beta_n)$, which is optimal in the mean square sense

$E[f(S_N)-X_N^{\pi}]^2,$ (2)

where X_N^{π} is the capital value at terminal moment N and interest rate $r=0$.

Theorem. In the model (1) of price evolution, optimal in sense(2) strategy is

$$\gamma_n = \frac{E[F_n(e^{h_n}-C_n)/F_{n-1}^S]}{S_{n-1}E[(e^{h_n}-C_n)^2/F_{n-1}^S]},$$

$$\beta_n = -\sum_{i=1}^n C_i\gamma_i S_{i-1} - \sum_{i=1}^n S_{i-1}\Delta\gamma_i,$$

where $C_n = e^{\frac{\Delta\langle M^{(1)}\rangle_n}{2}}\sum_{i=n}^N P_i^n + e^{\frac{\Delta\langle M^{(2)}\rangle_n}{2}}\sum_{i=1}^{n-1} Q_i^n - 1$, P_i^n and Q_i^n are given in [3].

References

1. Schweizer, M.: Variance-Optimal Hedging in Discrete Time. Mathematics of Operations Research, 20, (1995), 1--32.
2. Shiryaev, A.N.:Essentials of Stochastic Finance. Facts, models,Theory. World Scientific. Singapure, 1999.
3. Glonti, O., Khechinashvili, Z.: Geometric Gaussian Martingales With Disorder. Theory of Stochastic Processes, Vol. 16(32), no. 1, 2010, pp. 44-48.