MINIMAL PAIRS OF C.E. sQ- DEGREES

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Tennenbaum (see, [1, p.159]) defined the notion of Q-reducibility on sets of natural numbers as follows: a set A is *Q-redusible* to a set B (in symbols: $A \leq_Q B$) if there exists a computable function f such that for every $x \epsilon \omega$ (where ω denotes the set of natural numbers),

$$x \epsilon A \Leftrightarrow W_{f(x)} \subseteq B.$$

If, in addition, there exists a computable function g such that

$$(\forall x)(\forall y)(y \in W_{f(x)} \Rightarrow y < g(x)),$$

then we say that A is strongly *Q -reducible* (or *sQ-reducible*) to B (in symbols: $A \leq_{sQ} B$).

The relation of *sQ-reducibility* is reflexive and transitive so that it generates a degree structure on the subsets of ω.

Our notation and terminology are standard and can be found in [1, 2].

Theorem 1. If c.e. sQ-degrees **a** and **b** form a minimal pair in the c.e. sQ-degrees then **a** and **b** form a minimal pair in the sQ-degrees.

This is immediate from

Theorem 2. If **a** and **b** are c.e. sQ-degrees, then for every nonzero sQ-degree **c** such that **c** $\leq_{sQ}$ **a**, **b**, there exists a c.e. sQ-degree **d** such that **d** $\leq_{sQ}$ **a**, **b** and $(\forall C \in \mathbf{c})(\forall D \in \mathbf{d})(C \leq_Q D)$.

Theorem 3. For every simple set S such that K $\nleq_{sQ}$ S, where K is a creative set, there is a noncomputable nonspeedable set A which is sQ − incomparable with S and deg_{sQ}(A) and $deg_{sQ}(B)$ does not form a minimal pair.

Theorem 4. For every speedable set S such that K $\nleq_{sQ}$ S, where K is a creative set, there is a noncomputable nonspeedable set A which is sQ − incomparable with S and deg_{sQ}(A) and $deg_{sQ}(B)$ does not form a minimal pair.

References

1. Rogers, H.: Theory of recursive functions and effective computability. McGraw-Hill Book Co., New York, 1967.
2. Soare, R.I.: Recursively enumerable sets and degrees. Springer-Verlag, Berlin, 1987.