ON THE TESTING HYPOTHESIS OF EQUALITY DISTRIBUTION DENSITIES

PetreBabilua*, Elyzbar Nadaraya**, Grigol Sokhadze***
Iv. Javakhishvili Tbilisi State University, Faculty of Exact and Natural Sciences, Department of Mathematics, Tbilisi, Georgia
*p_babilua@yahoo.com, **elizbar.nadaraya@tsu.ge, ***giasokhi1@i.ua

Let $X^{(i)}=\left(X_1^{(i)},\dots,X_{n_i}^{(i)}\right)$, $i=1,\dots,p$, be independent samples with sizes $n_1,n_2,\dots,n_p$, from $p\geq 2$ general population with probability densities $f_1(x),\dots,f_p(x)$ and it is required to test two hypotheses, based on samples $X^{(i)}$, $i=1,\dots,p$: test of homogeneity

$$H_0:\ f_1(x)=\cdots=f_p(x)$$

and goodness-of-fit test

$$H_0':\ f_1(x)=\cdots=f_p(x)=f_0(x),$$

where $f_0(x)$ is a fully defined density function. In case of hypothesis H_0 the common density function $f_0(x)$ is unknown.

In this paper the test for checking hypothesis H_0 and H_0' is constructed against a sequence of "close" alternatives ([1], [2]):

$$H_1:\ f_i(x)=f_0(x)+\alpha(n_0)\varphi_i\left(\frac{x-l_i}{\gamma(n_0)}\right)+o\big(\alpha(n_0)\gamma(n_0)\big)$$
$$(\alpha(n_0),\gamma(n_0)\to 0),$$
$$\int\varphi_i(x)dx=0,\ \ n_0=\min\left(n_1,\dots,n_p\right)\to\infty.$$

We will consider criteria for testing hypotheses H_o and H_o' based on statistics

$$T(n_1,\dots,n_p)=\sum_{i=1}^{p}N_i\int\left[\hat{f}_i(x)-\frac{1}{N}\sum_{j=1}^{p}N_j\,\hat{f}_j(x)\right]^2 r(x)dx$$

Where $\hat{f}_i(x)$ is a kernel estimator of Rosenblatt-Parzen of density $f_i(x)$:

$$\hat{f}_i(x)=\frac{a_i}{n_i}\sum_{j=1}^{n_i}K\left(a_i\left(x-X_j^{(i)}\right)\right),\qquad N_i=\frac{a_i}{n_i},\qquad N=N_1+N_2+\cdots N_p.$$

Acknowledgement. This work was supported by Shota Rustaveli National Science Foundation Grant No FR/308/5-104/12.

References

1. Rosenblatt, M.A.: quadratic measure of deviation of two-dimensional density estimates and a test of independence. *Ann. Statist.***3** (1975), p. 1--14.
2. Nadaraya, È. A.: Nonparametric estimation of probability densities and regression curves. Translated from the Russian by Samuel Kotz. Mathematics and its Applications (Soviet Series), 20. *Kluwer Academic Publishers Group, Dordrecht,* 1989.