ON THE SMOOTHNESS OF CONDITIONAL MEAN OF SOME STOCHASTICALLY NONSMOOTH FUNCTIONALS

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It is well known, if a random variable is stochastically differentiable (in the Malliavin sense), then its conditional mathematical expectation is differentiable too ([1]). On the other hand, it is possible that conditional expectation can be smooth even if a random variable isn't stochastically smooth ([2]). If the functional is stochastically smooth then Clark-Ocone formula proves that the integrand from Clark representation of this functional, represents the conditional expectation of Malliavin derivative. Despite the fact, that Clark-Ocone formula gives integrand construction, there are problems with practical realizations (from the view point of stochastic derivative calculation, as well as of conditional mathematical expectation). We generalized this result ([2]) in case, when the functional isn't stochastically smooth, but its filter (conditional mathematical expectation) is stochastically differentiable and established the method of finding this integrand. Therefore, it is advisable to characterize as wide as possible class of such functionals. Below we will try to make the first step in this direction.

As it is known ([1]), an indicator of event is stochastically differentiable if and only if, the probability of this event equals 0 or 1. So, it is clear, that the indicator of $\{w_T > K\}$ has no Malliavin derivative. Accordingly, $F = f(w_T)I_{\{w_T>K\}}$ type functional is of the some kind, even if $f = f(x)$ is smooth in the classical sense.

Theorem. The conditional expectation of the functional $F = w_T^n I_{\{w_T>K\}}$ is stochastically smooth and we have the following relation:

$$D_s\{E[w_T^n I_{\{w_T>K\}} \mid \Im_t^w]\} = -\frac{1}{\sqrt{2\pi}\,(T-t)^{3/2}}\int_K^\infty y^n (w_t - y)\exp\{-\frac{(w_t-y)^2}{2(T-t)}\}dy \cdot I_{[0,t]}(s).$$

Moreover,

$$E[D_s\{E[w_T^n I_{\{w_T>K\}} \mid \Im_t^w]\} \mid \Im_s^w] = -\frac{w_s}{(t-s)\sqrt{2\pi(T-s)}}\int_K^\infty y^n \exp\{-\frac{(y-w_s)^2}{2(T-s)}\}dy \cdot I_{[0,t]}(s) +$$

$$+\frac{1}{(T-s)^{3/2}\sqrt{2\pi}}\int_K^\infty y^{n+1}\exp\{-\frac{(y-w_s)^2}{2(T-s)}\}dy \cdot I_{[0,t]}(s) +$$

$$+\frac{w_s(T-t)}{(t-s)(T-s)^{3/2}\sqrt{2\pi}}\int_K^\infty y^n \exp\{-\frac{(y-w_s)^2}{2(T-s)}\}dy \cdot I_{[0,t]}(s).$$

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