THE EFFECTIVE SOLUTIONS OF ONE CLASS OF INTEGRO-DIFFERENTIAL EQUATIONS

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The effective solutions for integro-differential equations related to problems of interaction of an elastic thin finite and infinite inclusion with a plate are considered. If the geometric and physical parameter of the inclusion is measured along its length according to the parabolic and linear law we have managed to investigate the obtained boundary value problems of the theory of analytic functions and to get exact solutions and establish behavior of unknown contact stresses at the ends of an elastic inclusion.

Let a finite or infinite non-homogeneous inclusion with modulus of elasticity $E_1(x)$, thickness $h_1(x)$ and Poisson's coefficient ν_1 be attached to the plate which is in the condition of a plane deformation. It is assumed that the inclusion has no bending rigidity, is in the uniaxial stressed state and is subject only to tension, the tangential stress $\tau_0(x)$ acts on the line of contact of the inclusion and the plate, the contact condition considers the existence of thin glue layer.

We are required to define the law of distribution of tangential contact stresses $\tau(x)$ on the line of contact, the asymptotic behavior of these stresses at the end of the inclusion and the coefficient of stress intensity.

To define the unknown contact stresses we obtain the following singular integro-differential equation

$$\frac{\varphi(x)}{E(x)} + \frac{\lambda}{\pi}\int_0^a \frac{\varphi'(t)dt}{t-x} - k_0\varphi''(x) = g(x), \qquad 0 \le x \le a,$$

$$\phi(0) = 0, \qquad \phi(a) = T_0,$$

where

$$\varphi(x) = \int_0^x \tau(t)dt, \qquad \int_0^a \tau(t)dt = T_0, \qquad T_0 = \int_0^a \tau_0(t)dt,$$

$$E(x) = \frac{E_1(x)h_1(x)}{1-\nu_1^2}, \quad g(x) = \frac{1}{E(x)}\int_0^x \tau_0(t)dt.$$

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