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THE WEAKLY CONSISTENT, STRONGLY CONSISTENT AND CONSISTENT ESTIMATES OF THE PARAMETERS

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Abstract. The problem of weakly consistency, strongly consistency and consistency of parameters of stationary processes are studied and the interrelations between the various concepts of consistencies are established.

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1 Introduction. Let there be given (E, S) measurable space and on this space there given $\{\mu_i, i \in I\}$ family of probability measures defined on S, the I is set of parameters.

Let us bring some definition (see [1]-[4]).

Definition 1. A statistical structure is called the set of objects $\{E, S, \mu_i, i \in I\}$, where (E, S) is a measurable space and $\{\mu_i, \ i \in I\}$ is a family of probability measures on it.

Definition 2. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called orthogonal (singular) if μ_i and μ_j are orthogonal for each $i \neq j$, $i \in I$, $j \in I$.

Definition 3. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called weakly separable if there exists a family of S-measurable sets $\{X_i, i \in I\}$ such that the realations are fulfilled:

$$\mu_i(X_j) = \begin{cases} 1, & if\ i = j; \\ 0, & if\ i \neq j. \end{cases}$$

Definition 4. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called separable if there exists a family of S-measurable sets $\{X_i, i \in I\}$ such that the realations are fulfilled:

$$1)\ \ \mu_i(X_j) = \begin{cases} 1, & if\ i = j; \\ 0, & if\ i \neq j. \end{cases}$$

$$2)\ \ card(X_i \cap X_j) < c, \ \ if\ i \neq j,$$

where c denotes the continuum power.

Definition 5. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called strongly separable if there exists a disjoint family of S-measurable sets $\{X_i, i \in I\}$ such that the realations are fulfilled:

$$\mu_i(X_i) = 1, \quad \forall\, i \in I.$$

Remark 1. It is known that weak separability of statistical structure follows from its strong separability. Moreover, orthogonality of statistical structure follows from its weakly separability but not vice versa (see [1]).

Let I be set of parameters and let $B(I)$ be σ-algebra of subsets of I which contains all its finite subsets. We denote by $\overline{\mu}_i$ the completion of the measure μ_i and $dom(\overline{\mu}_i)$ denotes the σ-algebra of all μ_i-measurable subsets of E.

Definition 6. Let's say that the statistical structure $\{E, S, \mu_i, i \in I\}$ admits a consistent estimator of parameters $i \in I$, if there exists at least one measurable map

$$\delta \; : \; (E, S) \; \longrightarrow \; (I, B(I)),$$

such that

$$\overline{\mu}_i(\{x : \delta(x)\}) = 1, \;\; \forall \, i \in I.$$

Definition 7. Let's say that the statistical structure $\{E, S, \mu_i, i \in I\}$ admits a consistent estimator of any parametric function if for any real bounded measurable function

$$g : (I, B(I)) \longrightarrow (R, B(R))$$

there exists at least one measurable function

$$f : (E, S) \longrightarrow (R, B(R))$$

such that

$$\overline{\mu}_i(\{x : f(x) = g(i)\}) = 1, \;\; \forall \, i \in I.$$

Definition 8. Let's say that the statistical structure $\{E, S, \mu_i, i \in I\}$ admits an unbiased estimator of any parametric function if for any real bounded measurable function

$$g : (I, B(I)) \longrightarrow (R, B(R))$$

there exists at least one measurable function

$$\beta : (E, S) \longrightarrow (R, B(R))$$

such that

$$\int_E \beta(x)\overline{\mu}_i(dx) = g(i), \;\; \forall \, i \in I.$$

Let (E, S) be a measurable space, S_n is an increasing sequence of σ-algebras such that $\cup_{n=1}^{\infty} S_n = S$. Let I be a metric space with metric ρ.

Definition 9. Let's say that the statistical structure $\{E, S, \mu_i, i \in I\}$ admits a weakly consistent estimator of parameters $i \in I$, if there exists a sequence of S_n-measurable functions

$$g_n(x) : E \longrightarrow I,$$

such that for any $\epsilon > 0$:

$$\lim_{n\to\infty} \mu_i\{x : \rho(g_n(x), i) \geq \epsilon\} = 0, \;\; \forall \, i \in I.$$

Definition 10. Let's say that the statistical structure $\{E, S, \mu_i, i \in I\}$ admits a strongly consistent estimator of parameters $i \in I$, if there exists a sequence of S_n-measurable functions

$$g_n(x) : E \longrightarrow I,$$

such that

$$\mu_i\{x : \lim_{n\to\infty} \rho(g_n(x), i) = 0\} = 1, \ \ \forall\, i \in I.$$

2 Main results.

Theorem 1. *If the statistical structure $\{E, S, \mu_i, i \in I\}$ admits a weakly consistent estimator of parameters $i \in I$, then this statistical structure is weakly separable.*

Proof. Let $g_n(x)$ be a weakly consistent estimators of parameters $i \in I$. We denote by $\{n_k(i)\}$ the subsequence of $\{n\}$ such that

$$\mu_i\{x : \lim_{k\to\infty} \rho(g_{n_k(i)}(x), i) = 0\} = 1, \quad \forall i \in I.$$

Let

$$X_i = \{x : \lim_{k\to\infty} \rho(g_{n_k(i)}(x), i) = 0\},$$

then $\mu_i\{X_i\} = 1$, $\forall i \in I$. If $i \neq j$, then $\mu_j\{X_i\} = 0$. Actually, we can choose such subsequence $n_k{'} \subset \{n_k(i)\}$ for which

$$\mu_{i'}\{x : \lim_{k\to\infty} \rho(g_{n_k^{'}}(x), i^{'}) = 0\} = 1.$$

It is clear that $X_i \cap X_{i'} = \emptyset$, $i \neq i^{'}$ and

$$\mu_i(X_{i'}) = \begin{cases} 1, & if\ i = i^{'}; \\ 0, & if\ i \neq i^{'}. \end{cases}$$

□

Theorem 2. *If the statistical structure $\{E, S, \mu_i, i \in I\}$ admits a strongly consistent estimators of parameters $i \in I$, then this statistical structure admits a consistent estimators of parameters $i \in I$.*

Proof. Because the statistical structure $\{E, S, \mu_i, i \in I\}$ admits a strongly consistent estimators of parameters $i \in I$, there exists a sequence of S_n-measurable functions g_n, such that

$$\mu_i\{x : \lim_{n\to\infty} \rho(g_n(x), i) = 0\} = 1, \ \ \forall\, i \in I.$$

Assuming now

$$g(x) = \begin{cases} i, & if \ \ \mu_i\{x : \lim_{n\to\infty} \rho(g_n(x), i) = 0\} = 1, \ \ \forall\, i \in I; \\ 0, & otherwise \end{cases}$$

we easily ascertain that

$$\mu_i(\{x : g(x) = i\}) = 1, \quad i \in I.$$

□

Theorem 3. *If the statistical structure $\{E, S, \mu_i, i \in I\}$ admits a consistent estimators of parameters $i \in I$, then this statistical structure admits both a consistent and an unbiased estimators of any parametric function.*

Theorem 4. *Let $\{\mu_i, i \in I\}$ be a family of Borel probability measures on S such that the following relations are fulfilled:*

1) I be a complete metric space, whose topological weights are not measurable in a wider sense, let S be a Borel σ-algebra in E and $cardI \leq c$;.

2) for all $\delta > 0$, i and j, $i \neq j$ one can indicate the set $X^{\delta}_{i,j}$ such that a) $\mu_i(X^{\delta}_{i,j}) = 1$, $\mu_j(X^{\delta}_{i,j}) < \delta$; b) the function $\mu_{j'}(X^{\delta}_{i,j})$is semicontinuous from above in a point $j^{'} = j$.

Then a statistical structure $\{E, S, \mu_i, i \in I\}$ is weakly separable.

Remark 2. If the statistical structure $\{E, S, \mu_i, i \in I\}$ admits a consistent estimator of parameters $i \in I$, then this statistical structure is strongly separable but not vice versa.

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