TSU 100 VIAM 50

THE THIRD INTERNATIONAL CONFERENCE

„MODERN PROBLEMS IN APPLIED MATHEMATICS“

BOOK of ABSTRACTS

თეზისების კრებული

მესამე საერთაშორისო კონფერენცია

„გამოყენებითი მათემატიკის თანამედროვე პრობლემები“

19.09.2018 - 21.09.2018

TBILISI თბილისი

THE THIRD INTERNATIONAL CONFERENCE „MODERN PROBLEMS IN APPLIED MATHEMATICS“

Dedicated to the Centenary of
I. Javakhishvili Tbilisi State University
and
50th Anniversary of I.Vekua Institute of Applied Mathematics

September 19-21, 2018
I. Vekua Institute of Applied Mathematics
of Iv. Javakhishvili Tbilisi State University (TSU)
University St. 2, Tbilisi, Georgia

Opening Ceremony will be held at the Main Building of TSU, Ilia Chavchavadze Ave. 1, on September 19 at 10:00 a.m.

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„გამოყენებითი მათემატიკის თანამედროვე პრობლემები"

ეძღვნება ი. ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტის 100
და
ი. ვეკუას სახელობის გამოყენებითი მათემატიკის ინსტიტუტის 50 წლისთავს

2018 წლის 19-21 სექტემბერი
ი. ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტის (თსუ)

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გახსნის ცერემონია გაიმართება 19 სექტემბერს 10:00-საათზე
თსუ პირველ კორპუსის სააქტო დაარბაზში, ილია ჭავჭავაძის გამზირი 1

ორგანიზატორები

ი. ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი
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რუხაია მ.

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George Jaiani

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The talk concerns 50 years long history of Ilia Vekua Institute of Applied Mathematics of Ivane Javaxishvili Tbilisi State University [1-3]. The Institute was founded by a Georgian mathematician and mechanist Ilia Vekua on October 29, 1968. The aim of the Institute was to carry out research on important problems of applied mathematics, to involve University professors, teachers and students in research activities on topical problems of applied mathematics in order to integrate mathematics into the educational processes and research, and to implement mathematical methodologies and computing technology in the non-mathematical fields of the University. In 1978, the Institute was named after its founder and first director Ilia Vekua. In December, 2006 - May, 2009 the Institute was functioning at the Faculty of the Exact and Natural Sciences. In June, 2009 - September, 2016 the Institute was directly subordinated to the University Administration. Since the end of September, 2016 the Institute has a status of the Independent Scientific-Research Institute. At present, the Institute successfully continues and develops activities launched by its founder in the following four main scientific directions:

- Mathematical problems of mechanics of continua and related problems of analysis;
- Mathematical modelling and numerical mathematics;
- Discrete mathematics and theory of algorithms;
- Probability Theory and mathematical Statistics.

The institute sees its mission as threefold:

- Carrying out fundamental and practical scientific research in applied mathematics, mathematical and technical mechanics, industrial mathematics and informatics, undertaking state and private sector contracts to provide expert services;
- Offering the university a high-level computer technology base for University professors and teachers, research employees and students undertaking their scientific research activities;
- Supporting PhD and post-graduate students to attain scientific grants, as well as through employment within the Institute and participation in scientific conferences.

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Mathematics at Ivane Javakhishvili Tbilisi State University

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The talk deals with the history of mathematics at the I. Javakhishvili Tbilisi State University (TSU). To a group of scientists who founded TSU, under the leadership Ivane Javakhishvili, belonged Andria Razmadze – the first Georgian scientist-mathematician. Along with A. Razmadze a crucial role in the development of the mathematical science and education at TSU was played by N. Muskhelishvili, G. Nikoladze, A. Kharadze, A. Benashvili and later I. Vekua and V. Kupradze. In the present talk, we will briefly describe a contribution of those scientists who are founders and organizers of scientific and pedagogical activity in mathematics at TSU. Presently, these traditions are continuing successfully in the Department of Mathematics of the Faculty of Exact and Natural Sciences of TSU, as well as in the I. Vekua Institute of Applied Mathematics and A. Razmadze Mathematical Institute.

Modeling of Plastics and Composite Materials

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There are several approaches to model the material behavior under mechanical loading:
- the deductive approach based on Continuum Mechanics and Material Theory,
- the inductive approach based on some experimental observations and a step-by-step generalization, and
- the method of rheological modeling

After some general statements with the help of the inductive approach combined with rheological modeling the mechanical behavior of plastics and composite materials will be presented.

On Some Goodness-of-Fit Tests Based on Wolverton–Wagner Type Estimates of Distribution Density

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Let $X_1, X_2, \dots, X_n$ be a sequence of independent, equally distributed random variables, having a distribution density $f(x)$. Based on sample $X_1, X_2, \dots, X_n$ it is required to check the hypothesis

$$H_0 : f(x) = f_0(x).$$

here we consider the hypothesis H_0 testing, based on the statistics

$$T_n = na_n^{-1} \int (f_n(x) - f_0(x))^2 r(x)\, dx,$$

where $f_n(x)$ is the recurrent Wolverton–Wagner kernel estimate of probability density defined by:

$$f_n(x) = n^{-1} \sum_{i=1}^{n} a_i K((a_i(x - X_i))),$$

where a_i is an increasing sequence of positive numbers tending to infinity, $K(x)$, $f_0(x)$ and $r(x)$ satisfy certain regularity conditions.

1. Question of consistency for the constructed criterion against any alternative $H_1 : f(x) = f_1(x)$, where $f_1(x)$ is such that $\int (f_n(x) - f_0(x))^2 r(x)\, dx > 0$ is studied.

2. The limiting behavior of the power is studied for sequence of close to hypothesis H_0 alternatives of Pitmen and Rosenblatt type [1] and it is shown that the tests based on T_n for the above mentioned alternatives are more powerfull in limits than the tests based of Bickel–Rosenblatt [2].

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Stochastic Equation of Fragmentation and Branching Processes Related to Avalanches

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We develop a method for the construction of continuous time fragmentation-branching processes on the space of all fragmentation sizes, induced either by continuous fragmentation kernels or by discontinuous ones. This construction leads to a stochastic model for the fragmentation phase of an avalanche. We introduce an approximation scheme for the process which solves the corresponding stochastic differential equations of fragmentation. Finally, we present numerical results that confirm the validity of a fractal property which is emphasized by our model for an avalanche.

The talk is based on joint works with **Madalina Deaconu** and **Oana Lupaşcu**.

Hyperbolic Model for the Helmholtz Equation with Impedance Boundary Conditions

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We propose a novel method that is motivated by first order system approach [6] introduced initially for the diffusion equation and by well balanced schemes for hyperbolic conservation laws with source terms pioneered in [1.4]. For a large wave number the numerically challenging task for the Helmholtz equation is achieving high accuracy with a small number of nodal points [3,5]. Solution of Helmholtz equation with impedance boundary condition on finite interval is equivalently reformulated as steady state of initial boundary value problem for the first order hyperbolic system of partial differential equations. Particularly interesting property of the proposed hyperbolic model is that steady state is achieved in finite time. Another competitive advantage of our hyperbolic model is that it allows stable computations with time step $O(\Delta x)$ that is typical for hyperbolic equations instead of time step $O((\Delta x)^2)$ that is typical for parabolic equations. Further improvements are made by means of spatial discretization that maintain discrete equilibrium states [2]. Numerical tests demonstrate excellent computational potential of the proposed method - achieving high accuracy with small number of nodal points in space and time, e.g. for wave number $k = 10^5$ relative error $\approx 10^{-3}$ can be obtained with just 11 nodal points in space and 20 time steps.

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The L^p-Dissipativity of First Order Partial Differential Operators

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In 2005 we proved, together with Vladimir Maz'ya, that the algebraic condition

$$|p-2|\,|\langle \mathscr{I}m\mathscr{A}\xi,\xi\rangle| \leq 2\sqrt{p-1}\,\langle \mathscr{R}e\mathscr{A}\xi,\xi\rangle$$

(for any $\xi \in \mathbb{R}^n$) is necessary and sufficient for the L^p-dissipativity of the Dirichlet problem for the differential operator $\nabla^t(\mathscr{A}\nabla)$, where $\mathscr{A}$ is a matrix whose entries are complex measures and whose imaginary part is symmetric. This condition characterizes the L^p-dissipativity individually, for each p, while usually the results in the literature concern the L^p-dissipativity for all p's simultaneously.

Later on we have determined necessary and sufficient conditions for the L^p-dissipativity of other partial differential operators of the second order, including some systems.

The aim of the present talk is to present recent results in this direction concerning first order partial differential operators. Also these results have been obtained together with Vladimir Maz'ya.

Decomposition Formulas for Cosine Operator Function with Two and Multi-addend Argument

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The paper presents new decomposition formulas for cosine operator function based on known trigonometric formulas. The validity of the constructed formula is proved when argument of cosine operator function is a sum of two bounded operators. High order decomposition formula is constructed, in case when there is a square root of the main operator in the argument of cosine operator function and the number of addends equals two. The decomposition formula is constructed using resolvents of the summand operators. There is also proposed an algorithm that allows to construct any order accuracy decomposition formula for cosine operator function. More precisely, the algorithm allows to construct 2p+2 accuracy order decomposition formula based on 2p order one.

Mellin Pseudodifferential Equations in Bessel Potential Spaces

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One dimensional Mellin pseudodifferential (ΨDOs) equations acting in Bessel potential spaces are considered. The study is based upon two results. The first one concerns the interaction of Mellin ΨDOs and Bessel potential operators (BPOs). In contrast to the Fourier ΨDOs, BPOs and Mellin ΨDOs do not commute and we derive an explicit formula for the corresponding commutator in the case of Mellin ΨDOs with meromorphic symbols. These results are used in the lifting of the Mellin ΨDOs operating between the Bessel potential spaces to operators in the Lebesgue spaces. The operators arising belong to an algebra generated by Mellin and Fourier convolutions acting on $\mathbb{L}_p$-spaces. Fredholm conditions and index formulae for such operators have been obtained earlier by the authors.

The results of the present work have numerous applications in boundary value problems for partial differential equations. We will demonstrate such application to the BVP for the Laplace-Beltrami equation with mixed Dirichlet-Neumann boundary conditions on a smooth hypersurface $\mathcal{C}$ in $\mathbb{R}^3$ with a smooth boundary

$$\begin{cases} \Delta_{\mathcal{C}} u(t) = f(t), \quad t \in \mathcal{C}, \quad u^+(\tau) = g(\tau), & \tau \in \Gamma_D, \\ (\partial_{\nu_\Gamma} u)^+(\tau) = h(\tau), \qquad \tau \in \Gamma_N, & \partial_{\nu_\Gamma} := \sum_{j=1}^{3} \nu_{\Gamma,j} \mathcal{D}_j. \end{cases} \tag{1}$$

Here $\partial\mathcal{C} = \Gamma = \Gamma_D \cup \Gamma_N$ and $\Gamma_D \cap \Gamma_N = \emptyset$ parts. $\nu(\omega) = (\nu_1(\omega), \nu_2(\omega), \nu_3(\omega))$, $\omega \in \overline{\mathcal{C}}$ is the unit normal vector field on the surface $\mathcal{C}$. The Laplace-Beltrami operator $\Delta_{\mathcal{C}} := \mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2$ is written in terms of Günter's tangent derivatives $\mathcal{D}_j := \partial_j - \nu_j \partial_\nu$, $j = 1, 2, 3$, $\partial_\nu = \sum_{j=1}^{3} \nu_j \partial_j$. The vector field $\nu_\Gamma(t) = (\nu_{\Gamma,1}(t), \nu_{\Gamma,2}(t), \nu_{\Gamma,3}(t))$, $t \in \Gamma$ is normal (orthogonal) to the boundary Γ and is tangential to the surface $\mathcal{C}$, points outside the surface. ∂_{ν_Γ} represents the normal derivative on the boundary $\partial\mathcal{C} = \Gamma$.

Lax-Milgram Lemma applied to the BVP (1) gives that it has a unique solution in the classical setting $f \in \widetilde{\mathbb{H}}^{-1}(\mathcal{C}), \quad g \in \mathbb{H}^{1/2}(\Gamma), \quad h \in \mathbb{H}^{-1/2}(\Gamma)$.

But in some problems, for example, in approximation methods, it is important to know the solvability properties in the non-classical setting

$$f \in \widetilde{\mathbb{H}}^{s-2}_p(\mathcal{C}), \quad g \in \mathbb{W}^{s-1/p}_p(\Gamma), \quad h \in \mathbb{W}^{s-1-1/p}_p(\Gamma), \quad 1 < p < \infty, \quad s > \frac{1}{p}. \tag{2}$$

Theorem. *Let $1 < p < \infty$, $s > 1/p$. The BVP* (1) *is not Fredholm in the non-classical setting* (2) *if and only if*

$$\cos^2 \pi\, s - \left| \sin 2\pi \left(s - \frac{1}{p} \right) \right| \neq 0, \tag{3}$$

Moreover, necessary and sufficient conditions are found on the pair (p, s) which ensure the unique solvability of the BVP (1).

The investigation was carried out in collaboration with V. Didenko (Brunei) and M. Tsaava (Georgia).

Quantum Hamiltonians Exhibiting a Spectral Transition

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The aim of this talk is to discuss several classes of Schrödinger operators with potentials that are below unbounded but their negative part is localized in narrow channels. A prototype of such a behavior can be found in Smilansky-Solomyak model devised to illustrate that an an irreversible behavior is possible even if the heat bath to which the systems is coupled has a finite number of degrees of freedom. We review its properties and analyze several modifications of this model, with regular potentials or a magnetic field, as well as another system in which $x^p y^p$ potential is amended by a negative radially symmetric term. All of them have the common property that they exhibit an abrupt parameter-dependent spectral transition: if the coupling constant exceeds a critical value the spectrum changes from a below bounded, partly or fully discrete, to the continuous one covering the whole real axis. We also discuss resonance effects in such models. The results come from a common work with Diana Barseghyan, Andrii Khrabustovskyi, Jiří Lipovský, Vladimir Lotoreichik, and Miloš Tater.

Deformation of Complex Structures of Generalized Analytic Functions Point of View

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We discuss relationship between pseudoanalytic functions of first and second kinds on complex plane and lthis known relations be applied for the study similar problem for nonregular CBV equations [1]. Periodisity problem for such system and related topics also will considered.

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On the Approximation of Continuous Functions of Two Variables by Fourier Angular Partial Sums

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The angular partial sums of double Fourier series of function $f(x_1, x_2)$ are defined by the following equality

$$\breve{S}_{n_1 n_2}(f, x_1, x_2) = S^{(1)}_{n_1}(f, x_1, x_2) + S^{(2)}_{n_2}(f, x_1, x_2) - S_{n_1 n_2}(f, x_1, x_2),$$

where $S^{(1)}_{n_1}(f, x_1, x_2)$ is the partial sum of Fourier series of the function f of n_1 - order with x_1 variable, $S^{(2)}_{n_2}(f, x_1, x_2)$ is the partial sum of n_2 - order with x_2 variable, and $S_{n_1 n_2}(f, x_1, x_2)$ is the partial sum of order n_1, n_2, with x_1 and x_2 variables. We have

Theorem. *Suppose $f \in C(T^2), T = [-\pi, \pi]$, then*

$$\|f - \breve{S}_{n_1 n_2}(f)\|_{C(T^2)} \le A H_{n_1 n_2}(f)_{C(T^2)} \ln n_1 \ln n_2,$$

where A is an absolute constant, and $H_{n_1, n_2}(f)_{C(T^2)}$ is the best approximation of the function f by the trigonometric quasipolynomials of order n_1, n_2.

Corollary 1.

$$\|f - \breve{S}_{n_1 n_2}(f)\|_{C(T^2)} \le B \omega \left(f, \frac{1}{n_1}, \frac{1}{n_2}\right)_{C(T^2)} \ln n_1 \ln n_2,$$

where B is an absolute constant and $\omega\big(f, \frac{1}{n_1}, \frac{1}{n_2}\big)_{C(T^2)}$ is the mixed modulus of continuity of the function f.

Corollary 2. If

$$\omega \left(f, \frac{1}{n_1}, \frac{1}{n_2}\right)_{C(T^2)} = o\left(\frac{1}{\ln n_1 \ln n_2}\right),$$

then angular partial sums of the function f are uniformly convergent to the function f.

Corollary 3. If

$$\omega^{(1)} \left(f, \frac{1}{n_1}\right)_{C(T^2)} = o\left(\frac{1}{\ln^2 n_1}\right), \quad or \quad \omega^{(2)} \left(f, \frac{1}{n_2}\right)_{C(T^2)} = o\left(\frac{1}{\ln^2 n_2}\right),$$

then for $\lambda \in [1, \infty]$,

$$\lim_{(n_1 n_2)_\lambda \to \infty} \|f - \breve{S}_{n_1 n_2}(f)\|_{C(T^2)} = 0,$$

where $\omega^{(1)}\big(f, \frac{1}{n_1}\big)_{C(T^2)}$ and $\omega^{(2)}\big(f, \frac{1}{n_2}\big)_{C(T^2)}$ are the partial moduli of continuity of the function f with x_1 and x_2 variables, respectively.

A Beam - Just a Beam in Plane Bending

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We derive one-dimensional beam theories from the three-dimensional theory of linear elasticity by a power-law expansion of the displacements in height and width direction. The strain energy and the potential of external forces are calculated and integrated over the cross-sectional area. Both appear as power laws of the small beam parameters $c^2 = h^2/(12\ell^2)$ and $d^2 = b^2/(12\ell^2)$, where ℓ is the characteristic dimension in length direction of the beam and h and b are height and width of the rectangular cross-section, respectively. Hierarchical beam theories arise from the consistent truncation of the elastic energy after a specific power $2N = 2n + 2m$ of the fast decaying factors $c^{2n}d^{2m}$. It turns out that the first-order ($N = 1$) approximation delivers the classical Euler-Bernoulli beam theory whereas the second-order approximation ($N = 2$) leads to a Timoshenko-type theory. A special feature of the derivation is that no a priori assumptions are invoked.

Deformation and Stability of Having Ribs Shells

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A new method of analysis of having breaks and ribs shells in conditions of non-linear deformation is developed. This method gives the possibility to study based on unified methodological foundation of discrete and continual systems. In addition, different types of irregularities are taken into account. Certain general methods of generation of differential equations, the methods of their simplification in private cases, for the analysis of shells and plates with discontinuous parameters in conditions of non-linear deformation are considered. The methods of differential equations with variable and impulsive coefficients solution are specified.

A new method of analysis of thin shells that are reinforced by same direction and orthogonally arranged ribs. The geometric and nonlinear tasks of the shells reinforced by restricted length ribs and discreetly attached ribs to the first time have been solved.

Numerical Treatment of Interfaces

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In this contribution we will consider moving interfaces and partial differential equations on moving interfaces in different contexts. First we will present the existence, uniqueness and numerical experiments for solutions of nonlinear conservation laws on moving surfaces. In addition to the "hydrodynamical" shocks, geometrically induced shocks will appear. In the second part we study the compressible two phase flow with phase transition on the basis of the Navier-Stokes-Korteweg- and a phasefield model. It turns out that it is extremely important for the numerical schemes of both models that they satify a discrete energy inequality to satisfy the second law of thermodynamics. Different numerical experiments will be presented. In the third part we will report on recent research on our experience of the application of the volume of fluid method (VOF) for the resolution of interfaces. The main advantage compared to level set methods is , that the VOF method is mass conserving. We will show different numerical experiments for the movement of droplets on solid walls.

These results have been obtained together with S. Burbulla, D. Diehl, J. Gerstenberger, M. Kränkel, T. Malkmus, T. Müller, M. Nolte, C. Rohde.

Interval Edge Coloring of Bipartite Graphs with Degree Bounded by 4

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An edge coloring of a graph G is called *interval edge coloring* if for each $v \in V(G)$ the set of colors on edges incident to v forms an interval of integers. A graph G is *interval colorable* if there is an interval coloring of G. For an interval colorable graph G, by the *interval chromatic index* of G, denoted by $\chi'_i(G)$, we mean the smallest number k such that G is interval colorable with k colors. The problem of interval edge coloring of bipartite graphs with the minimum number of colors has an application in the open shop scheduling problem with unit time operations, *no wait&idle* criterion and with the minimum makespan.

A bipartite graph G is called (α, β)-biregular if each vertex in one part has degree α and each vertex in the other part has degree β. A graph G is called (α^*, β^*)-bipartite graph if G is a subgraph of an (α, β)-biregular graph and the maximum degree in one part is α and the maximum degree in the other part is β.

The following problems of interval edge coloring are known to be $\mathcal{NP}$-complete: 6-coloring of $(6, 3)$-biregular graphs [1], 5-coloring of $(5^*, 5^*)$-bipartite graphs [2], and 5-coloring of $(5^*, 3^*)$-bipartite graphs [5]. In [3] the author showed that any $(3^*, 3^*)$-bipartite graph has an interval edge 4-coloring, which can be constructed in $O(n)$-time. In [2] the author proved that an interval edge k-coloring of every (k^*, k^*)-bipartite graph can be found in $O(n^{3/2})$-time (if it exists), for $k = 3, 4$. By the results of [4], every $(2k, 2)$-biregular graph admits an interval edge $2k$-coloring, and every $(2k + 1, 2)$-biregular graph admits a $(2k + 2)$-coloring, for every $k \geq 1$. In [5] the authors proved that every $(5^*, 2^*)$-bipartite graph admits an interval edge 6-coloring, which can be found in $O(n^{3/2})$-time, and the existing of an interval edge 5-coloring of a $(5^*, 2^*)$-bipartite graph can be verified in $O(n^{3/2})$-time. Moreover, they showed that every $(4^*, 2^*)$-bipartite graph admits an interval edge 4-coloring, which can be constructed in $O(n)$-time. In the paper we extend that result by showing that each bipartite graph of degree bounded by 4, where each vertex of degree ≥ 3 has neighbours of degree ≤ 2, admits 4-coloring, which can be found in $O(n^{3/2})$-time.

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Applications of the Riemann-Hilbert and $\mathbb{R}$-linear Problems to Determination of the Effective Properties of 2D Random Composites

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2D composites with non-overlapping inclusions randomly embedded in the matrix are investigated by means of the exact solution to the Riemann-Hilbert and $\mathbb{R}$-linear problems for a circular multiply connected domain. Symbolic-numerical computations are applied to estimate the effective properties [1, 2]. First, deterministic boundary value problems are solved for all locations of inclusions, i.e., for all events of the considered probabilistic space $\mathcal{C}$ by the generalized method of Schwarz. Second, the effective properties are calculated in the analytical form and averaged over $\mathcal{C}$. This method is related to the classic method based on the average probabilistic values involving the n-point correlation functions. However, we avoid computation of the correlation functions and compute their weighted moments of high orders by an indirect method which does not address to the correlation functions. The effective properties are exactly expressed through these moments. It is proved that the generalized method of Schwarz converges for an arbitrary multiply connected doubly periodic domain and for an arbitrary contrast parameter. The proposed method yields effective in symbolic-numeric computations formulae of high order in concentration.

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Dynamical Problems of Generalized Thermo-Electro-Magneto-Elasticity theory

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This talk is dedicated to the theoretical investigation of basic, mixed and crack type three-dimensional initial-boundary value problems of the generalized thermo-electro-magneto-elasticity theory associated with Green-Lindsay's model. The essential feature of the generalized model under consideration is that heat propagation has a finite speed. We analyze dynamical initial-boundary value problems and the corresponding boundary value problems of pseudo-oscillations which are obtained from the dynamical problems by the Laplace transform.

The dynamical system of partial differential equations generate a nonstandard 6×6 matrix differential operator of second order, while the system of partial differential equations of pseudo-oscillations generates a second order strongly elliptic formally non-selfadjoint 6×6 matrix differential operator depending on a complex parameter.

First we prove uniqueness theorems of dynamical initial-boundary value problems under reasonable restrictions on material parameters and afterwards we apply the Laplace transform technique to investigate the existence of solutions.

This approach reduces the dynamical problems to the corresponding elliptic problems for pseudo-oscillation equations.

The fundamental matrix of the differential operator of pseudo-oscillations is constructed explicitly by the Fourier transform technique, and its properties near the origin and at infinity are established. By the potential method the corresponding three-dimensional basic, mixed and crack type boundary value problems, and the transmission problems for composite elastic structures are reduced to the equivalent systems of boundary pseudodifferential equations. The solvability of the resulting boundary pseudodifferential equations are analyzed in appropriate Sobolev-Slobodetskii (W_p^s), Bessel potential (H_p^s), and Besov ($B_{p,q}^s$) spaces and the corresponding uniqueness and existence theorems of solutions to the boundary value problems under consideration are proved. The smoothness properties and singularities of thermo-mechanical and electro-magnetic fields are investigated near the crack edges and the curves where the different types of boundary conditions collide. It is shown that the smoothness and stress singularity exponents essentially depend on the material parameters and an efficient method for their computation is described.

By the inverse Laplace transform the solutions of the original dynamical initial-boundary value problems are constructed and their smoothness and asymptotic properties are analyzed in detail.

Structure and Mechanical Properties of Random Composites

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This paper is devoted to the study of the plastic deformation generated in friction stir processing on the changing concentration and distribution of SiC reinforcement particles in the cast composite A339/SiC/p, as well as determine its mechanical properties. Aluminum matrix reinforced with 10 % SiC particles was modified using friction stir processing method. Changing of distribution of the reinforcement particles was calculated and analysed using new mathematical RVE theory with Mityushev - Eisenstein - Rayleigh sums [1] and PointSel software. The mechanical properties are determined on the basis of compression and hardness tests. The significant changes in the concentration and distribution of SiC particles are observed. Mechanical testing of selected deformed areas showed significant differences in the values of the plastic flow stress and hardness in thermomechanical hardening areas.

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Quivers: Representations, Bundles, Applications

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Quivers or directed graphs and their representations were used to describe representations of finite dimensional algebras (see, e.g., [3]). However, they appear in quite a few numbers of applications, such as control theory ([4], [2]) and data analysis [5]. In algebraic geometry, quiver representations have been generalized to quiver bundles (see, e.g., [1]).

In the talk, I will give a brief overview over quivers and their representations. Then, I will sketch the applications mentioned above. In the final section, I will report on recent results of mine from [6] concerning quiver bundles.

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The Spectral Localizer for Even Index Pairings

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Even index pairings are integer-valued homotopy invariants combining an even Fredholm module with a K0-class specified by a projection. Numerous classical examples are known from differential and non-commutative geometry and physics. Here it is shown how to construct a finite dimensional selfadjoint and invertible matrix, called the spectral localizer, such that half of its signature is equal to the even index pairing. This makes the invariant numerically accessible. The index-theoretic proof heavily uses fuzzy spheres.

Steady Flows of Conductive Liquid in a Rectangular Channel

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In the present article we consider the steady flow of an incompressible viscous electrically conducting fluid in rectangular channel when there is a transverse magnetic field. An exact solution of the problem in general terms is obtained.

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The First Group of Necessary Conditions for the Boundary Value Problem of the Fourth-Order Partial Differential Equation

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My talk will be devoted to obtaining the first basic relation, which is obtained from the second Green's formula of the self-adjoint fourth-order partial differential equation and the fundamental solution of this equation. Then the necessary conditions are obtained from this basic relation.

On Physical Soundness of Von Kármán Type Systems and Cauchy Problems for Evolutionary Equations

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The report contains two parts.

I. One of the most principal objects in development of mechanics and mathematics is a system of nonlinear differential equations for an elastic isotropic plate constructed by von Kármán. In 1978 Truesdell expressed a doubt: "Physical Soundness" of von Kármán system. This circumstance generated the problem of justification of von Kármán system. Afterwards this problem has been studied by many authors, but with most attention it was investigated by Ciarlet. In particular, he wrote: "The von Kármán equations may be given a full justification by means of the leading term of a formal asymptotic expansion" ([1], p. 368). This result obviously is not sufficent for a justification of "Physical Soundness" of this system, because representations by asymptotic expansions is dissimilar and leading terms are only coefficients of power series without any "Physical Soundness." Based on our works, the method of constructing such anisotropic nonhomogeneous 2D nonlinear models of von Kármán-Mindlin-Reissner (KMR) type for binary mixtures; (poro/visco/piezo-electric/electrically conductive)elastic thin-walled structures with variable thickness is given, by means of which the terms become physically sound. The corresponding variables are quantities with certain physical meaning: averaged components of the displacement vector, bending and twisting moments, shearing forces, rotation of normals, surface efforts. The given method differs from the classical one by the fact that according to the classical method, one of the equations of von Kármán system represents one of Saint-Venant's compatibility conditions, i.e. it's obtained on the basis of geometry and not taking into account the equilibrium equations.

II. In the second one if we consider the problems connected with an extension(enlarge) of initial data for constructing by evident scheme to finding the approximate solution of evolutionary equations by high order of accuracy than Resolvent methods (or semi group operators theory) [see, for example, 2] or Courant, von Neumann, Lax direct methods for approximate solution some problems of mathematical physics [see, for example, 3]. As it's well known for Resolvent methods for solving by high order of accuracy lies in the best approximation of corresponding kerners while for Difference methods difficulties represent incorrectness of multipointing (high order of accuracy) schemes. In the report we construct the explicit schemes giving the approximate solution of some initial-boundary value problems by arbitrary order of accuracy depending only on order of smoothness of the desired solution.

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C O N T E N T S