CONORMAL SOLUTIONS OF HYPERBOLIC DIFFERENTIAL EQUATIONS OF VISHIK-GRUSHIN TYPE

Ingo Witt

Mathematical Institute of Georg-August University Goettingen, Goettingen, Germany

We proved in 2015 that the unique local solution u of the Cauchy problem

$$\begin{cases} u_{tt} - t^{2\ell_*}\Delta u = f(t,x,u), & (t,x) \in (0,T) \times \mathbb{R}^n, \\ u\big|_{t=0} = 0, \quad u_t\big|_{t=0} = H(x_1)\psi(x), & \end{cases}$$

where $2\ell_* \in \mathbb{N}_0$ and $\psi \in \mathscr{C}_c^\infty(\mathbb{R}^n)$, is conormal with respect to the two characteristic hypersurfaces $\{x_1 = \pm\, t^{\ell_*+1}/(\ell_* + 1)\}$ emanating from $\{x_1 = 0\}$. Key was to show that the solution of the corresponding linear equation belongs to $L^\infty((0,T) \times \mathbb{R}^n)$.

To generalize this result, we study the linear Cauchy problem for higher-order degenerate hyperbolic differential operators of Vishik-Grushin type with variable coefficients. We show that the unique solution is a sum of distributions, where each summand is conormal with respect to one of the characteristic hypersurfaces emanating from a hypersurface Σ in the initial hypersurface, provided the Cauchy data is conormal with respect to Σ. To this end, we extend the notion of conormal distribution to a class of distributions which exhibit the correct degenerate behavior at the initial hypersurface, and we develop a symbolic calculus for this new class of distributions.

This is joint work with Ruan Zhuoping (Nanjing University) and Yin Huicheng (Nanjing Normal University).