On some integral equations of the first kind in potential theory

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The classical potential method applied to three-dimensional BVPs leads to the study of two-dimensional singular integral equations.

In this talk we describe an indirect method, different from the classical one, proposed for the first time in [1] for the Dirichlet problem for Laplace equation

$$\begin{cases} \Delta u = 0, & \text{in } \Omega, \\ u = f, & \text{on } \Sigma \end{cases} \tag{1}$$

(Ω being a bounded simply connected domain of $\mathbb{R}^n$ with a Lyapunov boundary $\Sigma = \partial\Omega$), which gives the solution of (1) in the form of a simple layer potential. The boundary condition turns into the following integral equation of the first kind

$$\int_\Sigma \varphi(y) s(x,y) d\sigma_y = f(x), \qquad x \in \Sigma,$$

which can be reduced to an equivalent Fredholm equation. This approach can be considered as an extension of the one given by Muskhelishvili [5] in the complex plane.

The method hinges on the theory of reducible operators and on the theory of differential forms; it does not require the use of pseudo-differential operators nor the use of hypersingular integrals.

This approach has been applied to different BVPs for other PDEs, also for multiply connected domain. If $n = 2$, there are particular boundaries, that are said to be exceptional, for which the solution of the Dirichlet problem (1) cannot be represented by a simple layer potential; exceptional domains are avoided by scaling up.

We conclude with the generalization of the method to the basic boundary value problems of the three-dimensional Cosserat elasticity and to the ones of the linear theory of viscoelasticity for Kelvin-Voigt materials with voids.

References

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