XXXVII International Enlarged Sessions of the Seminar of Ilia Vekua Institute of Applied Mathematics of Ivane Javakhisvili Tbilisi State University

Book of Abstracts

April 19-22, 2023
Tbilisi

Organizing Committee

Jaiani George (Chair)
Avazashvili Nikoloz (Co-Chair)
Chinchaladze Natalia (Co-chair)
Jangveladze Temur (Co-chair)
Gulua Bakur (Scientific Secretary, Contact Person, bak.gulua@gmail.com)
Rukhaia Mikheil (Scientific Secretary, Contact Person, mrukhaia@yahoo.com)
Gvaramadze Manana (Technical Secretary)
Sharikadze Meri (Technical Secretary)
Tevdoradze Manana (Technical Secretary)
Amaghlobeli Mikheil
Antidze Jemal
Akhalaia George
Baaz Matias (Austria)
Bakuradze Malkhazi
Danelia Ana
Davitashvili Temur
Dundua Besik
Giorgadze Grigor
Goginava Ushangi
Gogoladze Leri
Kharazishvili Alekhander
Kharibegashvili Sergo
Khimshiashvili George
Kiguradze Zurab
Kipiani Archil
Koplatadze Roman
Nadaraya Elizbar
Natroshvili David
Omanadze Roland
Papukashvili Archil
Purtukhia Omar
Rogava Jemal
Shavadze Tea
Shavgulidze Ketevan
Tadumadze Tamaz
Vashakmadze Tamaz
Vepkhvadze Teimuraz

The present book of abstracts contains abstracts of talks given at XXXVII Enlarged Sessions (April 19-22, 2023) of the Seminar of I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University.
Each Section (there are 10 ones) is presented as separate Chapter of the book. The responsibility for the contents of each Chapter lies with leaders together with speakers.

SECTION OF MATHEMATICAL LOGIC AND FOUNDATIONS

Chairs: Alexander Kharazishvili, Roland Omanadze
Co-chair: Archil Kipiani

SOME APPLICATIONS OF FRAÏSSÉ LIMIT

Mariam Beriashvili
I. Vekua Institute of Applied Mathematics of I. Javakhishvil Tbilisi State University, Tbilisi, Georgia
Georgian Technical University, Tbilisi, Georgia
email: Mariam_beriashvili@yahoo.com

It is well known that Fraïssé Theory is a folklore of model theory. The most central property in investigation of the Fraïssé classes or Fraïssé limits is the amalgamation property, which says that two embeddings of a fixed object can be joined by further embeddings into a single one. Kubis and Solecki treated the special case of the class of finite-dimensional Banach spaces, essentially showing that their Fraïssé limit is the Gurarij space, which is therefore unique and universal, without ever actually uttering the phrase "Fraïssé limit" (and in a fashion which is very specific to Banach spaces).

In the presented talk, we show some applications of Fraïssé limits in the study of special objects of measure theory. In particular, we demonstrate several set-theoretical methods in our investigation of measure extension problem.

Acknowledgements. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG), Grant YS-21-1667.

ON SOME INEQUALITIES OF GEOMETRIC TYPE

Shalva Beriashvili
Georgian National University (SEU), Tbilisi, Georgia
email: s.beriashvili@seu.edu.ge

It is well known that many inequalities of geometric type help us to solve several concrete problems and questions in the convex, combinatorial and discrete geometry (see. [2], [3]). In the presented talk we discuss one geometric type inequality, so called the Erdös -Mordell inequality in the Euclidean plane R^2 and show, that the Erdös-Mordell inequality holds not true in the Euclidean space R^n, where $n \geq 3$ (see. [1], [4]).

Furthermore, let a non-degenerate triangle $[A, B, C]$ be given in the plane R^2 and r denote the radius of the inscribed circle of $[A, B, C]$, and let O be an arbitrary point in $[A, B, C]$. Let us put

$$R_A = |O - A|, \quad R_B = |O - B|, \quad R_C = |O - C|$$

Then the following inequality holds true:

$$R_A + R_B + R_C \geq 6r$$

Herewith, the equality is attained here only in the case when $[A, B, C]$ is an equilateral triangle and O coincides with the circumcenter of $[A, B, C]$. We show that the same relation holds true in the Euclidean space R^3, only after an appropriate changing of coefficients.

References:

1. P. Erdös, Problem 3740, Amer. Math. Monthly, 42 (1935) 396.
2. Kharazishvili A. *Elements of Combinatorial Geometry*, Part I, The Publishing House of Georgian National Academy of science, Tbilisi, 2016.
3. Kharazishvili A. *Elements of Combinatorial Geometry*, Part II, The Publishing House of Georgian National Academy of science, Tbilisi, 2020.
4. L. J. Mordell and D. F. Barrow, Solution 3740, Amer. Math. Monthly, 44 (1937) 252–254.

NOTES ON NOWHERE SIMPLE SETS AND CONJUNCTIVE REDUCIBILITY

Irakli Chitaia, Roland Omanadze
Department of Mathematics, Faculty of Exact and Natural Sciences,
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
emails: i.chitaia@gmail.com, roland.omanadze@tsu.ge

Shore [4] introduced the concept of nowhere simple set in the following way: let $A \subseteq \omega$, where ω denotes the set of natural numbers. A is a nowhere simple if A is c.e. and for every c.e. set B with $B–A$ infinite, there is an infinite c.e. set W such that $W \subseteq B–A$.

A set A is conjunctive reducible (or c-reducible) to a set B (in symbols: $A \leq_c B$) (see [1]), if there exist a computable function f such that for all $x \in \omega$, $x \in A \Leftrightarrow D_{f(x)} \subseteq B$, where D_u is the finite set with canonical index u.

Let $A \oplus B$ denotes the set $\{2x: x \in A\} \cup \{2x + 1: x \in B\}$.

Our notations and terminology are standard and can be found in [2] and [3]. In this talk we will present the following results:

Theorem 1. $A \oplus B$ is nowhere simple if and only if both of A and B are nowhere simple.

Theorem 2. Let A be a simple set, B an arbitrary set, C a nowhere simple set and suppose that $A \leq_c B \oplus C$. Then $A \leq_c B$.

Acknowledgments. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant number: N STEM-22-1837, Recursive Functions and Probabilistic Ontology Engineering].

References

1. Jockusch C.G.Jr.: Reducibilities in Recursive Functions Theory. PhD Thesis, Massachusetts 1. Institute of Technology. USA, Massachusetts, 1966. Massachusetts
2. Rogers H.: Theory of Recursive Functions and Effective Computability. MIT Press, Cambrige, MA, USA, 1967.
3. Soare R.I.: Recursively Enumerable Sets and Degrees. Springer-Verlag, Berlin, 1987.
4. Shore R.A.: Nowhere simple sets and the lattice of recursively enumerable sets. J. Symb. Logic, 43, 2 (1978), 322-330.

ON A PROBLEM OF GROUP REPRESENTATION

Mariam Gobronidze[1], Archil Kipiani[2]

[1]I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: mariami.gobronidze030@ens.tsu.edu.ge

[2]Department of Mathematics, Faculty of Exact and Natural Sciences, I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: archil.kipiani@tsu.ge

Kőnig's problem is discussed concerning the representation of abstract groups using the automorphisms group of a graph. It is proven that for every infinite group G and for every cardinal number α, if $Card(G) \leq \alpha$, then there exists a $H(G)$ graph with cardinality α, such that its automorphisms group is isomorphic to the G. Moreover, there exist 2^{α} many pairwise non-isomorphic graphs of this type, with cardinality α. This essentially strengthens the result of Gert Sabidussi, where the cardinality of the constructed graph is essentially greater than the cardinality of the group itself.

ON VARIOUS ASPECTS OF DEFINITIONS OF EQUIDECOMPOSABILITY OF SETS

Tamar Kasrashvili
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
Department of Mathematics, Georgian Technical University, Tbilisi, Georgia
email: tamarkasrashvili@yahoo.com

The present report is devoted to some aspects of geometrical and set-theoretical definetions of equidecomposability of point sets. Connections between the notions of finite equidecomposability and countable equidecomposability of point sets (figures) are shown. In particular:

(a) if sets X and Y are finitely equidecomposable, then they are also countably equidecomposable, but the converse assertion does not hold;

(b) if in the space R^n some sets X and Y are such that $\lambda_n(X) > 0$ and $\lambda_n(Y) = 0$, then these sets are not countably equidecomposable under the group of all affine transformations of R^n;

(c) in R^n there exist two sets X and Y such that $card(X) = card(Y) = c$ and X is not countably equidecomposable with Y, under a sufficiently large group of transformations of R^n;

(d) in R^n two point sets are countably equidecomposable if both of them have interior points.

The latter implies that in the space R^n there exists a non-measurable set with respect to the Lebesgue measure.

References

1. Hadwiger, H. Lectures on Volume, Surface Area, and Isoperimetry (in Russian, translation from German), Moscow, 1966.
2. Kharazishvili, A.B. *Transformation Groups and Invariant Measures*. Set-Theoretical Aspects. World Scientific Publishing Co., Inc., River Edge, NJ, 1998.

ON ALMOST NON-INVARIANT SETS

Marika Khachidze
Ph.D. student of Georgian Technical University, Tbilisi, Georgia
email: m.khachidze1995@gmail.com

In the presented talk we consider the notion of almost non-invariant sets and discuss several properties of them.

Let E be a basic space, which is equipped with a transformation group G, and let $X \subset E$. We say that X is almost non-invariant with respect to G, if for each transformation $g \in G$, we have

$$\text{card}\{g: g(X) \cap X \neq \emptyset\} < card(G).$$

According to Kunen's result, if the cardinal c of the continuum is real-valued measurable, then there exists a λ-nonmeasurable set $X \subset \mathbf{R}$ with $card(X) < c$ (see [3]). It is shown that such X is an almost non-invariant set (cf. [1], [2]).

References

1. M. Khachidze, A. Kirtadze, On one example of application of almost invariant sets, Reports of Enlarged Sessions of the Seminar of I. Vekua Institute of Applied Mathematics, Volume 32, 2018.
2. A.B. Kharazishvili, Nonmeasurable Sets and Functions, Elsevier, Amsterdam, 2004.
3. K. Kunen, Inaccessibility properties of cardinals, Ph.D. Thesis, Department of Mathematics, Stanford University, 1968.

DYNAMIC INTUITIONISTIC LOGIC

Antonio Di Nola[1], Revaz Grigolia[2], Giacomo Lenci[1], Gaetano Vitale[1]

[1]University of Salerno, Salerno, Italy

emails: adinola@inisa.it, gilenci@inisa.it, gvitale@inisa.it

[2]I. Javakhishvili Tbilisi State University,

Georgian Technical University, Tbilisi, Georgia

email: revaz.grigolia@tsu.ge

The dynamic intuitionistic logic DI_{KC}, is introduced that is generalization of monadic propositional Intuitionistic logic satisfying the weak law of excluded middle, and corresponding to it dynamic Heyting algebras, which are algebraic counterparts of the logic, that in turn represent two-sorted algebras (H, R, $\Diamond$, $\Box$) that combine the varieties of Heyting algebras (H, $\vee$, $\wedge$, $\rightarrow$, 0,1), satisfying the identity $\neg x \vee \neg\neg x = 1$, and regular algebras ($R$,$\cup$, ;, *) into a single finitely axiomatized variety resembling R-modules with two "scalar" multiplications $\Diamond$ and $\Box$. Kripke semantics is developed with application in neural networks. Having some formula and an estimation of its propositional variables, an activation function is defined. This is a way to represent how the neural networks change the information after some action that produces new information about the neural network state, including the states of all points of the Kripke frame.

ON THE STABILITY QUESTION FOR ONE PROBLEM OF COMBINATORIAL GEOMETRY

Tengiz Tetunashvili
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
Georgian Technical University, Tbilisi, Georgia
emails: tengiztetunashvili@gmail.com, t.tetunashvili@gtu.ge

In the talk the following two related questions are considered and answered:

1) Let d be an arbitrary fixed non-negative real number and let n be an arbitrary fixed natural number such that $n \geq 3$. Is there a non-negative real number $r_{n,d}$ having the property: if S is a subset of the Euclidean plane with $card(S) = n$ such that, for every straight line $L_{a,b}$ passing through two distinct points a and b of S, there exists a point c of S such that $a \neq c$ and $b \neq c$ and the distance between $L_{a,b}$ and c does not exceed d, then there exists a straight line L in the plane such that the distance from an arbitrary point of S to L is less than or equal to $r_{n,d}$?

2) Let N_3 be the set of all natural numbers greater than 2, let d be an arbitrary fixed positive real number, and let n be an arbitrary fixed natural number such that $n \geq 3$. Is there a function of two variables $f(m,y)$ acting from $N_3 \times (0,+\infty)$ into $(0,+\infty)$, satisfying $\lim_{y \to 0+} f(m,y) = 0$ for every $m \in N_3$, and if S is a subset of the plane with $card(S) = n$ such that, for every straight line $L_{a,b}$ passing through two distinct points a and b of S there exists a point c of S such that $a \neq c$ and $b \neq c$ and the distance from c to $L_{a,b}$ does not exceed d, then there exists a straight line L in the plane such that the distance from an arbitrary point of S to L is less than or equal to $f(n,d)$?

Besides, an interrelation between the presented questions and the Sylvester-Gallai well-known theorem (see, e.g., [1], [2], [3]) is considered.

References

1. Hadwiger H., Debrunner H. *Combinatorial Geometry in the Plane. Translated by Victor Klee. With a new chapter and other additional material supplied by the translator*, Holt, Rinehart and Winston, New York, 1964.
2. Kharazishvili A. *Elements of Combinatorial Geometry*, Part I, Tbilisi, 2016.
3. Sylvester J. J. Question 11851, *Educational Times,* **59** (1893), p. 98.

ON PROVABLE INEQUALITIES BETWEEN CARDINAL CHARACTERISTICS OF THE CONTINUUM

Thilo Weinert
Mathematical Institute of the University of Vienna, Vienna, Austria
email: thilo.weinert@univie.ac.at

Cardinal characteristics of the continuum are combinatorially defined uncountable cardinals of size at most that of the continuum. They often stem from problems in mathematical fields outside of logic. At the same time, they provide good test problems and further the development of forcing theory as more and more sophisticated techniques are developed to show that certain inequalities between them are unprovable by ZFC. I am going to present a few hitherto unknown inequalities provable by ZFC.

SECTION OF APPLIED LOGICS AND PROGRAMMING

Chair: Matthias Baaz (Austria)
Co-chair: Jemal Antidze, Besik Dundua, Mikheil Rukhaia

NON-CLASSICAL TECHNIQUES IN INDUCTIVE LOGIC PROGRAMMING WITH STATISTICAL RELATIONAL ARTIFICIAL INTELLIGENCE

Merihan Hazem Anwar Labib Bishara
International Black Sea University, Tbilisi, Georgia
emails: merihan.hazem20@gmail.com, merihan.hazem@yahoo.com

One of the fundamental outstanding challenges in artificial intelligence is merging descriptive knowledge representation formalisms like relational and first-order logic with principled probabilistic and statistical techniques for inference and learning. Relational and logical reasoning, probabilistic and statistical reasoning, and machine learning are considered traditional research fields in their own rights. In today's world, they are becoming highly connected.

Inductive logic programming is a combination of logic programming and machine learning. It is a type of machine learning that stimulates logic programs from data. Its attempt is to provide a formal framework and also practical algorithms for inductively learning relational descriptions (in the form of logic programs) from examples and background knowledge. However, it does not specifically handle uncertainty, like missing or noisy information.

The fundamental strategy for dealing with noise is the unification of logical and probabilistic reasoning in an inductive setting, which is the main focus of statistical relational artificial intelligence (StarAI). Since StarAI is an emerging domain, combining probability and logic would be a huge accomplishment.

The talk is about a future plan for my PhD thesis that aims to unify logical and probabilistic reasoning in an inductive setting which is the focus and concentration of statistical relational artificial intelligence (StarAI) and study how inductive logic programming with its formalisms, contexts, and methods can be expanded to deal with probabilities and develop the probabilistic ILP using statistical relational artificial intelligence (StarAI).

REASONING IN LOGIC WITH UNRANKED AND PROBABILISTIC PRIMITIVES

Merium Hazem Anwar Labib Bishara
International Black Sea University, Tbilisi, Georgia
email: merium.hazem20@gmail.com, meriumhazem@yahoo.com

All probabilistic logic formalisms studied so far permit only individual variables, i.e., variables that can be instantiated by a single term. On the other hand, theories and systems that use not only individual variables but also sequence variables have emerged.

The development of several formalisms and programming tools is needed in our daily lives since Probability theory deals with the challenges posed by uncertainty, while logic is more often used for reasoning with perfect knowledge. The idea is to combine both logical and probabilistic methods together in a single framework and develop an unranked probabilistic logic.

Since probabilistic logic formalisms only permit individual variables, extending probabilistic logic with sequence variables and flexible arity function and predicate symbols will be needed, as also developing reasoning methods for the unranked probabilistic logic.

The obtained formalism will have rich capabilities for multiple applications like Semantic Web, machine translation, etc.., we will be able to investigate applicability in e-shopping, randomized encryption, and computational linguistics, furthermore the expected results of this research can be used in many other scientific fields and in the industry field as well.

UNRANKED PROBABILISTIC THEORY: PROJECT PRESENTATION

Anriette Michel Fouad Bishara[1], Lia Kurtanidze[1,2], Mikheil Rukhaia[1], Lali Tibua[1,2]
[1]I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
[2]Georgian National University (SEU), Tbilisi, Georgia
emails: anriettehazem@yahoo.com, lia.kurtanidze@gmail.com, mrukhaia@logic.at, ltibua@gmail.com

Since the early days of Artificial Intelligence logical and probabilistic methods have been independently used in order to solve tasks that require some sorts of intelligence. Probability theory deals with the challenges posed by uncertainty, while logic is more often used for reasoning with perfect knowledge. Considerable efforts have been devoted to combining logical and probabilistic methods in a single framework, which influenced the development of several formalisms and programming tools.

All probabilistic logic formalisms studied so far permit only individual variables, that can be instantiated by a single term. On the other hand, theories and systems that use also sequence variables (these variables can be replaced by arbitrary finite, possibly empty,

sequences of terms) and unranked symbols (function and/or predicate symbols without fixed arity) have emerged. The unranked term is a first-order term, where the same function symbol can occur in different places with different number of arguments. Unranked function symbols and sequence variables bring a great deal of expressiveness in language. Therefore, it is actual to study extension of probabilistic logic with sequence variables and flexible-arity function and predicate symbols.

This talk is a presentation of the fundamental research project, that aims to develop an unranked probabilistic logic, study its properties and introduce reasoning method for it. We discuss preliminary results obtained during

Acknowledgements. This work was supported by Shota Rustaveli National Science Foundation of Georgia under the project FR-22-4254.

HIGHER-ORDER UNIFICATIN WITH REGULAR TYPES

Besik Dundua
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
Kutaisi International University, Tbilisi, Georgia
email: bdundua@gmail.com

In this talk we extend simply typed lambda calculus with regular types and study properties of the extended formalism. Moreover, we construct higher-order unification procedure for regularly typed lambda terms, and prove soundness and completeness theorems.

Acknowledgements. This work was supported by Shota Rustaveli National Science Foundation of Georgia under the project FR-21-7973.

SECTION OF ALGEBRA, GEOMETRY AND NUMBER THEORY

Chairs: Mikhail Amaghlobeli, George Khimshiashvili, Teimuraz Vepkhvadze, Malkhaz Bakuradze
Co-chair: Ketevan Shavgulide

ON THE VARIETIES OF EXPONENTIAL MR–GROUPS

Mikheil Amaglobeli[1], Alexei Myasnikov[2], Teona Nadiradze[3]
[1]I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
mikheil.amaglobeli@tsu.ge
[2]Stevens Institute of Technology, Hoboken, USA
amiasnikov@gmail.com
[3]I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
teona.nadiradze@tsu.ge

Definition of G exponential MR–group with respect to the ring R with unity is in [1]. Definition of free MR–groups is in [2]. In the section [3] there are the definitions of varieties of MR-group, Tensor completion of in verbal MR–group and varieties.

Theorem 1: 1) For any MR-group G, the R-commutant is the verbal MR-group defined by the word $x^{-1}y^{-1}xy$. 2) Let R be a field, then the α-commutator $(x,y)_\alpha = y^{-\alpha}x^{-\alpha}(xy)^{\alpha}$ generates an R-commutant as a verbal MR-group under the condition $\alpha \neq 0,1$.

Theorem 2: Let $G \in N_R$. Then a Tensor S-completion G_W^S -with respect to N_S exists, moreover $G_W^S = G^S / W(G^S)$.

Theorem 3: There exists a one-to-one correspondence between the lattice of bilateral ideals of a ring R and the lattice of verbal MR–groups of a free R–module.

Acknowledgements. The work is supported by the Shota Rustaveli National Science Foundation (SRNSF grant # FR 21-4713).

References

1. Myasnikov, A. G.; Remeslennikov, V. N. Degree groups. I. Foundations of the theory and tensor completions. (Russian) ; translated from Sibirsk. Mat. Zh. 35 (1994), no. 5, 1106–1118, rm iii Siberian Math. J. **35** (1994), no. 5, 986– 996.
2. Amaglobeli, M. G. Exponential MR-groups: faithful R-completion. Dokl. Math. **99** (2019), no. 3, 263–265.
3. Amaglobeli, M. G. Varieties of exponential MR-groups. Dokl. Math. **101** (2020), no. 1, 1–4.

ON THE LATTICE ISOMORPHISMS OF 2-NILPOTENT W-POWER GROUPS AND LIE ALGEBRAS

Tengizi Bokelavadze
Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia
email: Tengiz.bokelavadze@atsu.edu.ge

The paper deals with lattice isomorphisms of 2-nilpotent Hall W-power groups and Lie algebras. Analogues of the fundamental theorem of projective geometry are proved. A corresponding example is constructed.

Acknowledgements. The work is supported by the Shota Rustaveli National Science Foundation (SRNSF grant # FR 21-4713).

ON AN EXPLICIT CONSTRUCTION OF BICENTRIC QUADRILATERALS

Ana Diakvnishvili
Ilia State University, Tbilisi, Georgia
email: ana.diakvnishvili.1@iliauni.edu.ge

We present an effective method of constructing bicentric quadrilaterals. More precisely, we give an explicit construction of bicentric quadrilateral with prescribed two sides. In the symmetrical case, the distance between the two centers of the arising bicentric quadrilateral is computed, which in virtue of Fuss relation also gives an explicit formula for the radius of corresponding incircle. An analogous construction and some of its properties are given for bicentric polygons with arbitrary number of sides. In conclusion we present an interpretation of the main results in terms of the Kendall shape space and numerical results in several concrete cases.

A NOTE ON REPRESENTING DIMENSIONS

Johan Gielis
Universiteit Antwerpen, Departement Bio-ingenieurswetenschappen
Groenenborgerlaan 171, B-2020 Antwerpen, België
email: johan.gielis@ua.ac.be

The parametric description of 3D shapes via Gielis Transformations is based on two perpendicular planar cross sections. The planes can be real, complex, or the combination thereof. Only in the case of isotropic spaces, do (at least) two of these coincide, reducing to the spherical coordinate system. The perpendicular planes allow for different ways of

measuring in four different directions, which may be space or time, and which can vary in the course of movement or expansion in one or more directions, with some form of normalization as needed.

EXTREMAL PROBLEMS IN KENDALL SHAPE SPACES

Giorgi Khimshiashvili
Ilia State University, Tbilisi, Georgia
e-mail: giorgi.khimshiashvili@iliauni.edu.ge

We compute the extremal values of several geometric quantities in the loops of shapes of poristicpolygons in Kendall shape spaces.

Acknowledgements. This work was supported by Shota Rustaveli National Science Foundation of Georgia [grant FR-22-354 "Factorization problem and invariants of holomorphic bundles on Riemann surfaces"].

ON THE FUNDAMENTAL THEOREM OF GEOMETRIC ALGEBRA OVER SF-RINGS

Tamar Kvirikashvili
Department of Mathematics, Georgian Technical University, Tbilisi, Georgia
email: kvirikashvilitamar08@gtu.ge

For free modules over FS-rings, perspective maps are studied and thus the first ring version of the fundamental theorem of geometric algebra about the representation of perspective maps by the linear functions is proved.

References

1. Buekenhout F. Handbook of Incidence Geometry. Buildings and Foundations, Amsterdam: NorthHolland, 1995.
2. Lashkhi, A. A.; Bokelavadze, T. Z. A subgroup lattice and the geometry of Hall's W-power groups. (Russian); *translated from Dokl. Akad. Nauk* **429,** 6 (2009), 731-734 *Dokl. Math.,* **80,** 3 (2009), 891—894.

ON THE SPACES OF SPHERICAL POLYNOMIALS AND GENERALIZED THETA-SERIES FOR QUADRATIC FORMS OF FIVE VARIABLES

Ketevan Shavgulidze
I. Javakhishvili Tbilisi State University, Faculty of Exact and Natural Sciences, Tbilisi, Georgia
email: Ketevan.shavgulidze@tsu.ge

F. Gooding [1] constructed the spaces of spherical polynomials with respect to some positive definite binary quadratic forms and calculated the dimensions of the spaces of corresponding generalized theta-series are calculated. In [2], the upper bounds of the dimension of the space T(ν,Q) for some quadratic form of r variables are obtained. In this paper, some positive diagonal and non-diagonal quadratic forms of five variables are considered; the spaces of spherical polynomials with respect to these forms are constructed; corresponding generalized theta series are considered and the upper bounds of the dimensions (in some cases, the dimensions) of these spaces are obtained.

References

1. F. Gooding, Modular forms arising from spherical polynomials and positive definite quadratic forms, J. Number Theory **9**, (1977), 36-47.
2. K. Shavgulidze, On the space of generalized theta-series for certain quadratic forms in any number of variables, Mathematica Slovaca **69**, 1 (2019), 87-98.

ABOUT “CUTTING" OF THE GENERALIZED MÖBIUS-LISTING SURFACES AND BODIES

Ilia Tavkhelidze
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: ilia.tavkhelidze@tsu.ge

The report compares the results about “cutting” of the Generalized Möbius-Listing’s GML of bodies and surfaces obtained in different years 2006-2022 [1-2] . These results are formulated separately for surfaces and bodies. The notes show the common and different effects obtained after the "cutting" procedure. On the basis of the analytical representation of these objects, the causes of such differences are analyzed. In this report, GML-surfaces and bodies are mainly considered, the radial cross sections of which are: a) regular «stars» for surfaces and б) regular convex polygons for bodies "

References

1. Ricci P., Tavkhelidze I. - Classification of a Wide Set of Geometric Figures, Surfaces and Lines - (Trajectories) - Rendiconti Academia Nazionale delle Scienze detta dei XL, Memorie di Matematica e Applicazioni 124 °, (2006), vol. XXX, fasc. pp. 19-212.

2. Tavkhelidze I. About some properties of one special class of the generalized Mobius-Listing's bodies. Reports of enlarged sessions of the seminar of I. Vekua Institute of Applied Mathematics, **36** (2022), 91-94.

$T(Vn)$ TANGENT BUNDLE $T\big(T(Vn)\big)$ TANGENT BUNDLE'S I-FORMS

Gocha Todua
European University, Tbilisi, Georgia
Email: todua.gocha@eu.edu.ge

Consider a tangent bundle $T\big(T(Vn)\big)$ with the local coordinates $\left(x^i, y^{\bar{\imath}}, y^i, z^{\bar{\imath}}\right)$ where $\left(x^i, y^{\bar{\imath}}\right)$ are the coordinates of the basis $T(Vn)$, and $\left(y^i, z^{\bar{\imath}}\right)$ are those of the layer $T_z, z \in T(Vn)$, local coordinates of the point of the space $T\big(T(Vn)\big)$ are transformed as follows:

$$\bar{x}^i = \bar{x}^i(x^k), \bar{y}^i = x^i_k y^k, \bar{x}^{\bar{\imath}} = x^{\bar{\imath}}_{\bar{k}} y^{\bar{k}}, \bar{z}^{\bar{\imath}} = x^{\bar{\imath}}_{\bar{k}} z^{\bar{k}} + x^{\bar{\imath}}_{\bar{k}j} y^{\bar{k}} y^j.$$

The following I-forms can be defined of the space $T\big(T(Vn)\big)$:

$$\theta^i = dy^i + \omega^i_k y^k, \theta^{\bar{\imath}} = dy^{\bar{\imath}} + \omega^{\bar{\imath}}_{\bar{\jmath}} y^{\bar{\jmath}}, \vartheta^{\bar{\imath}} = dz^{\bar{\imath}} + \omega^{\bar{\imath}}_{\bar{\jmath}} z^{\bar{\jmath}} + \omega^{\bar{\imath}}_{\bar{\jmath}k} y^{\bar{\jmath}} y^k.$$

If we differentiate these equations externally, then based on the structural equations [1] of the I-forms $\omega^i_k, \omega^{\bar{\imath}}_{\bar{k}}, \omega^{\bar{\imath}}_{\bar{\jmath}k}$ we will have:

$D\theta^i = \theta^k \wedge \omega^i_k + \omega^k \wedge \theta^i_k, D\theta^{\bar{\imath}} = \theta^{\bar{k}} \wedge \omega^{\bar{\imath}}_{\bar{k}} + \omega^k \wedge \theta^{\bar{\imath}}_k, D\vartheta^{\bar{\imath}} = \vartheta^{\bar{k}} \wedge \omega^{\bar{\imath}}_{\bar{k}} + \theta^{\bar{k}} \wedge \theta^{\bar{\imath}}_{\bar{k}} + \omega^k \wedge \vartheta^{\bar{\imath}}_k,$
where $\theta^{\bar{\imath}}_k = \omega^{\bar{\imath}}_{\bar{k}k} y^{\bar{k}}, \theta^i_k = \omega^i_{kj} y^j, \theta^{\bar{\imath}}_{\bar{k}} = \omega^{\bar{\imath}}_{\bar{k}k} y^k,\ \vartheta^{\bar{\imath}}_j = \omega^{\bar{\imath}}_{\bar{k}j} z^{\bar{k}} + \omega^{\bar{\imath}}_{\bar{k}ij} y^{\bar{k}} y^i.$

Reference

1. Todua G. On the F structures of the space $T\big(Lm(Vn)\big)$. Transactions of A. Razmadze Matfematical Institute. **175**, 1 (2021).

INTEGERS REPRESENTED BY BINARY FORMS BELONGING TO MULTI-CLASS GENERA

Teimuraz Vepkhvadze
I. Javalhishvili Tbilisi State Universit, Tbilisi, Georgia
email: t-vepkhvadze@ hotmail.com

The formulae for the average number of representations of positive integers by the genus containing the positive binary quadratic form can be used to characterize positive integers of some kind represented by this form. In this paper we introduce this method in the case of binary forms belonging to multi-class genera.

SECTION OF REAL ANALYSIS

Chairs: Ushangi Goginava, Leri Gogoladze
Co-chair: Ana Danelia

APPROXIMATION OF NÖRLUND MEANS WITH RESPECT TO WALSH SYSTEM IN LEBESGUE SPACES

Nika Areshidze
Departament of mathematics, I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: nika.areshidze15@gmail.com, nika.areshidze804@ens.tsu.edu.ge

The talk is devoted to presenting some new methods to prove a similar estimate of Nörlund means with respect to the Walsh system but without any condition considered in the paper of Moricz and Siddiqi [1]. In particular, we prove approximation results for Nörlund means generated by non-decreasing sequence in Lebesgue spaces for some $1 \leq p < \infty$.

References

1. Móricz, F., and A. H. Siddiqi. Approximation by Nörlund means of Walsh-Fourier series. Journal of approximation theory **70**, 3 (1992), 375-389.
2. Persson Lars-Erik, George Tephnadze, and Ferenc Weisz. Martingale Hardy Spaces and Summability of One-Dimensional Vilenkin-Fourier Series. Birkhäuser, 2022.
3. Schipp. F., W. R. Wade, P. Simon, J. Pál, and Walsh Series. An introduction to dyadic harmonic analysis. Adam Hilger, Bristol, New York, 1990.

ON THE EXISTENCE OF AN INVARIANT MEASURE IN AN INFINITE-DIMENSIONAL POLISH VECTOR SPACE

Mariam Beriashvili[1,2], Aleks Kirtadze[2,3]
[1]I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
[2]Georgian Technical University, Tbilisi, Georgia
[3]A. Razmadze Mathematical Institute of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
emails: Mariam_beriashvili@yahoo.com, Kirtadze2@yahoo.com

In the presented talk we discuss the existence of an invariant measure in an infinite-dimensional linear Polish space. More precisely, by using the result presented in [3], we show that on every infinite-dimensional Polish linear space X there is a nonzero σ-finite Borel measure, which is invariant with respect to some everywhere dense linear subspace

of X (cf. [2]). Moreover, we show that for the same X there exists a class of nonzero σ-finite measures with analogous properties, which is of cardinality 2^{2^c}, where c denotes the cardinality of the continuum (see [1]).

Acknowledgements. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG), Grant YS-21-1667.

References

1. Beriashvili M. The cardinality number of the certain classes of measures, Transactions of A. Razmadze Mathematical Institute, **176**, 2 (2022), 269-271
2. Gill T., Kirtadze A. G. Pantsulaia, A. Plichko, Existence and uniqueness of translation invariant measures in separable Banach spaces, Functiones et Approximato, **50**, 2 (2014), 401-419.
3. Kharazishvili A. Invariant measures in the Hilbert space (Russian), Bull. Acad. Sci. Georgian SSR, **114,** 1 (1984), 45-48.

ABSOLUTE CONVERGENCE OF GENERAL FOURIER SERIES

Leri Gogoladze
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: lgogoladze1@hotmail.com

In the report the convergence of the series

$$\sum_{k=1}^{\infty} |c_k(f)|^r \gamma_k, \quad r \in (0,2)$$

is considered, where $c_k(f)$ are Fourier coefficients of the function $f \in L_2([0,1])$ with respect to the general orthonormal system and γ_k is the non-negative sequence of numbers, satisfying sufficiently general conditions. From the obtained results follow the theorems of S. B. Stechkin, A. A. Konyushkov, B. I. Golubov.

THE OPTIMAL CONVERGENCE FACTORS FOR GENERAL FOURIER COEFFICIENTS

Larry Gogoladze, Giorgi Cagareishvili
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
emails: lgogoladze1@hotmail.com, cagare@ymail.com

S. Banach proved that even the Fourier series of the function f(x) = 1 (x ∈ [0; 1]) might not be convergent for some orthonormal systems (ONS). Thus we can conclude that the Fourier series of functions belonging to a certain differential class cannot be convergent in general. On the other hand, for classical ONS (trigonometric, Haar, walsh systems,…) convergence of Fourier series of differentiable class functions is a simple subject. In this paper we investigate the sequence of positive numbers such that, multiplying the Fourier coefficients of Lipschitz class functions by these numbers, one obtains a convergent series of the special form. From the convergence of these special form series, we derive the convergence of the special Fourier series for Lipschitz class functions with respect to general ONS. The obtained results are best possible.

ON THE INTEGRABILITY OF MULTI-DIMENSIONAL RARE MAXIMAL FUNCTIONS

Irakli Japaridze[1], Giorgi Oniani[2]
[1] Departament of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia
email: irakli.japaridze@atsu.edu.ge
[2]School of Computer Science and Mathematics,
Kutaisi International University, Kutaisi, Georgia
email: giorgi.oniani@kiu.edu.ge

The translation invariant monotone collections of multi-dimensional intervals are characterized for maximal functions associated to which (known in the literature as rare maximal functions), the analogue of Stein's criterion for the integrability of the Hardy-Littlewood maximal function is true. Namely, the collections B of the mentioned type are characterized for which the conditions $\int_{[0,1]^d} M_B(f) < \infty$ and $\int_{[0,1]^d} |f| log^+ |f| < \infty$ are equivalent for functions f supported on the unit cube $[0,1]^d$. Here M_B denotes the maximal operator associated to a collection B.

ABSOLUTE CONVERGENCE OF DOUBLE FOURIER TRIGONOMETRIC SERIES

Rusudan Meskhia
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: rusudan.meskhia@tsu.ge

The sufficient conditions for the generalized absolute convergence of double Fourier trigonometric series are established in terms of mixed and partial moduli of variation of the function of two variables.

ON UNIVERSALITY OF RADEMACHER SERIES

Shakro Tetunashvili[1,2], Tengiz Tetunashvili [1,3]
[1]Georgian Technical University, Tbilisi, Georgia
[2]Andrea Razmadze Mathematical Institute of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
emails: stetun@hotmail.com, s.tetunashvili@gtu.ge
[3]Ilia *Vekua Institute of Applied Mathematics* of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
tengiztetunashvili@gmail.com, t.tetunashvili@gtu.ge

The existence of almost everywhere convergent and divergent Rademacher series such that they are universal in the sense of convergence to any given continuous function on the everywhere dense set is established.

Namely, it is proved that if

$$\sum_{k=0}^{\infty} a_k r_k(t)$$

is a Rademacher series such that $a_k \to 0$, when $k \to \infty$ and $\sum_{k=0}^{\infty} |a_k| = +\infty$, then for any function $f(t)$ such that $f(t)$ is continuous on (a,b), where $(a,b) \subset [0,1]$, there exists a set E of cardinality continuum such that E is an everywhere dense in (a,b) and

$$\sum_{k=0}^{\infty} a_k r_k(t) = f(t), \text{ for every } t \in E.$$

It is also shown that for the validity of theorem both above presented conditions imposed on the coefficients of the series are not only sufficient but also necessary.

SOME PROBLEMS OF CONVERGENCE OF GENERAL FOURIER SERIES

Giorgi Tutberidze[1], Vakhtang Tsagareishvili[2]
[1]The University of Georgia, Tbilisi, Georgia
email: g.tutberidze@ug.edu.ge
[2]I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: cagare@ymail.com

S. Banach proved that good differential properties of the function do not guarantee the a.e. convergence of the Fourier series of this function with respect to general orthonormal systems (ONS). On the other hand, it is very well known that a sufficient condition for the a.e. convergence of an orthonormal series is given by the Menshov-Rademacher Theorem.

The talk deals with a sequence of positive numbers (d_n) such that multiplying the Fourier coefficients $(c_n(f))$ of functions with bounded variation by these numbers one obtains a.e. convergent series of the $\sum_{n=1}^{\infty} d_n c_n(f)\varphi_n(x)$. It is established that the resulting conditions are best possible. These results are investigated in [3-4].

References

1. Banach S. Sur la divergence des series orthogonales, Stud. math., **9** (1940), 139-155.
2. Goginava U., Gogoladze L. Convergence in measure of logarithmic means of multiple Fourier series, J. Contemp. Math. Anal., **49**, 2 (2014), 70-77.
3. Gogoladze L., Tsagareishvili V. Some classes of functions and Fourier coefficients with respect to general orthonormal systems, English version: Proceedings of the Steklov Institute of Mathematics, **280** (2013), 156-168.
4. Tsagaraeishvili V., Tutberidze G. Multipliers of Absolute Convergence, Mat. Zametki, **105**, 3 (2019), 433-443.
5. Tsagaraeishvili V., Tutberidze G. Absolute convergence factors of Lipshitz class functions for general Fourier series, Geo. Math., J., https://doi.org/10.1515/gmj-2021-2107.
6. Tsagareishvili V. Functional conditions for the convergence of Fourier series with respect to general orthonormal systems, Russian Mathematics (Izvestiya VUZ. Matematika), (2011), **55**, 5 (2011), 56-62.

SECTION OF COMPLEX ANALYSIS AND APPLICATIONS

Chair: Grigor Giorgadze
Co-chair: George Akhalaia

GENERALIZED MEROMORPHIC FUNCTIONS

Giorgi Akhalaia[1], Nino Manjavidze[2], Giorgi Makatsaria[3]
[1]I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: giaakha@gmail.com
[2]Ilia State University, Tbilisi, Georgia
email: ninomanjavidze@iliuni.edu.ge
[3]Vladimer Chavchanidze Institute of Cybernetics of the Georgian Technical University, Tbilisi, Georgia
email: giorgi.makatsaria@gmail.com

The generalized solutions of a special class of regular Carleman-Vekua equations (generalized meromorphic functions) will be studied from the point of view of the theory of functions as well as from the point of view of the analysis of boundary value problems of the theory of functions; sufficiently important information about the behavior of the generalized meromorphic functions in the neighborhood of a point at infinity (and therefore the structure) is established; for the generalized meromorphic functions in some sense natural boundary problems are studied and their complete analysis is obtained.

It should be also mentioned that the obtained results are new for the classical meromorphic functions as well. All the above results are essentially based on the iterative transformation of the meromorphic functions studied by the authors. The obtained results continue the research started in the monograph [1].

Acknowledgements. This work was supported by grant # FR 22- 354 from the Shota Rustaveli National Science Foundation.

References

1. Akhalaia G., Giorgadze G., Jikia V., Kaldani N., Makatsaria G., Manjavidze N. *Elliptic Systems on Riemann Surfaces*. Lecture Notes of TICMI. **13** (2012), 1-154.

ON CARLEMAN-VEKUA'S METHOD OF REGULARIZATION OF SINGULAR INTEGRAL EQUATION

Nikoloz Avazashvili
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: nikola.avazashvili@gmail.com

The singular integral equation with degenerate kernel

$$a(t)\varphi(t)-\frac{1}{\pi i}\int_{\Gamma}\sum_{k=1}^{n}b_k(t)c_k(\tau)\frac{\varphi(\tau)d\tau}{\tau-t}=f(t),\qquad t\in\Gamma, \tag{1}$$

Γ is a simple closed smooth curve, a, b_k, c_k, f, are given and φ is unknown $H_{\mu}(\Gamma)$ class functions, is considered in [1].

If $n=1$, from equation (1) can be obtained the characteristic and the union characteristic equations, with the help of each of them the full singular integral equation can be reduced to the equivalent Fredholm integral equation (the so called Carleman-Vekua's method of equivalent regularization (see, for example, [2])).

In the present report the possibility of the application of the equation (1) for the equivalent regularization of the full singular integral equation is considered in the case when $n>1$.

References

1. Авазашвили Н.Д. О сингулярном интегральном уравнении с вырожденным ядром. //Научные труды Грузинского политехнического института. 1980. №1(222). с.55-57.
2. Мусхелишвили Н.И. Сингулярные интегральные уравнения. М.: Наука, 1968.

THE ACCESSORY PARAMETER PROBLEMS

Nino Bregvadze
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: nbregvadze97@gmail.com

We consider unitarity conditions for monodromy groups of Lame and Heun differential equations (see [1], [2]). We also set and solve the accessory parameter problems for these equations.

Acknowledgments. This work was supported by grant N FR 22- 354 from the Shota Rustaveli National Science Foundation.

References

1. Beukers F. Unitary monodromy of Lame differential operators. Mathematics Institute, University of Utrecht, 2007.
2. Chen Eric, Zitzewitz Alex. Unitary conditions for Lame and Heun differential operators. Wayzata High School, Minnesota, United States of America, Proof School, California, United States of America.

RIEMANN-HILBERT BOUNDARY VALUE PROBLEM WITH SHIFT FOR GENERALIZED ANALYTIC FUNCTIONS

Mariam Chakhoiantsi
Faculty of Exact and Natural Sciences,
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: mariami.chakhoiantsi375@ens.tsu.edu.ge

In the talk we give the generalization of Riemann-Hilbert boundary value problem with shift for generalized analytic function and prove that the solution of the problem depends on the complex structures of the Riemann surfaces induced from the shift function.

ON SOLVABILITY CONDITION OF THE RIEMAN-HILBERT BOUNDARY PROBLEM BY QUADRATURES

Gia Giorgadze, Gega Gulagashvili
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University,
Tbilisi, Georgia
emails: gia.giorgadze@tsu.ge, gega.tsu.mathematic@gmail.com

We discuss the solvability condition of the Riemann-Hilbert boundary value problem by generalized quadratures a for piecewise constant transition function and from this we obtain the criterion of the solvability of one old problem.

Namely, it is known that the Fuchsian system of differential equations is solvable by generalized quadratures if and only if the identity component of its monodromy group is solvable. From this it follows that solvability or not-solvability of Fuchsian system by generalized quadratures depends only on monodromy matrices. Therefore, the family of system of differential equations induced from the family of this monodromy matrices is simultaneously solvable or non-solvable by generalized quadratures.

Therefore we have

Theorem. The factorization problem for a piecewise constant matrix function is solvable by generalized quadratures, if and only if the identity component of algebraic

group generated by monodromy matrices of the corresponding Fuchsian system is a solvable group.
This theorem is the solution of the factorization problem for piecewise constant matrix functions.

Acknowledgments. This work was supported by grant N FS 22- 354 from the Shota Rustaveli National Science Foundation.

References

1. Giorgadze G., Gulagashvili G. Riemann-Hilbert Boundary Value Problem with Piecewise Constant Transition Function. Journal of Dynamical and Control systems. **28**, (2022), 109-119, https://doi.org/10.1007/s10883-020-09524-z.

ON THE ALGORITHM OF INTEGER PARTITION

Giorgi Kakulashvili
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
Email: giorgik1994@gmail.com

This report discusses a non-negative integer partition algorithm [1]. Our algorithm aims to represent a given natural number as a sum of other natural numbers. We will illustrate the process through examples, highlighting how n integers can be devided into k parts. Furthermore, we will derive a formula for determining the number of elements in the set of partitions, with k values ranging from 2 to 5.

Acknowledgments. This work was supported by grant N FR 22- 354 from the Shota Rustaveli National Science Foundation.

References

1. Andrews, G. E. The Theory of Partitions. Cambridge University Press, 1998.

RENORMSTATISTICS IN MULTIPARTICLE PRODUCTION PROCESSES

Nugzar Makhaldiani
Joint Institute for Nuclear Research, Dubna Moscow region, Russia
Email: mnv@jinr.ru

Motion equations of the Renormdynamics (RD) for the higher energy Multiparticle production processes formulated. Perturbative RD motion equations need nonperturbative modifications at low energy hadronization phase. Modifications by methods of statistical physics - Renormstatistics with some applications given.

THE ABEL-JACOBI THEOREM

Irakli Sikharulidze
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: irakli.sikharulidze407@ens.tsu.edu.ge

For a compact Riemann surface X the image of the mapping

$$\pi_1(X) \to \Gamma(X, \Omega_X)^*;$$
$$\gamma \mapsto \left(\omega \mapsto \int_\gamma \omega\right)$$

$\mathrm{Per} := \left\{\omega \mapsto \int_\gamma \omega \,\middle|\, \gamma \in \pi_1(X)\right\} \subseteq \Gamma(X, \Omega_X)^*$is called the period lattice of X; it indeed is a lattice in the complex vector space $\Gamma(X, \Omega_X)^* = H^0(X, \Omega_X)^*$, whose quotient space by this lattice $J(X) := {H^0(X, \Omega_X)^*}/{\mathrm{Per}}$ is called the Jacobian variety of Xor also the Jacobian period torus; it is a complex Lie group, whose dimension equals the genus g of X. The following mapping from thegroup of divisors of degree zero of Xinto its Jacobian variety

$$\mathrm{Div}(X)_0 \to J(X),$$
$$\sum_{i\in I} P_i - \sum_{i\in I} Q_i \mapsto \left(\omega \mapsto \sum_{i\in I} \int_{P_i}^{Q_i} \omega\right)$$

is called the Abel-Jacobi mapping; it is a group homomorphism and induces the homomorphism from the divisor class group of degree zero of X into the Jacobian variety:

$$\mathrm{DCG}(X)_0 \to J(X),$$
$$\left[\sum_{i\in I} P_i - \sum_{i\in I} Q_i\right] \mapsto \left(\omega \mapsto \sum_{i\in I} \int_{P_i}^{Q_i} \omega\right);$$

the Abel-Jacobi theorem states that this homomorphism is an isomorphism.

Acknowledgments. This work was supported by grant № FR-22- 354 from the Shota Rustaveli National Science Foundation.

SECTION OF ORDINARY DIFFERENTIAL EQUATIONS AND OPTIMAL CONTROL

Chairs: Roman Koplatadze, Tamaz Tadumadze
Co-chair: Tea Shavadze

ON THE CRITERION OF WELL-POSEDNESS OF THE CAUCHY PROBLEM WITH WEIGHT FOR SYSTEMS OF LINEAR ORDINARY DIFFERENTIAL EQUATIONS WITH SINGULARITIES

Besarion Anjaparidze
I. Javakhishvili Tbilisi State University,
Department of Mathematics, Tbilisi, Georgia
email: besarion.anjaparidze305@ens.tsu.edu.ge

On the interval $I=[a,b]$ the Cauchy problem with weight for the system of singular ordinary differential equation

$$\frac{dx}{dt}=P(t)x+q(t), \tag{1}$$

$$\lim_{t\to t_0}(\Phi^{-1}(t)x(t))=0 \tag{2}$$

is considered, where $t_0\in I$, $P\in L_{loc}(I_{t_0},R^{n\times n}), I_{t_0}=I\setminus\{t_0\}$, $q\in L_{loc}(I_{t_0},R^n)$; $\Phi(t)$ is a continuous positive defined $n\times n$ dimensional diagonal matrix-function having an inverse. Along with system (1) we consider the sequence of singular systems

$$\frac{dx}{dt}=P_m(t)x+q_m(t),\ m=1,2,... \tag{3}$$

with the condition (2), where $P_m\in L_{loc}(I_{t_0},R^{n\times n})$, $q_m\in L_{loc}(I_{t_0},R^n)$. In the paper, the well-possednes of the problem (1)-(2) is investigated. Namely, what proximity conditions must satisfy the matrix P_m and q_m vector functions, respectively, with respect to P and q, so that problem (3)-(2) has a unique solution for sufficiently large m and it is uniformly close to solution of problem (1)-(2). Necessary and sufficient conditions, as well as effective sufficient conditions, which provide the well-possednes of problem (1)-(2) are established.

OPTIMAL CONTROL OF ONE MARKETING RELATION MODEL CONSIDERING DEMAND-SUPPLY LINEAR FUNCTIONS AND VARABLE DELAY

Phridon Dvalishvili, Lela Alkhazihvili
I. Javakhishvili Tbilisi State University,
Department of Computer Sciences, Tbilisi, Georgia
emails: phridon.dvalishvili@tsu.ge; Lela.Alkhazishvili@tsu.ge

The dynamical model of marketing relation with variable delay in controls is considered. For the corresponding optimal problem the existence of an optimal control and necessary conditions of optimality for control are proved on the bases of results given in [1]. All possible controls that are doubtful on optimality are written out. Analogous problem was discussed in [2] in the case of constant delay.

References

1. Kharatishvili, G. L., Tadumadze, T. A. Formulas for the variation of a solution and optimal control problems for differential equations with retarded arguments. *J. Math. Sci.*, **140,** 1 (2007), 1-175.
2. Dvalishvili1, Ph., Tadumadze, T. Optimization of one marketing relation model with delay. *Journal of Modern Technology and Engineering*, **4,** 1 (2019), 5-10.

PREDICTING THE SPREAD OF THE CORONAVIRUS (COVID-19): CHALLENGEES AND PRACTICAL EXPERIENCE

Akaki Gabelaia
Georgian Technical University,
Department of Computation Mathematics, Tbilisi, Georgia
email: agabelaia@gtu.ge

The author's practical experience in predicting coronavirus (Covid-19) (for both, the World and Georgia) based on modern mathematical models and the well-known computer program EViews-10 ("Econometric Views") is discussed. For the sake of clarity, it should be noted that, initially, in terms of forecasting, we considered such a key indicator of the spread of the coronavirus as the total number of cases of infection at the moment (in terms of days). However, for forecasting we used the so-called ARMA ("Autoregression and Moving avereages") type models with the addition of trendy components. As we have seen, these types of models showed a sufficiently high prediction accuracy for a maximum of a month (then their accuracy dropped). On the other hand, given that the virus is "not going to stop" in the near future, the problem of increasing the forecast horizon is on the agenda. It should be noted, however, that according to the central probability theory of probability, the

distribution of the "average daily increase in the number of infection cases" should be close to normal, which, in addition to analyzing the spread of the virus, should allow the "pure" statistical method to predict its spread (on the basis of building confidence intervals for the number of infected). However, as our practical experience has shown, using such an indicator as "the total number of of infection cases at the end of the period (in this case, month)" is more appropriate to find more reliable current prognostic estimates of the process (due to the monotonicity (non-decreasing!) of this indicator), while "average daily increase in the number of infection cases during the month" provides better opportunities for analyzing the course of the pandemic. The difficulties of forecasting caused by the Omicron strain of Covid, which is characterized by dramatically high rates of spread, and the possibilities of describing the corona spread process as a whole with a logistic curve are analyzed.

ON THE EXISTENCE OF AN OPTIMAL ELEMENT FOR THE QUASI-LINEAR NEUTRAL OPTIMAL PROBLEM WITH THE TWO TYPES CONTROLS AND SEVERAL DELAYS

Nika Gorgodze[1], Eka Bokhua[2]
[1]Akaki Tsereteli State University, Kutaisi, Georgia
email: nika.gorgodze@atsu.edu.ge;
[2]I. Javakhishvili Tbilisi State University,
Department of Mathematics, Tbilisi, Georgia
email: eka.bokhua164@ens.tsu.edu.ge

For the optimization problem with the general boundary conditions and functional

$$\dot{x}(t) = \sum_{i=1}^{m} A_i(t, v(t))\dot{x}(t-\sigma_i) + f\left(t, x(t), x(t-\tau_1), \cdots, x\left(t-\tau_p\right), u(t)\right), \quad t \in [t_0, t_1],$$

$$x(t) = \varphi(t), \quad t < t_0, \quad x(t_0) = x_0,$$

$$q^i\left(t_0, t_1, \sigma_1, \dots, \sigma_m, \tau_1, \dots, \tau_p, x_0, x(t_1)\right) = 0, \quad i = 1, \dots, l,$$

$$q^0\left(t_0, t_1, \sigma_1, \dots, \sigma_m, \tau_1, \dots, \tau_p, x_0, x(t_1)\right) \to min,$$

the existence theorems of an optimal element $(t_0, t_1, \sigma_1, \dots, \sigma_m, \tau_1, \dots, \tau_p, x_0, v(\cdot), u(\cdot))$ are proved.

Analogous question is investigated in [1] for the case of fixed t_0 and t_1.

References

1. Bokhua, E., Dvalishvili, Ph., Gorgodze, N., Tadumadze, T. Existence of an Optimal Element for the quasi-linear Neutral Optimal Control Problem. *Proceedings of the 8 th International Conference on Control and Optimization with Industrial Applications,* 24-26 August, 2022, Baku, Azerbaijan, 132-134.

LOCAL PRESENTATION FORMULAS OF SOLUTION AND OPTIMIZATION PROBLEMS FOR THE CONTROLLED FUNCTIONAL-DIFFERENTIAL EQUATION WITH THE MIXED INITIAL CONDITION

Medea Iordanishvili
I. Javakhishvili Tbilisi State University, Department of Computer Sciences,
St. Andrews Georgian University, Tbilisi, Georgia
emails: medea.iordanishvili@tsu.ge; m.iordanishvili@sangu.edu.ge

For the perturbed controlled functional-differential equation the representation formulas of solution are proved in the neighborhood of the endpoint of an interval. In the formulas the effects of the initial data variation and the mixed initial condition are derived. Under the initial data we mean the collection of the initial moment, constant delays containing in the phase coordinates, a constant delay containing in the continuous differentiable control, an initial vector, theintial functions, the control functions. Here, the essential novelty is that such a wide class of an initial data is considered firstly. Representation formulas of solution are usedin the optimization problems [1] and in the finding of an approximation solution of the perturbed functional-differential equation. Analogous formulas of representation are proved in [2] when the initial moment is fixed. Besides, for the initial data optimization problems necessary conditions of optimality are obtained.

References

1. Tadumadze, T. Variation formulas of solutions for functional differential equations with several constant delays and their applications in optimal control problems. *Mem. Differ. Equ. Math . Phys.* **70** (2017), 7-97.
2. Alkhazishvili, L., Iordanishvili, M. The local formula of representation of a solution for a functional differential equation with the mixed initial condition considering perturbations of delays containing in the phase coordinates and in controls. *Georgian Math. J.* **29**, 1 (2022), 1-12.

ON THE OSCILATORY PROPERTIES OF THE SECOND ORDER EMDEN-FOWLER TYPE DIFFRENCE EQUATIONS WITH DEVIATING ARGUMENT

Roman Koplatadze
I. Javakhishvili Tbilisi State University, Department of Mathematics
and I. Vekua Institute of Applied Mathematics, Tbilisi, Georgia
email: roman.koplatadze@tsu.ge

Consider the Emden-Fowler type difference equation

$$\Delta\Big(r(k)\big|\Delta u(k)\big|^{\alpha}\operatorname{sign}\Delta u(k)\Big)+p(k)\Big|u\big(\sigma(k)\big)\Big|^{\lambda}\operatorname{sign}u\big(\sigma(k)\big)=0,$$

where $r:\mathrm{N}\to(0,+\infty)$, $\alpha\geq 1$, $\lambda>0$, $\sigma:\mathrm{N}\to\mathrm{N}$, $p:\mathrm{N}\to\mathrm{R}_{+}$ and $\lim\limits_{k\to+\infty}\sigma(k)=+\infty$.

New type sufficient conditions for the oscillation of all solutions are established.

ON THE WELL-POSEDNESS OF THE CAUCHY PROBLEM FOR ONE CLASS OF CONTROLLED NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATION CONSIDERING PERTURBATION OF THE INITIAL DATA

Tea Shavadze[1], Ia Ramishvili[2]
[1]I. Javakhishvili Tbilisi State University,
I. Vekua Institute of Applied Mathematics, Tbilisi, Georgia
email: tea.shavadze@gmail.com
[2] Georgian Technical University, Department of Mathematics, Tbilisi, Georgia
email: ia.ramis@yahoo.com

For the quasi-linear neutral functional differential equation

$$\dot{x}(t)=A\big(t,x(t),x(t-\theta_0),v_0(t)\big)\dot{x}(t-\sigma_0)+f\big(t,x(t),x(t-\tau_0),u_0(t)\big),\quad t\in[t_{00},t_{10}]$$

with the initial condition

$$x(t)=\varphi(t),\ \dot{x}(t)=g(t),\ t<t_{00},\ x(t_{00})=x_{00},$$

Using the scheme given in [1], theorems on the continuous dependence on the initial data are proved. Under the initial data we mean the collection of the initial moment, delay parameters, the initial vector, initial and control functions. Analogous theorems for the fixed initial moment are given in [1].

Acknowledgements. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG), Grant No. YS-21-554.

References

1. Tadumadze, T., Gorgodze, N. Variation formulas of a solution and initial data optimization problems for quasi-linear neutral functional differential equations with

discontinuous initial condition. *Mem. Differential Equations Math.Phys.*,63 (2014), 1-77.
2. Shavadze, T. On the well-posedness of the Cauchy problem for the quasi-linear controlled neutral functional-differential equation. *International Conferenceon Mathematical Analysis and Applicationsin Scienceand Engineering*, Porto-Portugal, June 27 - 29, 2022, *Book of Extended Abstracts*, 329-332.

SENSITIVITY COEFFICIENTS OF A CONTROLLED FUNCTIONAL-DIFFERENTIAL MODEL OF THE IMMUNE RESPONSE CONSIDERING OF THE MIXED INITIAL CONDITION AND VARIATION OF THE INITIAL MOMENT

Tamaz Tadumadze[1], Abdeljalil Nachaoui[2], Mourad Nachaoui[3]
[1]I. Javakhishvili Tbilisi State University, Department of Mathematics and
I. Vekua Institute of Applied Mathematics, Tbilisi, Georgia
email: tamaz.tadumadze@tsu.ge
[2]Laboratoire de Mathematiques Jean Leray, Nantes Universit´e, Nantes, France.
email: Abdeljalil.Nachaoui@univ-nantes.fr
[3](EMI), Universit´e Sultan Moulay Slimane, Beni-Mellal, Morocco
email: m.nachaoui@usms.ma

In the paper a simple modified Marchuk's model [1],

$$\begin{cases} \dot{x}_1(t) = p_1 x_1(t) - p_2 x_1(t) x_3(t), \\ \dot{x}_2(t) = p_3 x_1(t-\tau) x_3(t-\tau) - p_4 (x_2(t) - x_2^*) + u_1(t), \\ \dot{x}_3(t) = p_5 x_2(t) - p_6 x_3(t) - p_7 x_1(t) x_3(t) + u_2(t), \\ t \in [t_0, t_1], \tau \in (0, \hat{\tau}) \end{cases}$$

is considered with the mixed initial condition

$$x_1(t) = \varphi_1(t), t < t_0, \ x_1(t_0) = x_{10}; \ x_i(t) = \varphi_i(t), t \le t_0.$$

In the case of variation of the initial moment and delay, the initial and control functions, in the neighborhood of the initial moment t_1, a form of the system of differential equations in variations is established, which satisfies the sensitivity coefficients. On the whole interval a form of equation in variations is established in [2], when t_0 is fixed.

References

1. Marchuk, G. I. Mathematical modelling of immune response in infectious diseases. MIA **395**, *Kluwer Academic Publishers, Dordrecht*, 1997.
2. Tadumadze, T., Nachaoui, A., Shavadze, T. On the coefficient of sensitivity of a controlled differential model of the immune response. *Semin. I. Vekua Inst. Appl. Math., Rep.*, **48** (2022), 57-64.

SECTION OF PARTIAL DIFFERENTIAL EQUATIONS

Chairs: Temur Jangveladze, Sergo Kharibegashvili, David Natroshvili
Co-chair: Zurab Kiguradze

A BOUNDARY VALUE PROBLEM FOR A CLASS OF HIGHER-ORDER NONLINEAR HYPERBOLIC EQUATIONS

Teona Bibilashvili
Faculty of Informatics and Control Systems,
Georgian Technical University, Tbilisi, Georgia
email: teonabibilashvili12@gmail.com

For one class of higher-order nonlinear hyperbolic equations in the angular domain, a boundary value problem with Dirichlet and Neumann type boundary conditions is considered. The concept of a generalized solution of this problem in the class of continuous functions is introduced. This problem is equivalently reduced to a nonlinear functional equation in the mentioned space. Under certain conditions imposed on the nonlinear terms, an a priori estimate for the solution of a functional equation is obtained, from which its existence follows. The question of uniqueness and nonexistence of a solution to this problem is also discussed.

INVESTIGATION OF THE FOURTH-ORDER NONLINEAR INTEGRO-DIFFERENTIAL EQUATION AND ITS CORRESPONDING DIFFERENCE SCHEME

Teimuraz Chkhikvadze
I. Javakhishvili Tbilisi State University,
I. Vekua Institute of Applied Mathematics, Tbilisi, Georgia
email: m.zarzma@gmail.com

For the fourth-order nonlinear integro-differential equation, the initial-boundary value problem is considered [1-3]. The stability and uniqueness of the solution are investigated. A difference scheme is built and the question of its convergence is discussed. Numerical experiments are fulfilled and graphical illustrations of the results are given.

Acknowledgements. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant number FR-21-2101].

References

1. Jangveladze, T. Investigation and numerical solution of nonlinear partial differential and integro-differential models based on system of Maxwell equations. Mem. Differential Equations Math. Phys., **76** (2019), 1-118.
2. Jangveladze T. On one class of nonlinear integro-differential parabolic equations. Semin. I.Vekua Inst. Appl. Math. Rep., **23** (1997), 51-87.
3. Chkhikvadze, T. On one nonlinear integro-differential parabolic equation. Rep. Enlarged Sess. Semin. I.Vekua Inst. Appl. Math., **35** (2021), 19-22.

NUMERICAL SOLUTION OF ONE NONLINEAR PARTIAL DIFFERENTIAL MULTIDIMENSIONAL SYSTEM

Mikheil Gagoshidze

I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia

email: MishaGagoshidze@gmail.com

A multidimensional analogue of one non-linear two-dimensional system of partial differential equations describing some biological processes is studied [1]. The averaged model of sum approximation [2] and the variable direction difference scheme [3] for an initial-boundary problem are considered. Various numerical experiments have been conducted and a comparative analysis of the obtained results is given.

Acknowledgements. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant number FR-21-2101].

References

1. Mitchison, G.J. The polar transport of auxin and vein patterns in plants. Philos. Trans. R. Soc. Lond. B Biol. Sci., **295** (1981), 461-471.
2. Dzhangveladze, T.A. Averaged model of sum approximation for a system of nonlinear partial dierential equations. Proc. I. Vekua Inst. Appl. Math., **19** (1987), 60-73 (Russian).
3. Jangveladze, T., Kiguradze, Z., Gagoshidze, M., Nikolishvili, M. Stability and convergence of the variable directions dierence scheme for one nonlinear two-dimensional model. International Journal of Biomathematics. **8**, 5 (2015), 1550057 (21 pages), DOI: 10.1142/S1793524515500576.

ON THE SYSTEM OF MAXWELL'S NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

Temur Jangveladze
I. Vekua Institute of Applied Mathematics,
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
Department of Mathematics, Georgian Technical University, Tbilisi, Georgia
email: tjangv@yahoo.com

Two one-dimensional models based on Maxwell's well-known system of non-linear partial differential equations [1] are considered. Some exact solutions are constructed. Several properties of the corresponding initial-boundary value problems are studied. Finite-difference schemes are built, numerical experiments are conducted and their analysis is given. The obtained results are a continuation of some studies of papers [2-4].

Acknowledgements. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant number FR-21-2101].

References

1. Landau, L., Lifschitz E. Electrodynamics of Continuous Media. Course of Theoretical Physics, Moscow, 1957.
2. Abuladze, I.O., Gordeziani, D.G., Dzhangveladze, T.A., Korshiya, T.K. Discrete models for a nonlinear magnetic-field-scattering problem with thermal conductivity. Differential'nye Uravnenyia, **22**, 7 (1986), 1119-1129. English translation: Differential Equations, **22**, 7 (1986), 769-777 (Russian).
3. Jangveladze, T. Investigation and numerical solution of nonlinear partial differential and integro-differential models based on system of Maxwell equations. Mem. Differential Equations Math. Phys., **76** (2019), 1-118.
4. Jangveladze T. On investigation and approximate solution of two systems of nonlinear partial differential equations. Rep. Enlarged Sess. Semin. I.Vekua Inst. Appl. Math., **36** (2022), 35-38.

ON THE NUMERICAL SOLUTION OF ONE SYSTEM OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

Nino Mzhavanadze
I. Javakhishvili Tbilisi State University,
I. Vekua Institute of Applied Mathematics, Tbilisi, Georgia
email: Ninomzhavanadze2@gmail.com

Linear stability of the stationary solution of the initial-boundary value problem of one system of non-linear partial differential equations [1-3] is studied. The possibility of the

occurrence of Hopf-type bifurcation is obtained. The difference scheme is constructed, and the numerical experiments are carried out. An analysis of the obtained results is given.

References

1. Dzhangveladze, T.A. Stability of the stationary solution of a system of nonlinear partial differential equations, *Sovremennye problemy matematicheskoi fiziki. (Proceeding of AU-Union Sympozium. The Modern Problems of Mathematical Physics)*. Tbilisi, **1** (1987), 214-221 (Russian).
2. Jangveladze, T. Investigation and numerical solution of nonlinear partial differential and integro-differential models based on system of Maxwell equations. *Mem. Differential Equations Math. Phys.,* **76** (2019), 1-118.
3. Mzhavanadze, N. On one nonlinear diffusion system. *Rep. Enlarged Sess. Semin. I.Vekua Inst. Appl. Math.,* **36** (2022), 63-66.

ON THE INITIAL-BOUNDARY VALUE PROBLEM OF THE NONLINEAR FOURTH ORDER INTEGRO-DIFFERENTIAL MODEL AND ITS CORRESPONDING DIFFERENCE SCHEME

Tamar Paikidze
I. Javakhishvili Tbilisi State University,
I. Vekua Institute of Applied Mathematics, Tbilisi, Georgia
email: Tamofaiqidze98@gmail.com

The stability and uniqueness of the solution of the initial-boundary value problem for one system of fourth-order nonlinear parabolic integro-differential equations [1,2] is studied. The corresponding difference scheme is constructed and numerical experiments are conducted. The analysis of obtained results is presented.

References

1. Jangveladze, T., Kiguradze, Z., Neta, B. *Numerical Solution of Three Classes of Nonlinear Parabolic Integro-Differential Equations*. Elsevier, 2016, ACADEMIC PRESS, ISBN: 978-0-12-804628-9. Elsevier/Academic Press, Amsterdam, 2015.
2. Paikidze, T. On one system of fourth-order nonlinear integro-differential parabolic equation. *Rep. Enlarged Sess. Semin. I.Vekua Inst. Appl. Math.,* **36** (2022), 71-74.

A FORM OF A GENERAL SOLUTION FOR ONE CLASS OF HIGH-ORDER HYPERBOLIC EQUATIONS AND SOME OF ITS APPLICATIONS

Irine Sigua[1], Ana Dzirkvadze[2]
[1]Department of mathematics, Georgian Technical University, Tbilisi, Georgia
email: irinasigua@gtu.ge
[2]Faculty of Informatics and Control Systems,
Georgian Technical University, Tbilisi, Georgia
email: dzirkvadze.a@gtu.ge

For one class of high-order hyperbolic equations with two independent variables, a general solution formula is obtained. This formula allows solving initial, initial-boundary and some non-local problems. In particular, in some cases solutions of Cauchy problems are written in quadratures.

SECTION OF PROBABILITY THEORY AND MATHEMATICAL STATISTICS

Chairs: Elizbar Nadaraya, Omar Purtukhia

AMERICAN OPTION WITH DISCOUNTING

Besarion Dochviri, Zaza Khechinashvili
I. Javakhishvili Tbilisi State University, Department of Mathematics, Tbilisi, Georgia
email: besarion.dochviri@tsu.ge, zaza.khechinashvili@tsu.ge

Financial (B, S) market in discrete time model is considered and for the American call option the problem of pricing, hedging and optimal executing moment is investigated. In particular, pay-off function is of the type $f_n(x) = b^n(x-K)^+$, $0 < b < 1$, where K is a strike price. The representations of option price process $P_n(x), n = 0,1, \dots, N-1$, hedging strategy (γ_n, β_n) and optimal stopping moment τ are derived.

References

1. Melnikov A. *Financial markets. Stochastic Analysis and Pricing of Derivative Securities*. Moscow, 1997, 1-125.
2. Shiryaev A.N. *Essential of Stochastic Finance. Facts, Models, Theory*. World Scientific, 1999, 1-834.

ON THE SUBGAUSSIAN RANDOM ELEMENTS IN A BANACH SPACE

George Giorgobiani, Vakhtang Kvaratskhelia
N. Muskhelishvili Institute of Computational Mathematics
of the Georgian Technical University, Tbilisi, Georgia
emais: giorgobiani.g@gtu.ge, v.kvaratskhelia@gtu.ge

The concept of Subgaussian random variable was first considered in 1960 by the famous French mathematician J.P. Kahane [1] for the study of convergence of trigonometric series. Later, Subgaussian random variables and processes were discussed by many authors. Despite the rather wide range of Subgaussian random variables and processes, their behavior is, in a certain sense, analogous to the behavior of Gaussian random variables and processes.

A numerical random variable ξ is called subgaussian if there exists a non-negative number a such that for every real number t the inequality

$$\mathbb{E}\, e^{t\xi} \leq e^{\frac{1}{2}a^2t^2},$$

holds, where $\mathbb{E}$ is a symbol of mathematical expectation.

If ξ is a centered Gaussian random variable with variance σ^2, then $\mathbb{E}\, e^{t\xi} = e^{\frac{1}{2}\sigma^2 t^2}$, and, therefore, ξ is a subgaussian random variable. Another well-known example of a subgaussian random variable is the Bernoulli random variable.

In the case of infinite-dimensional space, there is no unique approach to defining subgaussian random vectors. In this talk, the concepts of a weak subgaussian, T–subgaussian and F–subgaussian random vectors for the case of an infinite-dimensional Banach space are discussed and the connections between them are explored.

References

1. Kahane J.P. Proprietes locales des fonctions a series de Fourier aleatoires. *Studia Math.*, **19** (1960), 1–25.

QUASI-OPTIMAL RULE OF TESTING DIRECTIONAL HYPOTHESES

Kartlos Kachiashvili
Georgian Technical University, Faculty of Informatics and Control Systems,
Tbilisi, Georgia,
I. Javakhishvili Tbilisi State University, I. Vekua Institute of Applied Mathematics,
Tbilisi, Georgia
Georgian Technical University, N. Muskhelishvili Institute of Computational Mathematics,
Tbilisi, Georgia
emails: k.kachiashvili@gtu.edu.ge, kkachiashvili@gmail.com

The problem of testing directional hypotheses is examined using the consideration of the basic and alternative hypotheses in pairs, allowing implementing computation easily and faster with guaranteed reliability. The concept of mixed directional false discovery rate (*mdFDR*) is used for the decision rule optimality. The fact of guaranteeing the quality of a decision (in the developed approach) on the desired level theoretically is proved and practically is demonstrated by computation of practical examples. The developed method is applied for testing multiple hypotheses that guarantee the restriction of the total *mdFDR* on the desired level. It is also shown that the offered method can be used for solving the problems of intersection-union, union-intersection hypotheses. The offered method is adapted for testing large numbers of the subsets of individual hypotheses at testing multiple hypotheses that saves computational time and resources. Reliability and convenience of the developed method for big data are demonstrated.

References

1. Bahadur, R. R. (1952). A property of the t-statistics. *Sankhya*, 12, 79-88.
2. Bansal, N. K., Hamedani, G. G. & Maadooliat, M. Testing Multiple Hypotheses with Skewed Alternatives, *Biometrics*, **72,** 2 (2016), 494-502.
3. Kachiashvili K. J. The Methods of Sequential Analysis of Bayesian Type for the Multiple Testing Problem. *Sequential Analysis*, **33**, 1 (2014), 23-38.

4. Kachiashvili, K. J. (2018) *Constrained Bayesian Methods of Hypotheses Testing: A New Philosophy of Hypotheses Testing in Parallel and Sequential Experiments*. Nova Science Publishers, Inc., New York, 361 p.
5. Kachiashvili K. J., Kachiashvili J. K. & Prangishvili I. A. (2020) CBM for Testing Multiple Hypotheses with Directional Alternatives in Sequential Experiments. *Sequential Analysis: Design Methods and Applications.* (ID: 1727166 DOI:10.1080/07474946.2020.1727166).

ON CONDITIONAL DISTRIBUTION OF m DEPENDENT VECTOR SUM

Zurab Kvatadze[1], Beqnu Parjiani[1,] Tsiala Kvatadze[2]
[1]Georgian Technical University, Department of Mathematics, Tbilisi, Georgia
emails: zurakvatadze@yahoo.com, beqnufarjiani@yahoo.com
[2]International Black Sea University, Tbilisi, Georgia
email: ttkvatadze@gmail.com

On the (Ω, F, P) probability space the two-component stationary is considered in the narrow concept sequence. $\{\xi_i, Y_i\}_{i\geq 1}$. $Y_i : \Omega \to R^k$ is conditionally m dependent vector sequence ([1]). The decomposition $S_n = \sqrt{\frac{1}{n}}\sum_{i=1}^{n}[Y_i - EY_1] = S_{n1} + S_{n2}$ is applied, where $S_{n1} = \sqrt{\frac{1}{n}}\sum_{i=1}^{n}\left[Y_i - E\left(Y_i|\xi_i\right)\right]$ and $S_{n2} = \sqrt{\frac{1}{n}}\sum_{i=1}^{n}\left[E\left(Y_i|\xi_i\right) - EY_1\right]$. It is proved that if $F_{S_{n2}}(\cdot) \xrightarrow[n\to\infty]{} Q(\cdot)$ and $Q(\cdot)$ is a nondegenerate distribution, then for each $x, y \in R^k$ $P\left(\Phi_{R_m}(x-y) \leq F_{S_n|\bar{\xi}_{1n}}(x) \leq \Phi_{R_m}(x+y)\right) \xrightarrow[n\to\infty]{} Q(y) - Q(-y)$ where, $R_0^{(p)} = E\operatorname{cov}\left(Y_1, Y_{1+p}|\bar{\xi}_{1n}\right)$, $\bar{\xi}_{1n} = \left(\xi_1, \xi_2, ..., \xi_n\right)$ is a fixed trajectory of control sequence. When $\{\xi_i\}_{i\geq 1}$ is a finite ergodic Markov chain with one class of ergodicity, then it is shown that $Q(\cdot) = R_{T_\mu}(\cdot)$, where T_μ is expressed by chain parameters.

References

1. Bokuchava I., Kvatadze Z., Shervashidze T. Central Limit Theorem for Conditionally m-Dependent Vectors Controlled by a Finite Markov Chain. Proceeding of 4th Iranian International Statistics Conference. 23-25. VIII. 1998. SHAHID BEHESHTI UNIVERSITY PRESS « 292 », Vol. 1. Theory of Statistics, Theory of Probability. pp. 249-260. VIII-1999.

ON THE NADARAYA-WATSON TYPE NONPARAMETRIC ESTIMATOR OF THE POISSON REGRESSION FUNCTION

Elizbar Nadaraya[1], Petre Babilua[2]
[1]I. Javakhishvili Tbilisi State University, Department of Mathematics
and I. Vekua Institute of Applied Mathematics, Tbilisi, Georgia
email: elizbar.nadaraya@tsu.ge
[2]I. Javakhishvili Tbilisi State University, Department of Mathematics,
Tbilisi, Georgia
email: petre.babilua@tsu.ge

In the paper, the limiting distribution is established for an integral square deviation of the Nadaraya-Watson type nonparametric estimate of the Poisson regression function. The test for the hypothesis on the Poisson regression function is constructed. The question of consistency of the constructed test is studied and for some close alternatives, the asymptotic behavior of the test power of the constructed test is investigated.

ON SOME PROPERTIES OF ONE NONPARAMETRIC ESTIMATE OF THE POISSON REGRESSION FUNCTION

Elizbar Nadaraya[1], Petre Babilua[2], Mzevinar Patsatsia[3]
[1]I. Javakhishvili Tbilisi State University, Department of Mathematics
and I. Vekua Institute of Applied Mathematics, Tbilisi, Georgia
email: elizbar.nadaraya@tsu.ge
[2]I. Javakhishvili Tbilisi State University, Department of Mathematics,
Tbilisi, Georgia
email: petre.babilua@tsu.ge
[3]Sokhumi State University, Tbilisi, Georgia
email: mzevinar.patsatsia@tsu.ge

Kernel-type nonparametric estimates of the Poisson regression function are considered. We establish the conditions of uniform consistency and the limit theorems for continuous functionals connected with this function on $C[a, 1-a], 0 < a < \frac{1}{2}$.

NONLINEAR FILTERING PROBLEM AND MARTINGALE REPRESENTATION

Vakhtang Jaoshvili[1], Valeriane Jokhadze[2], Omar Purtukhia[3]
[1]Vladimir Komarov Tbilisi School of Physics and Mathematics N199, Tbilisi, Georgia
email: vakhtangi.jaoshvili@gmail.com
[2]I. Javakhishvili Tbilisi State University, Faculty of Economics and Business, Tbilisi, Georgia
email: jokhadze.valeriane@gmail.com
[3]I. Javakhishvili Tbilisi State University, Department of Mathematics and
A. Razmadze Mathematical Institute, Tbilisi, Georgia
email: o.purtukhia@gmail.com

The problem of nonlinear filtering is as follows: we are interested in the estimation of a signal process ξ_t which cannot be observed directly but we have an observation process η_t which is related to ξ_t. The best estimate in the mean square sense of $f(\xi_t)$ based on the natural σ-algebra of observations $\Im_t^\eta = \sigma\{\eta_s : 0 \le s \le t\}$ is given by the conditional mathematical expectation $E[f(\xi_t) \mid \Im_t^\eta]$. In the general case this estimate depends nonlinearly on observations and is called a nonlinear filter. A practical and mathematically more appealing method for solving the filtering problem is to derive a stochastic differential equation for the filter and use Ito's stochastic calculus.

If ξ_t is the solution of a stochastic differential equation and f is a C^2-function, then according to the Ito's formula $f(\xi_t)$ is a semimartingale and hence, under appropriate conditions, $E[f(\xi_t) \mid \Im_t^\eta]$ is a right-continuous semimartingale with respect to σ-algebras $\Im_t^\eta$. Therefore, if every right continuous L^2-martingale can be represented as a stochastic integral with respect to a Wiener process, then we can derive a stochastic differential equation for $E[f(\xi_t) \mid \Im_t^\eta]$. Thus, the question of the stochastic integral representation of martingales is very important for filtering problems. Therefore, the following question naturally arises: can any $\Im_t$-martingale be represented as a stochastic integral? It turned out that we have a positive answer to this question (Clark, 1970) when $\Im_t = \Im_t^w$, but in general this is not so. This is shown in the example of Kallianpur (1980) (to whom M. Ior described it, and the latter, in turn, attributes the example to H. Kunita): let (w_t^1, w_t^2) be a Wiener process in R^2, and let $M_t = \int_0^t w_s^1 dw_s^2$. Choose $\Im_t = \Im_t^M$. Then $N_t = (w_t^1)^2 - t = 2\int_0^t w_s^1 dw_s^1$ is a $\Im_t^M$ - martingale, but it cannot be represented as a stochastic integral with respect to M_t.

On the other hand, taking into account the needs of modern financial mathematics, it is not enough to know only the existence of an integral representation, it is necessary to be able to find the explicit form of the integrand of the integral representation. It is known that for stochastically smooth functionals, the integrand is calculated by Ocone's formula (1984), which was later generalized by Glonti and Purtukhia (2017), when only the filter of the functional is stochastically smooth. Here we study functionals whose filter is no longer smooth and propose a method for finding the integrand.

Acknowledgements. The work was partially supported by the grant STEM-22-226.

SOME PROPERTIES OF HAAR STATISTICAL STRUSTURES IN BANACH SPACE OF MEASURES

Zurab Zerakidze[1], Tamar Chkonia[2]
[1]Gori State University, Gori, Georgia
email: zura.zerakidze@mai.ru
[2]I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: tamari.chkonia@tsu.ge

In this paper we define Haar statistical structures. We prove necessary and sufficient conditions to be weakly separable and strongly separable Haar statistical structures in Banach space of measures.

SOME PROPERTIES OF HAAR STATISTICAL STRUSTURES IN THE HILBERT SPACE OF MEASURES

Zurab Zerakidze[1], Mimoza Tkebuchava[2]
[1]Gori State University, Gori, Georgia
email: zura.zerakidze@mai.ru
[2]I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: mimozatkebuchava@yahoo.com

In this paper we define Haar statistical structures. We prove necessary and sufficient conditions to be weakly separable and strongly separable Haar statistical structures in the Hilbert space of measures.

SECTION OF MECHANICS OF CONTINUA

Chairs: George Jaiani
Co-chair: Natalia Chinchaladze

NUMERICAL ANALYSIS OF THE STRESSED STATE OF PARABOLIC ROTATIONAL THREE LAYERED SHELL

Edison Abramidze
N. Muskhelishvili Institute of Computational Mathematics of the Georgian Technical University, Tbilisi, Georgia
email: edisoni.abramidze@mail.ru

A problem of nonlinear deformation by the surface force of truncated parabolic notational three layered shell is considered. For the numerical solution of the paraboloidal shell deformation process a variant of the non-linear theory of shells is applied, which is constructed on a basis of hypothesis of broken lines.

A particular example of a parabolidal rotational shell deformation is given. A numerical realization of this particular example is given. The obtained numerical results are compared with results obtained by means of the linear theory.

INVESTIGATION OF THREE-PHASE-LAG NONCLASSICAL MODEL OF THERMOELASTIC SOLIDS

Gia Avalishvili[1], Mariam Avalishvili[2]
[1]I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: gavalish@yahoo.com
[2]University of Georgia, Tbilisi, Georgia
email: mavalish@yahoo.com

In the present paper nonclassical dynamic three-dimensional model of thermoelastic solids, which depend on three phase-lag parameters, is studied. The considered model of thermoelastic solids was proposed by Roy Choudhuri [1], where the classical Fourier's law of heat conduction is replaced by an approximation of a generalization that depends on three relaxation times, which are phase-lags of heat flux, temperature gradient, and thermal displacement gradient. This model is a generalization of the Chandrasekharaiah-Tzou model [2], which depends on two phase-lags, and modification and generalization of the Lord-Shulman model [3], which depends only on phase-lag of heat flux. Variational formulation of the general three-dimensional initial-boundary problem with mixed boundary conditions in corresponding spaces of vector-valued distributions with respect to

the time variable with values in Sobolev spaces is obtained, the existence and uniqueness of solution, energy equality, and the continuous dependence of the solution on given data, are proved.

References

1. Roy Choudhuri, S.K. On a thermoelastic three-phase-lag model. *J. Thermal Stresses*, **30** (2007), 231-238.
2. Lord, H.W., Shulman, Y. A generalized dynamical theory of thermoelasticity. *J. Mech. Phys. Solids*, **15** (1967), 299-309.
3. Chandrasekharaiah, D.S. Hyperbolic thermoelasticity: a review of recent literature. *Appl. Mech. Reviews*, **51** (1998), 705-729.

A DYNAMICAL PROBLEM OF ZERO APPROXIMATION OF HIERARCHICAL MODELS FOR FLUIDS

Natalia Chinchaladze

I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia

email: natalia.chinchaladze@tsu.ge

We investigate dynamical problem of zero approximation of hierarchical models for fluids [1]. Applying the Laplace transform technique, we reduce the dynamical problem to the elliptic problem which depends on a complex parameter τ and prove the corresponding uniqueness and existence results. Further, we establish uniform estimate for solutions and their partial derivatives with respect to the parameter τ at infinity and via the inverse Laplace transform show that the original dynamical problem is uniquely solvable.

References

1. Jaiani, G. Mathematical Hierarchical models for fluids. *Book of Abstracts of XIII Annual International Meeting of the Georgian Mechanical Union*, (2022), p. 151.

BASIC BOUNDARY VALUE PROBLEMS FOR THE PLANE THEORY OF ELASTICITY OF POROUS COSSERAT MEDIA FOR CIRCULAR RING WITH VOIDS

Bakur Gulua[1,2], Ucha Todria[2]
[1]I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: bak.gulua@gmail.com
[2]Sokhumi State University, Tbilisi, Georgia
email: utodria@gmail.com

We assume that the body under consideration is an elastic Cosserat media with voids [1, 2]. The two-dimensional system of equations, corresponding to a plane deformation case is written in a complex form and its general solution is presented with the use of two analytic functions of a complex variable and two solutions of the Helmholtz equations [3]. On the basis of the constructed general representation, specific boundary value problems are solved for a circular ring.

References

1. Cowin, S.C., Nunziato, J.W. Linear elastic materials with voids. *Journal of Elasticity*, **13**, 2 (1983), 125-147.
2. Cosserat, E., Cosserat, F. *Theorie des corps deformables*, Hermann, Paris, 1909.
3. Janjgava, R., Gulua, B., Tsotniashvili S. Some boundary value problems for a micropolar porous elastic body. *Arch. Mech.* **72**, 6 (2020), 485–509.

A REMARK ON RELATIONS OF EXPERIMENTAL RESULTS OF J. NIKURADZE AND THE KELDYSH BVP

George Jaiani
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: george.jaiani@gmail.com

In the zero approximation of hierarchical models for fluids [1] the full accordance of peculiarities of setting of the Dirichlet and Keldysh type boundary conditions by motion of the fluids in pipes of angular cross-sections with the results of experiments carried out by J. Nikuradze (see [2] and also [3]) in L. Prandtl's Laboratory at University of Göttingen.

References

1. Jaiani, G. Mathematical Hierarchical models for fluids. *Book of Abstracts of XIII Annual International Meeting of the Georgian Mechanical Union*, (2022), p. 151.

2. Nikuradze, J. *Untersuchungen über turbulente*. Strömungen in nichtkreisförmigen Rohren. Ing. Arch. B. I. 1930. S306.
3. Kavtaradze, R. *Johann (Iwane) Nikuradze. Mythos and Realitaet*. Tbilisi, (2023), 154-155 (in Georgian).

APPROXIMATE SOLUTION OF SOME BOUNDARY VALUE PROBLEMS OF STRESS CONCENTRATION FOR PERFORATED PLATES WITH VOIDS

Roman Janjgava
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University,
Tbilisi, Georgia
email: roman.janjgava@gmail.com

We consider tension-compression problems for rectangular porous aluminum plates with one or two circular holes. The corresponding two-dimensional system of equilibrium equations is obtained from the linear three-dimensional Cowin-Nunziato model [1] by the method of successive differentiation [2]. Boundary value problems are solved by an approximate method, for which general representations of the solution of a system of equilibrium equations and the method of fundamental solutions are used [3], [4]. Stress concentration factors on the contours of the hole are calculated.

References

1. Cowin, S. C., Nunziato, J. W. Linear elastic materials with voids. *J. Elas.*, **13** (1983), 125–147.
2. Vekua, I. N. *Shell Theory: General Methods of Construction*. Pitman Advanced Publishing Program, Boston-London-Melbourne, 1985.
3. Aleksidze, M. A. *Solution of Boundary Value Problems by Expansion in Non-Orthogonal Functions*. Nauka, Moscow, Russia, 1978 (Russian).
4. Janjgava, R. The approximate solution of some plane boundary value problems of the moment theory of elasticity. *Adv. Math. Phys.*, Article ID 3845362 (2016), 12 p.

ON ONE PROBLEM OF THE PLANE THEORY OF VISCOELASTICITY FOR CIRCULAR PLATE WITH A POLYGONAL HOLE

Giorgi Kapanadze
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
A. Razmadze Mathematical Institute of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: kapanadze.49@mail.ru

The problem of the plane theory of viscoelasticity for a circular plate with a polygonal hole is considered according to the Kelvin-Voigt model. It is assumed that the outer boundary of the plate is subjected to a normal compressive force (pressure), and a rigid smooth washer of a slightly larger size is inserted into the hole in such a way that the normal displacements of the boundary points are constant and there is no friction.

Using the methods of conformal mappings and boundary value problems of analytic functions, the required complex potentials are constructed efficiently (in an analytic form). Estimates of these potentials near the corner points are given.

ON THE SOLUTION OF THE BITSADZE-SAMARSKII PROBLEM FOR THE TWO DIMENSIONAL EQUATION OF STATES OF THE MOMENT ELASTICITY THEORY BY VARIATIONAL METHOD

Kosta Svanadze
Akaki Tsereteli States University Department of Mathematics, Kutaisi, Georgia
email: kostasvanadze@yahoo.com

In the paper the Bitsadze-Samarskii non-local problem for the equation of statics of the moment elasticity theory in a rectangle is solved by the variation method. The uniqueness theorem is proved and the necessary and sufficient condition indicating when the vector-function minimizing the specially constructing functional is a solution of the considered problem is given.

STUDY OF THE PROPAGATION OF ACTION POTENTIALS IN HEART TISSUE USING CABLE EQUATION

Natela Zirakashvili[1], Teona Zirakashvili[2,3]
[1]I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: natela.zirakashvili@tsu.ge
[2]Ilia State University, Tbilisi, Georgia
email: teona.zirakashvili.1@iliauni.edu.ge
[3]"Tbilisi Heart and vascular Clinic" LTD, Tbilisi, Georgia
email: zirakashvilit@gmail.com

Cardiovascular diseases are still the leading factors of mortality worldwide. It is particularly noteworthy, that the major reason of death is caused by the heart failure, for example, due to a heart attack and development of a fatal arrhythmia. The direct cause of fatal cardiac arrhythmias is still not completely investigated, however, in many cases the cause can be traced to a failure of the cardiac action potential to propagate correctly. Remarkably, the propagation of action potential is still not completely understood in spite of many years of investigation. Therefore, the study remains a critical topic of many modern scientific studies. The aim of the current work is to investigate the prevalence of action potentials in cardiac tissue using the cable equation. In the variation of the cable equation, developed by Lord Kelvin for modeling the propagation of electrical signals of underwater telegraphs, the passive one-dimensional cable equation is obtained, which is a model of monodomain and bidomain, which describes the electrical behavior of the cell membrane of the heart tissue and the propagation of the action potential. A homogeneous representation of heart tissue includes a large number of identical cells, which can be imagined as two interconnected spaces - intracellular and extracellular. Cells are connected to each other by gap junctions. The paper discusses 1D model of continuously connected myocytes. Here, due to the assumption of continuity, the electrical behavior in the tissue is average for many cells, so we will study the behavior of the transmembrane potential for a single cell. For the monodomain, in the absence of current at the beginning and end of the cable (cell), numerical modeling in Matlab is carried out. Figures of the corresponding contours (isolines), 2D and 3D graphs of the obtained numerical results are presented.

SECTION OF MATHEMATICAL MODELING AND NUMERICAL ANALYSIS

Chairs: Teimuraz Davitashvili, Jemal Rogava, Tamaz Vashakmadze
Co-chair: Archil Papukashvili

ON THE APPROXIMATE SOLUTION OF THE BOUNDARY VALUE PROBLEM FOR ORDINARY DIFFERENTIAL EQUATIONS

Giorgi Buzhgulashvili[1], Tamaz Vashakmadze[2]
[1]I. Javakhishvili Tbilisi State University, Master's student
of Department of Mathematics Exact and Natureall Faculty, Tbilisi, Georgia
email: giorgi.buzhghulashvili570@ens.tsu.edu.ge
[2] I. Vekua Institute of Applied Mathematics
of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: tamazvashakmade@gmail.com

The problem of approximate solution for ordinary second-order nonlinear differential equations with Sturm- Liouville boundary conditions by the multi-point difference method, when the sufficient conditions for the existence and uniqueness of the solution are fulfilled according to [1], will be studied. The method developed in [1] and its modification is used to find both -the solution and its derivative- for a class of less smooth functions. The case when the right-hand side of the differential equation is an oscillating type functions is studied separately. In this case, the differential analog solution is performed at each iteration stage by summing the finite part of the trigonometric series with variable coefficients, which is related to the selection of the optimal method during the numerical implementation of the "Fast Fourier Transform". The report will be presented as well as some results obtained in this direction.

References

1. Vashakmadze, T.S. *The Theory of Anisotropic Elastic Plates*, Kluwer Academic Publishers, Dordrecht/ Boston/ London, 1999, xv+240p. https://doi.org/10.1007/978-94-017-3479-0, Publisher Springer, Hardcover ISBN 978-0-7923-5695-0, Softcover ISBN978-90-481-5215.

MATHEMATICAL MODEL FOR THE PROTO-KARTVELIAN POPULATION DYNAMICS

Temur Chilachava, Gia Kvashilava, George Pochkhua
Sokhumi State University, Tbilisi, Georgia
emails: temo_chilachava@yahoo.com, gia.kvashilava@tsu.ge,
g.pochkhua@sou.edu.ge

According to Homer and Diodorus Siculus, the Pelasgians lived in Bronze Age Crete. Hecateos Miletus says that "before the Hellenes, the Peloponnese peninsula was inhabited by barbarians". According to Herodotus, "the Pelasgians spoke a barbarous non-Greek language... the Pelasgian tribe forgot its old language when it became Hellenic."

The Pelasgian tribes spoke the Proto-Kartvelian language and were primarily found in the Peloponnese peninsula, Crete, and other islands, in ancient Asia and the Caucasus. At a certain stage of development (XXX-XXV centuries BC), the Pelasgurian tribes experienced strong harassment from nomadic Indo-European and Semitic warlike tribes, as a result of which their territory of residence was significantly reduced and the population was divided into three parts over time: Proto-Svan; Speaking the Colchian-Georgian language, with the corresponding area of residence, and the third part, speaking Proto-Kartvelian, dispersed on the European continent. Subsequently, the process of dividing the Proto-Kartvelian language continued and today it is represented in the form of four languages: Georgian, Megrelian; Laz and Svan.

From a historical point of view, we consider its mathematical modeling as an innovative approach to describe the area of the population speaking the Proto-Kartvelian language and the process of language transformation, determining the number of the population speaking the relevant language in each period of time.

This work discusses the first period (50^{th} – (30^{th} – 25^{th}) centuries BC) when the entire population spoke one Proto-Kartvelian language. This period is described by a Pearl-Verhulst-type mathematical model. For the unknown function of variable coefficients in the general case, which determines the number of Proto-Kartvelian speaking population at a given moment in time, the exact analytical solution of the Cauchy problem for the Bernoulli equation is found in quadratures. Analytical solutions have been found for specific values of coefficients (constant; qualitative; exponential; trigonometric functions) and the results have been analyzed. The generalization of the mathematical model is also discussed when in the area where the population lives, the assimilation of the neighboring tribes or the unnatural reduction of the population as a result of clashes with the neighboring tribes takes place. In this case, for the unknown function, the Cauchy problem is obtained for the Riccati-type equation. In some particular cases, when the inhomogeneous term has a special form, an analytical solution to the Riccati equation has been found.

MODELING THE DYNAMICS OF A MIXTURE OF NATURAL GAS AND HYDROGEN IN PIPELINE

Teimuraz Davitashvili, Giorgi Rukhaia
I. Vekua Institute of Applied Mathematics of
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
emails: tedavitashvili@gmail.com, TheGR1992@gmail.com

Hydrogen is currently considered one of the most promising fuels of the future. It is expected to be used in a wide variety of applications such as the generation and storage of electricity, automotive fuels and reactive devices, various industries and even our domestic energy needs [1]. At present, the problems of efficient production, storage and transportation of gaseous hydrogen are the main focus of many researchers around the world. The study of the behavior of a mixture of natural gas and hydrogen substances during flow in pipelines has become an urgent task of our time and has attracted the attention of a number of scientists [1-4]. This work is devoted to one mathematical model describing the flow of a mixture of natural gas and hydrogen substances in a pipeline. A quasi-nonlinear system of two-dimensional partial differential equations is considered, which describes the unsteady flow of a mixture of natural gas and hydrogen substances in a pipe. The distribution of pressure and gas flow through a branched gas pipeline has been studied. Some results of numerical calculations of a mixture of natural gas and hydrogen in a gas pipeline are presented.

Acknowledgements. The research was funded by Shota Rustaveli National Scientific Foundation Grant No. FR-22-18445.

References

1. Ball, M., Basile, A., Veziroglu, T.N. Compendium of Hydrogen Energy: Hydrogen Use, Safety and the Hydrogen Economy; Woodhead Publishing: Cambridge, UK, 2015.
2. Hosseini, S.F., Wahid, M.A. Hydrogen production from renewable and sustainable energy resources: Promising green energy carrier for clean development. Renew. Sustain. Energy Rev. **57** (2016), 850–866.
3. Davitashvili, T. On liquid phase hydrates formation in pipelines in the course of gas non-stationary flow. E3S Web of Conferences **230**, 01006 (2021), DOI: https://doi.org/10.1051/e3sconf/202123001006.

THE MATHEMATICAL MODEL OF ELECTRON AFFINITY (DRIVE) AND ELECTRODE POTENTIAN FUNCTIONAL CONNECTION

Giorgi Gobejishvili
email: gobejishviligiorgi01@gmail.com

Magnitudes: electron affinity (drive) – Ee, electrode potential φ^0 of important characteristics, atoms, molecules. Until now, it was considered that they have no direct connection with each other, but we show that they have a definite connection both with each other and with the serial number of elements in Mendeleev's periodic system; The values of the electrode potentials are arranged in the group (row) of the corresponding elements, which is caused by the periodic repetition of the electronic configurations of the elements in the row. Such a regular arrangement of electronic potentials in a group once again reveals the deep physical meaning of Mendeleev's periodic system of elements. The general form of electronic potential that we have obtained is expressed by the ratio$\varphi^0 = K_0^m \cdot n_e^n \cdot 0{,}04 \cdot E_e$n turn, this equation is related to the Gibbs free energy by the following equation:

$$-\Delta G = nF\varphi^0$$

By combining these two equations and the equation for electronegativity (X=(Ee+I)/2) (on the Muller scale), one can obtain an equation for the rate of a chemical reaction:

$$K = \frac{kT}{h}\, e^{K^0(2X-I)/RT}$$

where the rate of a chemical reaction is represented by two quantities: electronegativity (X) and ionization energy (I)

References

1. Lukashvili, N., Giorgadze, K., Pachulia, Z. (2009) General Chemistry 1985.

ON THE EXACT SOLUTIONS OF THE ZAKHAROV-KUZNETSOV DYNAMICAL EQUATION IN AN ELECTRON-POSITRON-ION PLASMA

David Kaladze, Luba Tsamalashvili
I. Vekua Institute of Applied Mathematics
of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
emails: datokala@yahoo.com, luba_tsamal@yahoo.com

Using the exp-function method the traveling wave special exact solutions of the (2+1)D nonlinear Zakharov-Kuznetsov partial differential equation in an electron-positron-ion plasma are represented. The results are expressed in the forms of hyperbolic, trigonometric, exponential and rational functions and have spatially isolated structural forms. Traveling wave velocity is defined as the function of dynamic parameters.

ON THE ACCURACY OF THE DIFFERENCE SCHEME FOR A NONLINEAR MODEL OF THE DYNAMIC BEAM

Zviad Kalichava[1], Jemal Peradze[2], [3]Zviad Tsiklauri
[1]Muskhelishvili Institute of Computational Mathematics
of the Georgian Technical University, Tbilisi, Georgia
email: zviadi.kalichava@gmail.com
[2]I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: j_peradze@yahoo.com
[3]Georgian Technical University, Tbilisi, Georgia
email: zviad_tsiklauri@yahoo.com

The initial boundary value problem is posed for a nonlinear integro-differential inhomogeneous equation that describes the dynamic behavior of the beam. To approximate the solution with respect to a time variable the Crank–Nicolson type difference scheme is used, the error of which is estimated.

MATHEMATICAL AND COMPUTER MODEL OF EFFECTIVE FIGHT AGAINST DISINFORMATION

Nugzar Kereselidze
Sokhumi State University, Tbilisi, Georgia
email: nkereselidze@sou.edu.ge

The mathematical and computer model of disinformation is considered. The dynamic system of dissemination of false information and its opposite - objective information is described in the model. A function has been introduced in which the cost of disseminating objective information is "weighted". The number of people under the influence of misinformation in the society, which do not create a critical point in terms of information security, is determined. With this in mind, the task of control the fight against disinformation is built. The task of control is solved by numerical methods and computer experiments.

ON THE AXI-SYMMETRIC FLUID FLOW IN CASE OF SMALL REYNOLDS NUMBER

Nino Khatiashvili
I. Vekua Institute of Applied Mathematics
of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: ninakhatia@gmail.com

We consider the axi-symmetric incompressible unsteady fluid flow over the axis of symmetry for the small Reynolds number. The velocity components of the flow satisfy the nonlinear Navier - Stokes equations (NSE) with the suitable initial-boundary conditions [1].For the small Reynolds number NSE can be reduced to the Stokes linear system (STS) [1]. We have studied the Stokes system in the axi-symmetric case when the pressure depends on time exponentially. By the separation of variables the exact solutions of STS are obtained.

Non-symmetric Stokes flows were studied by the author in [2-6].

References

1. Batchelor, G.K, *An Introduction to Fluid Dynamics.* Cambridge Univ. Press, Cambridge, 2000.
2. Khatiashvili, N., On the Free Boundary Problem for the Creepping Flows. *Lecture Notes in Engineering and Computer Science*: *Proceedings of the World Congress on Engineering*, London, UK, (2021), 29-34.
3. Khatiashvili, N., Janjgava, D. On the 3D Stokes Flow in the Infinite Domains. *Proc. of I. Vekua Inst. of Appl. Math.,* **71** (2021), 19-28.
4. Khatiashvili, N. On the Stokes Flow in Pipes with the Polygonal Cross-Section. *Lecture Notes in Engineering and Computer Science, Proceedings of the World Congress on Engineering* 2022, 6-8 July, London, UK, (2022), 1-7.
5. Khatiashvili, N. On the Creeping Flow in Prismatic Pipes. *IAENG International Journal of Applied Mathematics*, **52** (2022), 1144-1149.
6. Khatiashvili, N. On the Free Boundary Problem for the Low Reynolds Number. In: Ao, SI., Gelman, L. (eds) *Transactions on Engineering Technologies. Lecture Notes in Electrical Engineering,* Springer, Singapore, **919** (2023),17-31.

ON THE NUMERICAL SOLUTION TO J. BALL'S BEAM EQUATION IN THE CASE WHERE THE MATERIAL'S EFFECTIVE VISCOSITY IS DEPENDENT ON ITS VELOCITY

Archil Papukashvili[1,2], Giorgi Geladze[1], Zurab Vashakidze[2], Meri Sharikadze[1]
[1]I. Vekua Institute of Applied Mathematics
of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
emails: archil.papukashvili@tsu.ge, giorgi.geladze@tsu.ge, meri.sharikadze@tsu.ge
[2]School of Science and Technology the University of Georgia
Tbilisi, Georgia
email: zurab.vashakidze@gmail.com

The current article is a continuation of the previously published papers [1]-[3], which examine the initial-boundary value problem for J. Ball's integro-differential equation. The equation models the dynamic behaviour of a beam. To obtain an approximate solution, a combination of the Galerkin method, a stable symmetric difference scheme, and the Jacobi iteration method is utilized. In papers [1]-[2], the numerical algorithm is validated using numerical samples. The present paper, along with [3], focuses on the application of the numerical solution to a practical problem. In particular, the numerical results of the initial-boundary value problem for a specific iron beam are presented, where the effective viscosity of the material depends on its velocity. The results are summarized in a table.

References

1. Papukashvili, A., Papukashvili, G., Sharikadze, M. Numerical calculations of the J. Ball nonlinear dynamic beam. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.* **32** (2018), 47-50.
2. Papukashvili, A., Papukashvili, G., Sharikadze, M. On a numerical realization for a Timoshenko type nonlinear beam equation. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.* **33** (2019), 51-54.
3. Papukashvili, A. Geladze, G., Vashakidze, Z., Sharikadze, M. On the Algorithm of an Approximate Solution and Numerical Computations for J. Ball Nonlinear Integro-Differential Equation. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.* **36** (2022), 75-78.

ON THE SOLUTION OF AN INITIAL-BOUNDARY VALUE PROBLEM OF A STRING

Jemal Peradze[1], Archil Papukashvili[2,3], Giorgi Papukashvili[4],
Meri Sharikadze[2]
[1]Faculty of Exact and Natural Sciences,
I. Javakhishvili Tbilisi State University, Tbilisi, Georgia
email: j_peradze@yahoo.com
[2]I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University
emails: archil.papukashvili@tsu.ge, meri.sharikadze@tsu.ge
[3]School of Science and Technology the University of Georgia
[4]V. Komarovi no. 199 Public School
email: gagapapukashvili@gmail.com

A three–step method for a nonlinear integro-differential hyperbolic equation which describes the behavior of a dynamic string is presented. The method has been tested on an example.

ON A LOCAL CONVERGENCE OF A SYMMETRIC THREE-LAYER SEMI-DISCRETE SCHEME FOR THE NONLINEAR DYNAMIC STRING EQUATION OF KIRCHHOFF-TYPE WITH TIME-VARYING COEFFICIENTS

Jemal Rogava[1], Zurab Vashakidze[2]
[1]Faculty of Exact and Natural Sciences, I. Javakhishvili Tbilisi State University,
I. Vekua Institute of Applied Mathematics of TSU, Tbilisi, Georgia
email: jemal.rogava@tsu.ge
[2]Institute of Mathematics, School of Science and Technology,
The University of Georgia,
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University,
Tbilisi, Georgia
emails: zurab.vashakidze@gmail.com, z.vashakidze@ug.edu.ge

In this talk, we shall examine an initial-boundary value problem that is addressed to the Kirchhoff-type nonlinear dynamic string equation with time-varying coefficients, which has been discussed in the paper [1]. Our objective is to develop a temporal discretization algorithm that can provide an approximation for a solution to the initial-boundary value problem. For this purpose, we employ a symmetric three-layer semi-discrete scheme with respect to the temporal variable, wherein the value of the nonlinear term is evaluated at the middle node point. This technique allows us to obtain numerical solutions per temporal step by inverting linear operators, resulting in a system of second-order linear ordinary

differential equations. We have established the local convergence of the proposed scheme, which exhibits quadratic convergence concerning the step size of the discretization of time on the local temporal interval.

Acknowledgements. The second author of this work was supported by the Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant number: FR-21-301, project title: "Metamaterials with Cracks and Wave Diffraction Problems"].

References

1. Rogava J, Vashakidze Z. On Convergence of a Three-layer Semi-discrete scheme for the Nonlinear Dynamic String Equation of Kirchhoff-type with Time-dependent Coefficients. *arXiv preprint* arXiv:2303.10350. 2023. DOI: 10.48550/arXiv.2303.10350.

CONTENT

SECTION OF MATHEMATICAL LOGIC AND FOUNDATIONS

SECTION OF APPLIED LOGICS AND PROGRAMMING

SECTION OF ALGEBRA, GEOMETRY AND NUMBER THEORY

SECTION OF REAL ANALYSIS

SECTION OF COMPLEX ANALYSIS AND APPLICATIONS

SECTION OF ORDINARY DIFFERENTIAL EQUATIONS AND OPTIMAL CONTROL

SECTION OF PARTIAL DIFFERENTIAL EQUATIONS

SECTION OF PROBABILITY THEORY AND MATHEMATICAL STATISTICS

SECTION OF MECHANICS OF CONTINUA

SECTION OF MATHEMATICAL MODELING AND NUMERICAL ANALYSIS