XXXVI International Enlarged Sessions of the Seminar of Ilia Vekua Institute of Applied Mathematics of Ivane Javakhisvili Tbilisi State University

Book of Abstracts

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Foreword

The present book of abstracts contains abstracts of talks given at XXXVI Enlarged sessions (April 19-21, 2022) of the Seminar of I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University.

Each Section (there are 10 ones) is presented as separate Chapter of the book. The responsibility for the contents of each Chapter lies with leaders together with speakers.

SECTION OF MATHEMATICAL LOGIC AND FOUNDATIONS

Chairs: Alexander Kharazishvili, Roland Omanadze
Co-chair: Archil Kipiani

CERTAIN CLASSES OF MEASURES, THEIR CARDINALITY AND APPLICATIONS

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If M is a given class of σ-finite measures on E, then all real-valued functions f defined on E can be of the following three categories: absolutely nonmeasurable functions with respect to M, relatively measurable functions with respect to M and absolutely (or universally) measurable functions with respect to M.

In the presented talk we consider various families of the measures, their characterizations in the sense of the set theory and the cardinality of some classes of measures be presented.

DEGREE STRUCTURE OF CONJUNCTIVE REDUCIBILITY

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A set A is c-reducible to a set B (in symbols: $A \leq_c B$) if there exists a computable function f such that for all $x \in \omega$ (where ω denotes the set of natural numbers),

$$x \in A \Leftrightarrow D_{f(x)} \subseteq B,$$

where D_u is the finite set with canonical index u. In this case, we say that $A \leq_c B$ via f.

If $A \leq_c B$ via f and for all x and y,

$$x \neq y \Rightarrow D_{f(x)} \cap D_{f(y)} = \emptyset,$$

then we say that A is c_1-reducible to B, denoted by $A \leq_{c_1} B$.

Our notation and terminology are standard, and can be found e.g., in [1, 2].

In this talk we will present the following results.

Theorem 1. For every noncomputable c.e. incomplete c-degree, there exists a nonspeedable c-degree incomparable with it.

Theorem 2. The c-degree of a hypersimple set includes an infinite collection of c_1-degrees linearly ordered under $\leq_{c_1}$ with order type of the integers and consisting entirely of hypersimple sets.

Theorem 3. There exist two c.e. sets having no least upper bound in the c_1-reducibility ordering.

Theorem 4. The c.e. c_1-degrees are not dense.

References:
1. Rogers, H.Jr. Theory of Recursive Functions and Effective Computability: McGraw-Hill, New York, 1967.
2. Soare, R.I. Recursively Enumerable Sets and Degrees: Springer Verlag, Berlin, 1987.

DYNAMIC LUKASIEWICZ LOGIC WITH APPLICATION IN NEURAL NETWORKS

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It is introduced neural network dynamic Lukasiewicz logic $NND\text{Ł}$ and corresponding to it dynamic MV-algebra, which are algebraic counterparts of the logic, that in turn represent two-sorted algebras $(\mathcal{M}, \mathcal{R}, \diamond)$ that combine the varieties of MV-algebras $\mathcal{M} = (M, \oplus, \odot, \sim, 0, 1)$ and regular algebras $\mathcal{R} = (R, \cup, ;, ^*)$ into a single finitely axiomatized variety resemblig R-module with "scalar" multiplication $\diamond$. Kripke semantics is developed for application to neural networks.

SOME PROPERTIES OF ALGEBRAIC CURVES

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In this talk we consider several properties of the algebraic curves, which are of interest from geometric, combinatorial, algebraic, and number-theoretical points of view and somehow illustrates the role of algebraic curves in various mathematical topics.

ON AN EXISTENCE OF NON-MEASURABLE SET

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Let E be a basic space, which is equipped with a transformation group G and $X \subset E$. We say that X is almost G-invariant (with respect to μ measure), if for each transformation $g \in G$ we have the equality $(\forall g)(g \in G \Rightarrow \mu(g(X) \triangle X) = 0)$ (see [1, 2]) .

Theorem. If a set X is not almost invariant, but its measurable hull (measurable kernel) is almost invariant, then X is a non-measurable set.

References:
1. P. Halmos, Measure theory, Princeton, Van Nostrad, 1950.
2. A. Kharazishvili, Topics in Measure Theory and Real Analysis, Atlantis Press/World Scientific, 2009.

CONNECTIONS BETWEEN K-PARTITIONS OF BAIRE SPACES AND PRECIPITOUS IDEALS AS WELL AS MEASURABLE AND REAL-MEASURABLE CARDINALS

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Our main point of interest will be such partitions of Baire space into meager sets that sum of any subfamily of it has the Baire property. As will be presented the existence of such partitions is deeply connected with the existence of measurable and real-measurable cardinals. What's more when reasoning about them precipitous ideals quite often naturally emerge.

ON SOME VERSIONS OF MAZURKIEWICZ TYPE SETS

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In 1914 Stefan Mazurkiewicz proved that there exists a subset of the Euclidean plane such that every straight line in the plane meets the subset in exactly two points (see [1]). Later on, Waclaw Sierpinski generalized this result. Namely, he proved that, for any natural number $k \geq 2$, there exists a subset of the Euclidean plane such that every straight line in the plane meets the subset in exactly k points. This theorem shows the existence of a Mazurkiewicz type set for every natural $k \geq 2$.

The above-mentioned theorem of Mazurkiewicz initiated a number of research works related to the existence of certain versions of Mazurkiewicz type sets and the study of different aspects of such sets. Nowadays, there is extensive literature reflecting results of those studies. Some results of this sort are given, e.g., in [2] and [3] (they are primarily considered from the geometric view-point).

In this talk one purely set-theoretical theorem and its consequences are presented and discussed. The theorem shows how to produce various versions of Mazurkiewicz type sets. One of the direct corollaries of the theorem is Sierpinski's above-mentioned result. Among straightforward consequences of the theorem are also some of results presented in [2] and [3].

References:

1. Mazurkiewicz S. Sur un ensemble plan qui a avec chaque droite deux et seulement deux points communs, C.R. Varsovie, 7 (1914), 382-384.

2. Kharazishvili A. B., Tetunashvili T. Sh. On some coverings of the Euclidean plane with pairwise congruent circles, Amer. Math. Monthly, 117, 5 (2010), 414-423.

3. Kharazishvili A. B. Elements of Combinatorial Geometry, Part I, Tbilisi, 2016.

SECTION OF APPLIED LOGICS AND PROGRAMMING

Chair: Matthias Baaz
Co-chairs: Jemal Antidze, Besik Dundua, Mikheil Rukhaia

UNIVERSAL FIRST-ORDER SAFETY PROPERTIES

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We show that every universal first-order safety property can be compiled into a universal invariant of a first-order transition system using quantifier-free substitutions only. We apply this insight to prove that every universal first-order safety property is decidable for large classes of stratified guarded first-order transition systems.

APPROXIMATE REASONING TECHNIQUES

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In this talk we consider reasoning with incomplete, imperfect information, which is very common in human communication. For such problems, exact equality/equivalence is replaced by its approximation. This kind of reasoning is a highly nontrivial task and remains an important issue in applications of artificial intelligence.

Modeling the incomplete and imprecise information is achieved using so called tolerance relations, which are reflexive and symmetric, but not necessarily transitive relations. This idea goes back to Poincaré, who viewed tolerance as the notion of fundamental importance in distinguishing mathematics applied to the physical world from ideal mathematics.

In this presented talk we discuss several tolerance relations, starting from crisp and ending with fuzzy tolerance relations.

AN OVERVIEW OF UNCERTAIN REASONING IN PREDICATE LOGIC

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In this talk we survey uncertainty reasoning in predicate logic, where formulas are interpreted over [0,1] interval. We discuss different calculi for reasoning with uncertainty, in particular probabilized sequent calculus and natural deduction

SECTION OF NUMBER THEORY, ALGEBRA AND GEOMETRY

Chairs: Mikheil Amaglobeli, Malkhaz Bakuradze, Giorgi Khimshiashvili, Teimuraz Vepkhvadze
Co-chair: Ketevan Shavgulidze

ON IRREDUCIBLE ALGEBRAIC SETS OVER FREE 2-NILPOTENT GROUPS

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In this talk we give a pure algebraic description on the coordinate groups of irreducible algebraic sets ower non-abelian free 2-nilpotent group N. As a corollary we describe finitely generated groups H which are universally equivalent to the group N (with constants from N in the language). Besides, we give a pure algebraic criterion when a group H, containing N as a subgroup, and N-separated by N, is in fact N-discriminated by N.

SUBGROUP LATTICES OF HALL'S W-POWER GROUPS

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We study Hall's W-power nilpotent groups from the lattice standpoint. A relationship between the structure of a W-power group G and the structure of the lattice of its subgroups $L(G)$ is established.

References:
1. Amaglobeli, M., Bokelavadze, T. Abelian and nilpotent varieties of power groups. Georgian Math. J. 18 (2011), no. 3, 425–439.
2. Bokelavadze, T. Lattice isomorphism of nilpotent W-power groups. Bull. Georgian Natl. Acad. Sci. 173 (2006), no. 3, 463–465.
3. Hall, Ph. The Edmonton notes on nilpotent groups. Queen Mary College Mathematics Notes. Queen Mary College, Mathematics Department, London, 1969 iii+76 pp

LAMÉ-GIELIS CURVES AND RVACHEV'S R-FUNCTIONS

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Gielis transformations (GT) are a generalization of Lamé curves, and hence of the circle and the Pythagorean theorem. Its origin is in modeling natural shapes and phenomena [1, 2], and in mathematics it has opened the door to using Fourier methods to solve boundary value problems in natural shapes [3] and in the study of Generalized Möbius-Listing bodies [4]. To combine domains, we can make use of the natural alliance between Lamé's work [5] and Rvachev's R-functions [6]. GT define normal polar domains, and for non-integer symmetries, self-intersecting curves are generated, leading to separate domains, which can be defined as single domains using R-functions [7, 8]. This combination also gives the possibility to a continuous transformation between logical functions. A logical next step is the generalization from 2-valued (Boolean) logic to n-valued logic defining different partitions, which could be useful in the study of boundaries and shells.

Reference:

1. Gielis J. A generic geometric transformation that unifies a large range of natural and abstract shapes. American Journal of Botany 90, 3 Invited Special (2003), 333-338.

2. Gielis J., Shi P., Beirinckx B., Caratelli D., Ricci P.E. Lamé and Gielis curves in geometry and nature. Mihai A., Mihai I. (Eds). Proceedings of the Conference RIGA 2021 Riemannian Geometry and Applications, Bucharest, Romania, (2021), 139-166.

3. Ricci PE., Gielis J. From Pythagoras to Fourier and from geometry to Nature. Athena Publishing, Amsterdam (2022).

4. Gielis J., Ricci P.E, Tavkhelidze I. The Möbius phenomenon in Generalized Möbius-Listing surfaces and bodies, and Arnold's Cat phenomenon. Advanced Studies: Euro-Tbilisi Mathematical Journal, 14, 4 (2021), 17-35.

5. Lamé G. Examen des différentes méthodes employées pour résoudre les problèmes de géométrie: par G. Lamé... Mme Ve Courcier (1818).

6. Rvachev V.L. Geometric Applications of Logic Algebra, Naukova Dumka, 1967. (In Russian).

7. Fougerolle Y., Truchetet F., Gielis J. Potential Fields of Self Intersecting Gielis Curves for Modeling and Generalized Blending Techniques Atlantis Transactions in Geometry Volume 2, 67-81. Atlantis-Springer (2017).

8. Gielis J., Tavkhelidze I. The General Case of Cutting of GML surfaces and bodies. (arXiv:1904.01414 preprint), (2020).

PROJECTIVE GEOMETRIES OVER LATTICES AND THEIR MORPHISMS

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In this talk, we study ring projective geometries, their association with projective lattices, and morphisms between geometries and between lattices. Factor-geometries of projective geometries on the rings, morphisms between them, and their main properties are also studied.

References:

1. Faure C. A., Fr¨olicher A. Morphisms of projective geometries and semilinear maps// Geom. Dedicata. 53, 3 (1994), 237-262.

2. Lashkhi A. A. General geometric lattices and projective geometry of modules. Geometry, 1// J. Math. Sci. 74, 3 (1995), 1044–1077.

3. Buekenhout F. Handbook of Incidence Geometry. Buildings and Foundations. — Amsterdam: NorthHolland, 1995.

4. Lashkhi, Alexander; Bokelavadze, Tengiz. Affine geometry of Hall's w-power groups. Bull. Georgian Natl. Acad. Sci. (N.S.) 1 (175) (2007), 4, 41-43.

5. Bokelavadze, Tengiz; Lashkhi, Alexander. Affine isomorphisms of power groups. Bull. Georgian Natl. Acad. Sci. (N.S.) 6, 3 (2012), 25-29.

ON THE GENERALIZED THETA-SERIES FOR POSITIVE DEFINITE NONDIAGONAL QUADRATIC FORMS OF R VARIABLES

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Gooding [1] calculated the upper bound of the dimension of the space generalized theta-series for some diagonal quadratic form of r variables. In [2] we calculated the upper bound of the dimension of the space T(v,Q) for some quadratic form of r variables.

In this talk the spaces of generalized theta-series with respect to nondiagonal quadratic forms are considered and the upper bound for the dimension of these spaces is obtained.

References:

1. F. Gooding, Modular forms arising from spherical polynomials and positive definite quadratic forms, J. Number Theory, 9, (1977), 36-47.

2. K. Shavgulidze, On the space of generalized theta-series for certain quadratic forms in any number of variables, Mathematica Slovaca 69, 1 (2019), 87-98.

A NECESSARY AND SUFFICIENT CONDITION FOR THE EXISTENCE OF A MONOID FACTORIZATION

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In this talk monoid factorizations and descent 1-cocycles for monoids will be discussed. A neccessary and sufficient condition in terms of descent 1-cocycles for a monoid to be factorized through its two submonoids will be shown.

ON THE GENERALIZED LIFEGUARD PROBLEM

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To solve this main problem we use reconcept of the T-rescue range of lifeguard which consists of the points in H+ which lifeguard can reach from A(0; a) within time T. Let us denote by B(A, T) the boundary of R(A, T) which consists of the points which lifeguard can reach from A(0; -a) along a minimal time path exactly in time T. A closely related notion is the T-rescuer's area of swimmer, $L_T(B)$, consisting of all points in H- from which point B(0; b) can be reached within time T. Obviously, this set is the union of t-levels of minimal rescue time for $0 < t < T$.

Knowledge of this sets makes the situation more visual and may be used in planning safe beach structures. To determine them we use the analogy between the optimal path of lifeguard and refraction of light, which enables us to use the concept of refraction wavefront from geometric optics. Since both

settings are based on the refraction formula the following results and their proofs are mathematically rigorous. The crucial observation is that the boundary B(A, T) of R(A, T) coincides the refraction wavefront with coefficients which is known to be a straight line segment.

ABOUT SOME PROPERTIES OF ONE SPECIAL CLASS OF THE GML BODIES

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The report discusses Generalized Möbius-Listing (GML) bodies whose radial section represents the flat areas bounded by Rhodonea curves (Grandi roses). The paper provides an analytical representation of such bodies and establishes some properties of the bodies based on it.

References:
1. Gielis J., Ricci P.E, Tavkhelidze I. The Möbius phenomenon in Generalized Möbius-Listing surfaces and bodies, and Arnold's Cat phenomenon. Advanced Studies: Euro-Tbilisi Mathematical Journal, 14, 4 (2021), 17-35.

POSITIVE INTEGERS REPRESENTED BY SOME BINARY FORMS

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The formulae for the average number of representations of positive integers by a genus of positive binary quadratic forms are given. It gives us the opportunity to characterize all the primes or the primes multiplied by natural power of two which can be represented by some binary quadratic forms.

SECTION OF REAL ANALYSIS

Chairs: Ushangi Goginava, Leri Gogoladze
Co-chair: Ana Danelia

LEBESGUE'S TEST FOR GENERAL DIRICHLET'S INTEGRALS

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It is well-known the Lebesgue ([1], [4]) test for trigonometric Fourier series. Taberski ([2], [3]) considered real-valued Lebesgue locally integrable functions f, such that

$$\lim_{T\to\infty}\int_T^{T+c}|f(t)|dt = 0, \quad \lim_{T\to\infty}\int_{-T-c}^{-T}|f(t)|dt = 0,$$

for every fixed $c > 0$. For this class of functions, he defined generalized Dirichlet's integrals. Taberski ([2], [3]) investigated problems of convergence and $(C, 1)$-summability of this integrals.

In this talk the analogous of the Lebesgue test for the generalized Dirichlet's integrals is formulated and proved.

References:
1. Lebesgue, H. Recherches sur la onvergence des séries de Fourier. Math. Ann., 61 (1905), 251-280.
2. Taberski, R. Convergence of some trigonometric sums. Demonstratio Mathematica, 5 (1973), 101-117.
3. Taberski, R. On general Dirichlet's integrals. Anales soc. Math Polonae, Series I: Prace mathematyczne, (1974), XVII, 499-512.
4. Zygmund A. Trigonometric Series. Cambridge University Press, Vol. 1 (1959).

MATRIX SUMMABILITY OF WALSH-FOURIER SERIES

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The report will establish the necessary and sufficient conditions for the sequence obtained as a result of the matrix transformation of a sequence of partial sums of Walsh-Fourier series to be convergent in norm.

ON THE UNCONDITIONAL CONVERGENCE OF GENERAL FOURIER SERIES FOR LIP1 CLASS OF FUNCTIONS

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The questions of unconditional convergence of Lip1 class functions with respect to general orthonormal systems are considered. The sufficient conditions are obtained to be satisfied by the functions of orthonormal systems so that the Fourier series of every function from Lip1 class are unconditionally

convergent with respect to this system. Some of the obtained results are proved to be best possible. It is also proved that from every orthonormal (φ_n) system a subsequence (φ_{n_k}) can be taken with respect to which the Fourier series of every function from Lip1 class will be unconditionally convergent.

GENERALIZED ABSOLUTE CONVERGENSE OF DOUBLE FOURIER-HAAR SERIES

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The generalized absolute convergence of double Fourier-Haar series is considered. In particular, the sufficient conditions for the convergence of the series

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \gamma_{mn} |\widehat{f}(m,n)|^r, \ \ 0 < r < 2,$$

are established, where $\{\gamma_{mn}\}_{m\geq 1,\, n\geq 1}$ is a definite multiple sequence of nonnegative numbers and $\widehat{f}(m,n)$ denotes the Fourier-Haar coefficients of the function $f(x,y) \in L(I^2)$, where $I^2 = [0,1] \times [0,1]$.

The sufficient conditions are formulated in terms of mixed and partial moduli of variation for the function of two variables.

ON MALIGRANDA'S PROOF OF AM-GM INEQUALITY

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We will discuss a proof of AM-GM inequality appeared in Lech Maligranda's note of 2012.

ON A 1906'S RESULT OF L. FEJER

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We plan to discuss a result which appeared in Fejer, L. Sur la serie de Fourier (French). C. R. 142, 501-503 (1906).

ON RECONSTRUCTION OF COEFFICIENTS OF SOME ALMOST EVERYWHERE DIVERGENT ORTHOGONAL SERIES

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In this talk the problem of reconstruction of coefficients of a certain given orthogonal series by the means of the values of the sum of the series is considered.

It is shown that if a Rademacher series converges at least at one point of $[0, 1]$, then, the series converges on a countable subset H of $[0, 1]$. Formulas to calculate coefficients of the series are presented. It is also shown that every coefficient of the series can be calculated by means of the values of the sum of the series at certain two points of H depending on the index of the coefficient.

Namely, it is established that:

if a Rademacher series

$$\sum_{k=0}^{\infty} a_k r_k(t) \qquad (1)$$

converges at a point $t_0 \in [0, 1]$, then, the series (1) converges at any point of the set $H(t_0) = \left\{ t_0, t_0 \dot{+} \frac{1}{2}, t_0 \dot{+} \frac{1}{2^2}, \ldots \right\}$,

also, if $\sum_{k=0}^{\infty} a_k r_k(t) = f(t)$, when $t \in H(t_0)$, then,

$$a_k = \frac{1}{2} r_k(t_0) \left[f(t_0) - f\left(t_0 \dot{+} \frac{1}{2^{k+1}} \right) \right] \qquad (2)$$

for every nonnegative integer k.

According to Rademacher's and Kolmogorov's well-known theorems a Rademacher series converges almost everywhere on $[0, 1]$ or diverges almost everywhere on $[0, 1]$.

If the series (1) converges almost everywhere on $[0, 1]$ then, above presented formulas (2) coincide with the Fourier formulas to calculate the coefficients of the series (1).

In the case when the series (1) converges at least at one point of $[0, 1]$ and diverges almost everywhere on $[0, 1]$ the formulas (2) are valid, while the Fourier formulas to calculate the coefficients of the series (1) are not.

Similar statements hold for some Walsh series with gaps.

ERASING OF SINGULARITIES OF MEAN PERIODIC FUNCTIONS ON $\mathbb{S}^2$

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We study the problem of erasing singularities of smooth functions with zero integrals over circles of fixed radius in $\mathbb{S}^2 = \{\xi \in \mathbb{R}^3 : |\xi| = 1\}$. It is shown that under a certain condition such functions admit a smooth extension to $\mathbb{S}^2$ with the class preserved. In this case the condition, generally speaking, cannot be omitted.

Let ξ_1, ξ_2, ξ_3 be the Cartesian coordinates of a point $\xi \in \mathbb{S}^2$, $\mathbb{S}' = \{\xi \in \mathbb{S}^2 : \xi_3 \neq -1\}$, $\mathcal{S} = \{\xi \in \mathbb{S}^2 : \xi_3 \neq \pm 1\}$. We introduce spherical coordinates φ, θ on $\mathcal{S}$ as follows: $\xi_1 = \sin\theta \sin\varphi$, $\xi_2 = \sin\theta \cos\varphi$, $\xi_3 = \cos\theta$, $\varphi \in [0, 2\pi)$, $\theta \in (0, \pi)$. The distance $d(\xi, \eta)$ between points $\xi, \eta \in \mathbb{S}^2$ is calculated by the formula $d(\xi, \eta) = \arccos(\xi_1\eta_1 + \xi_2\eta_2 + \xi_3\eta_3)$. In particular, $d(\xi, o) = \arccos\xi_3 = \theta$, where $o = (0, 0, 1)$, $\xi \in \mathcal{S}$. The set $S_r(\eta) = \{\xi \in \mathbb{S}^2 : d(\xi, \eta) = r\}$ is called a geodesic circle of radius r on $\mathbb{S}^2$ centered at the point η. Note that the Euclidean radius of this circle is $\sin r$. In addition, for the element of length $dl(\xi)$ of a circle $S_r(o)$, the following relation holds: $dl(\xi) = \sin r \, d\varphi$.

Let $E = \mathbb{S}'$ or $E = \mathbb{S}^2$, $\mathcal{U}_r(E)$ be the set of all infinitely differentiable functions on E that have zero integrals with respect to the measure $dl(\xi)$ over any geodesic circle of radius r on $\mathbb{S}^2$ lying in E. Let also $SO(3)$ be the rotation group of the sphere $\mathbb{S}^2$.

Theorem 1. Let $f \in \mathcal{U}_r(\mathbb{S}')$ and $f \circ \tau = f$ for all $\tau \in SO(2)$. Then $f \in \mathcal{U}_r(\mathbb{S}^2)$.

Note that the invariance condition for the function f in Theorem 1 cannot be omitted. For other singularity erasure theorems for functions with zero spherical means, see [1 - 4] and the bibliography there.

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SECTION OF COMPLEX ANALYSIS AND APPLICATIONS

Chair: Gia Giorgadze
Co-chair: Giorgi Akhalaia

THE SUB-RIEMANNIAN GEOMETRY OF THE 3-DIM SPHERE

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In the report we consider quaternionic description of the 3-dim sphere and we obtain a noncommutative Lie group. After this we construct horizontal vector fields (distribution) on 3-dim sphere and proof horizontality condition of the smooth curve on this sphere [1].

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1. Calin O., Chang D.-C. Sub-Riemannian geometry: general theory and examples. Encyclopedia of Mathematics and Its Applications, vol. 126, Cambridge University Press, 2009.

ON THE INDEX OF GRADIENT OF REAL INVERTIBLE POLYNOMIAL

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We present a number of observations concerned with the so-called invertible polynomials introduced and studied in a series of papers on mathematical physics and singularity theory. Specifically, we consider real versions of invertible polynomials and investigate invariants of the associated isolated hypersurface singularities. By the very definition such a polynomial is weighted homogeneous and its gradient vector field $grad\, f$ has an isolated zero at the origin hence its index $ind_0 grad\, f$ is well defined. This index, referred to as the gradient index of polynomial, is our main concern. In particular, we give an effective estimate for the absolute value of the gradient index $ind_0 grad\, f$ in terms of the weighted homogeneous type of f and investigate its sharpness. For real invertible polynomials of two and three variables, we give the whole set of possible values of the gradient index. As an application, in the case of three variables we give a complete list of possible topological types of Milnor fibres of real invertible polynomials, which generalizes recent results of L.Andersen "On real isolated singularities." I. rXiv: 2110. 04407 [math.AG], 2021 on the topology of isolated real hypersurface singularities. Finally, we present a few open problems and conjectures suggested by our results.

This report is based on the joint work with G. Khimshiashvili.

REGULARIZATION OF THE TWO-PARTICLES COULOMB SCATTERING PROBLEM. THE COULOMB T MATRIX

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Within the classical analytics the scattering T matrix of the two charged particles in the screened potential approximation is studied. Using the convergence factor, it is shown, that the regular Coulomb T matrix outside the energy shell is proportional to the Coulomb potential. The value of the above mentioned matrix on the energy shell is also calculated, its equality to zero on the energy shell and its

vicinity is shown. It is also shown that the Coulomb T matrix under consideration belongs to the Hilbert space.

THE MAXIMUM PRINCIPLE FOR CARLEMAN-VEKUA IRREGULAR EQUATIONS

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The The maximum principle for Carleman-Vekua irregular equations has been formulated and approved, the coefficients of which belong to sufficiently wide range of functions. These spaces represent an extension of the classical spaces introduced by Vekua and a generalization of examples of functions he has explored. Find the classes of Irregular Equations of Carlemagne-Vekua for which the principle of maximum is not fulfilled. The definitions of the spaces of these functions and their properties are given.

ON THE CONSTRUCTION OF INVERSE AND INVERT FORMAL POWER SERIES

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In the report, we discuss the possibility of constructing inverse and invert series for the analytic functions using formal power series. We will provide examples and discuss how the partition of integer participates in the construction of these coefficients [1].

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ON THE SOLUTIONS SPACE OF SPECIAL TYPE RIEMANN EQUATIONS

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The report deals with the issue of finding a linearly independent solution from the known solution of Legendre differential equation

$$(1 - z^2)y''(z) - 2zy'(z) + k(k+1)y(z) = 0$$

for fixed k, and is established its divergence on a circumcircle with radius 1, centered at 0. "The series comparison test" for investigation of divergence of corresponding series is used. In the report we also investigate the relation between the coefficient α, β, γ of hypergeometric function, which represents a solution of the

$$z(1 - z)y''(z) + (\gamma - (\alpha + \beta + 1))y'(z) - \alpha\beta y(z) = 0.$$

The differential equation, and the order p of the space L^p, and a complete answer is given to the question, when the solution belongs to L^p space.

COMPLETE SINGULAR ELLIPTIC SYSTEMS

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The report deals with some of the results of the authors authors' research on the theory of singular elliptic systems. We consider higher order multidimensional complete elliptic systems on a complex plane with higher order inside singularity [see. 1, 2, 3]. Based on the analysis of the solution (which is understood in the classical sense), their global structure and specifics of behaviour in the neighbourhood of the singularity are studied; one of the main results of the research (Theorem 1) has the characteristic form of the basic theorems of classical complex analysis; namely, from the antecedent of local nature the succedent of global nature is obtained. In the case of a nonhomogeneous system (for a sufficiently general form of the right-hand side), the existence and uniqueness Theorem 2 without any additional boundary conditions is obtained. The obtained results are in some sense complete since Theorems 1 and 2 are not generally valid in case the conditions are not fulfilled.

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NEGATIVE BINOMIAL DISTRIBUTION IN MULTIPARTICLE PRODUCTION PROCESSES AND PRIMORDIAL BLACK HOLES COUNTING RULES

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Negative binomial distribution (NBD) provides the best description of the high energy multiparticle production processes, has very clear physical interpretation [1] and corresponds to the intermediate states of independently radiating primordial black holes (PBH).

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THE SERRE DUALITY FOR GENERALIZED ANALYTIC FUNCTIONS

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Sheaves of generalized analytic functions and differential forms on a Riemann surface are considered, several propositions regarding them are proven, Cech cohomology groups of these sheaves are characterized. A proof of a Serre-type duality theorem that relates zeroth and first cohomology groups of the

sheaf of generalized analytic functions and differential forms of a certain kind on a compact Riemann surface is given, the aforementioned proof utilizes facts about Cech cohomology of a sheaf. The way these sheaves are defined makes it possible to consider them on a complex manifold of any dimension.

NEW PROPERTIES OF SOLUTIONS OF CONVOLUTION EQUATIONS

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Let $\mathcal{D}'(\mathbb{R}^n)$ be the space of distributions in $\mathbb{R}^n$, $\mathcal{E}'(\mathbb{R}^n)$ the space of distributions with compact supports in $\mathbb{R}^n$ ($n \geq 2$). Denote by $\mathcal{E}'_\natural(\mathbb{R}^n)$ the set of all radial distributions $T \in \mathcal{E}'(\mathbb{R}^n)$. The simplest example of a distribution from the class $\mathcal{E}'_\natural(\mathbb{R}^n)$ is the Dirac delta function δ_0 supported at zero. We study the properties of solutions of the convolution equation $f * T = 0$ on a given ray in $\mathbb{R}^n$. For $T \in \mathcal{E}'(\mathbb{R}^n)$ we put $\mathcal{D}'_T(\mathbb{R}^n) = \{f \in \mathcal{D}'(\mathbb{R}^n) : f * T = 0\}$. Theorem 1. Let $T \in \mathcal{E}'_\natural(\mathbb{R}^n)$ and $T \neq c\delta_0$, $c \in \mathbb{C}\backslash\{0\}$. Then for any ray L in $\mathbb{R}^n$ there exists a function $f \in (C^\infty \cap \mathcal{D}'_T)(\mathbb{R}^n)$ such that the set of values of the function f on L is dense in $\mathbb{C}$. In particular, for any $\lambda \in \mathbb{C}$ there exists a solution u of the Helmholtz equation $\Delta u + \lambda u = 0$ in $\mathbb{R}^n$ for which the set of values of the function $u(t, 0, \ldots, 0)$ on $[0, +\infty)$ is dense in $\mathbb{C}$. It is easy to see that the condition for the radiality of the distribution $T \in \mathcal{E}'(\mathbb{R}^n)$ in Theorem 1 is essential. Indeed, if for example $T = \frac{\partial}{\partial x_1}\delta_0$, then any solution of the equation $f * T = 0$ does not depend on the variable x_1 and the indicated statement is not fulfilled. We also note that Theorem 1 is generally not true for the one-dimensional convolution equation $f * T = 0$, where the natural analog of the radiality condition is the condition that the distribution $T \in \mathcal{E}'(\mathbb{R}^1)$ is even. The corresponding counterexample is easy to construct, for example, for the distribution $T = \left(\frac{d}{dx_1}\right)^2 \delta_0$. For other results related to the properties of solutions of convolution equations, see [1]-[3].

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SECTION OF ORDINARY DIFFERENTIAL EQUATIONS AND OPTIMAL CONTROL

Chairs: Roman Koplatadze, Tamaz Tadumadze
Co-chair: Tea Shavadze

ONE CLASS OF THE OPTIMAL PROBLEM TAKING INTO CONSIDERATION FACTOR OF DELAY AND MIXED INITIAL CONDITION

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For the optimal problem

$$\dot{x}(t) = (\dot{p}(t), \dot{q}(t))^T = f(t, x(t), p(t - \tau), q(t - \sigma), u(t), u(t - \theta)), \ \ t \in [t_0, t_1],$$

$$\begin{cases} x(t) = (\varphi(t), g(t))^T, \ t < t_0, \\ x(t_0) = (p_0, g(t_0))^T, \end{cases}$$

$$z^i(\tau, \sigma, \theta, p_0, x(t_1)) = 0, \ i = \overline{1, l},$$

$$z^0(\tau, \sigma, \theta, p_0, x(t_1)) \to \min$$

the necessary optimality conditions of delay parameters τ, σ, θ, initial vector p_0, initial $(\varphi(t), g(t))$ and control $u(t)$ functions are obtained.

ON PRACTICAL EXPERIENCE IN PREDICTING THE SPREAD OF CORONAVIRUS (COVID-19)

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Authors' practical experience in predicting coronavirus (Kovid-19) (for both, the World and Georgia) based on modern mathematical models and the well-known computer program EViews-10 ("Econometric Views") is discussed.

For the sake of clarity, it should be noted that, initially, in terms of forecasting, we considered such a key indicator of the spread of the coronavirus such that the total number of cases of infection at the moment (in terms of days). However, for forecasting we used the so-called ARMA ("Autoregression and Moving avereages") type models with the addition of trendy components.

As we have seen, these types of models showed a sufficiently high prediction accuracy for a maximum of a month (then their accuracy dropped). On the other hand, given that the virus is "not going to stop" in the near future, the problem of increasing the forecast horizon is on the agenda.

Therefore (in order to increase the forecast horizon), we considered indicators such as "average daily increase in the number of infected during the month" and "total number of infected by the end of the period (in this case, the month)". This allows a forecast of this figure to be made for a horizon containing several months.

It should be noted, however, that according to the central probability theory of probability, the distribution of the "average daily increase in the number of infected" should be close to normal, which,

in addition to analyzing the spread of the virus, should allow the "pure" statistical method to predict its spread (on the basis of building confidence intervals for the number of infected). On the other hand, as our practical experience has shown, the use of an indicator such as the "total number of infected by the end of the period (in this case, the month)" may be more appropriate to find more reliable current prognostic assessments (due to the monotonicity of this indicator). In addition, it is clear that the accuracy of the forecast should increase as the volume of relevant statistics increases.

The present report is dedicated to the study of these possibilities and the experience accumulated by the author in the field of coronavirus prediction.

DEVELOPMENT OF COMPUTER SYSTEM FOR KNOWLEDGE TESTING USING MULTICRITERIA DISCRETE OPTIMIZATION

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A bicriteria model of discrete optimization is considered for the task of building the optimal knowledge testing test.

ON THE EXISTENCE OF AN OPTIMAL ELEMENT FOR THE QUASI-LINEAR NEUTRAL OPTIMAL PROBLEM WITH SEVERAL DELAYS

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For the quasi-linear neutral optimal problem

$$\dot{x}(t) = \sum_{i=1}^{m} A_i(t)\dot{x}(t - \sigma_i) + f(t, x(t), x(t - \tau_1), \ldots, x(t - \tau_s)), \quad t \in [t_0, t_1],$$

$$x(t) = \varphi(t), \quad t < t_0, \quad x(t_0) = x_0,$$

$$q^i(\sigma_1, \ldots, \sigma_m, \tau_1, \ldots, \tau_s, x_0, x(t_1)) = 0, i = 1, \ldots, l,$$

$$q^0(\sigma_1, \ldots, \sigma_m, \tau_1, \ldots, \tau_s, x_0, x(t_1)) \to \min$$

existence of an optimal element $(\sigma_1, \ldots, \sigma_m, \tau_1, \ldots, \tau_s, x_0, u(\cdot))$ is proved.

n-TH ORDER ESSENTIAL NONLINEAR DIFFERENCE EQUATIONS WITH PROPERTY A AND B

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Consider the difference equation

$$\Delta^{(n)}u(k) + F(u)(k) = 0,$$

where $\Delta^{(n)}u(k)$ is n-th order difference operator. $F : S(N;R) \to S(N;R)$ ($S(N;R)$ denote the set of discrete functions whose set of values is R).

Sufficient conditions for the above equation to have Property A and B are established.

ON THE EXISTENCE OF AN OPTIMAL INITIAL DATA FOR THE CONTROLLED QUASI-LINEAR NEUTRAL EQUATION

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For the quasi-linear neutral optimal problem with the two type controls $v(t)$ and $u(t)$

$$\dot{x}(t) = A(t, x(t), v(t))\dot{x}(\sigma(t)) + f(t, x(t), u(t)), t \in [t_0, t_1],$$

$$x(t) = \varphi(t), t \in [\sigma(t_0), t_0], x(t_0) = x_0,$$

$$q^i(t_0, t_1, x_0, x(t_1)) = 0, i = 1, \ldots, l,$$

$$q^0(t_0, t_1, x_0, x(t_1)) \to \min$$

the existence of an optimal initial data $(t_0, t_1, x_0, v(t), u(t))$ is proved.

OSCILLATION CRITERIA FOR LINEAR ORDINARY DIFFERENTIAL EQUATION

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Consider the differential equation:

$$u^{(n)} + p(t)u = 0, \qquad (1)$$

where $p \in L_{loc}(R_+; R)$, $n \geq 2$. New oscillation criteria generalizing a series of earlier results are established for the differential equation (1).

THE INTEGRAL EQUATION EQUIVALENT TO ONE CLASS OF THE CONTROLLED NEUTRAL DIFFERENTIAL EQUATION AND PROPERTIES OF ITS KERNEL

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The quasi-linear neutral differential equation

$$\dot{x}(t) = A(t, x(t), x(t-\tau), u(t))\dot{x}(t-\tau) + f(t, x(t), x(t-\tau), u(t)), t \in [t_0, t_1] \quad (1)$$

with the initial condition

$$x(t) = \varphi(t), t \in [t_0 - \tau, t_0), \ x(t_0) = x_0 \quad (2)$$

and the integral equation

$$y(t) = x_0 + \int_{t_0}^{t_0+\tau} Y(\xi; t, y(\cdot), u(\cdot)) A(\xi, y(\xi), y(\xi-\tau), u(\xi))\dot{\varphi}(\xi-\tau)d\xi +$$

$$+ \int_{t_0}^{t} Y(\xi; t,), y(\cdot), u(\cdot)) f(\xi, y(\xi), y(\xi-\tau), u(\xi))d\xi, t \in [t_0, t_1] \quad (3)$$

with the initial condition

$$y(t) = \varphi(t), t \in [t_0 - \tau, t_0), \quad (4)$$

are considered, where $u(t)$ is a control function and $Y(\xi; t, y(\cdot)), u(\cdot)$ is the kernel of integral equation (3) satisfying the difference equation. Properties of the kernel $Y(\xi; t, y(\cdot)), u(\cdot)$ are established on the basis of which equivalence of problems (1)-(2) and (3)-(4) is proved.

A MEASURABLE SOLUTION OF THE FUNCTIONAL EQUATION AND THE EXISTENCE OF AN OPTIMAL ELEMENT IN THE OPTIMAL PROBLEM WITH DEVIATING ARGUMRNT

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For the functional equation

$$g(t, y(t), y(t-\theta), y(t+\theta)) = z(t), \quad t \in [t_0, t_1]$$

the sufficient condition of the existence of a measurable solution $y(t), \quad t \in [t_0 - \theta, t_1 + \theta]$ is established. For the optimal problem with the deviating argument

$$\dot{x}(t) = f(t, x(t), x(t-\tau), u(t), u(t-\theta), u(t+\theta)), \quad t \in [t_0, t_1],$$

$$x(t) = \varphi(t), \quad t < t_0, \quad x(t_0) = x_0,$$

$$q^i(\tau, x_0, x(t_1)) = 0, \quad i = 1, \ldots, l,$$

$$q^0(\tau, x_0, x(t_1)) \to \min$$

the existence of an optimal element $(\tau_0, x_{00}, \varphi(\cdot), u(\cdot))$ is proved.

SECTION OF PARTIAL DIFFERENTIAL EQUATIONS

Chairs: Temur Jangveladze, Sergo Kharibegashvili, David Natroshvili
Co-chair: Zurab Kiguradze

DARBOUX TYPE PROBLEM FOR ONE NONLINEAR HYPERBOLIC EQUATION OF THE FOURTH ORDER

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For a fourth-order nonlinear hyperbolic equation with an iterated wave operator in the main part in the angular domain a Darboux type boundary value problem is considered. The concept of a weak generalized solution of the problem posed in the class of continuous functions is introduced. This problem is equivalently reduced to a nonlinear functional equation in the indicated space. Under certain conditions imposed on the nonlinear terms, an apriori estimate for the solution of the functional equation is proved, which implies its existence. Under certain conditions, this solution is a classic one. The questions of uniqueness of the solution to this problem are also considered.

ON ONE SYSTEM OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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One system of nonlinear partial differential equations [1] is considered. Uniqueness and stability of solution of initial-boundary value problem is studied.

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ON A SYSTEM OF FOURTH-ORDER NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS

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The uniqueness and stability of the solution of the initial-boundary value problem for one system of fourth-order nonlinear parabolic integro-differential equations [1, 2] are shown.

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1. Gordeziani, D.G., Dzhangveladze, T.A., Korshiya, T.K. Existence and uniqueness of a solution of certain nonlinear parabolic problems. Differential'nye Uravnenyia, 19, 7 (1983), 1197-1207. English translation: Differential Equations, 19, 7 (1984), 887-895 (Russian).

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ON NUMERICAL SOLUTION OF ONE NONLINEAR PARTIAL DIFFERENTIAL MATHEMATICAL MODEL

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A multidimensional analogue of a two-dimensional system of nonlinear partial differential equations [1] describing the vein formation in plant leaves is studied. A variable direction difference scheme and an average decomposition model are constructed and investigated [2, 3]. A comparative analysis of the results of numerical experiments is carried out.

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ON INVESTIGATION AND APPROXIMATE SOLUTION OF TWO SYSTEMS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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Two one-dimensional models based on Maxwell's well-known system of nonlinear partial differential equations [1], describing the process of penetration of a magnetic field in a substance are considered. The uniqueness of the solutions of the corresponding initial-boundary value problems and the convergence of the finite-difference schemes are studied, which are an extension of some of the results obtained in [2, 3].

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ON ONE NONLINEAR DIFFUSION SYSTEM

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The asymptotic behavior, as time variable tends to infinity, of a solution for a nonlinear diffusion system [1, 2] is considered. It is shown that the stationary solution of the system is linearly stable, and the possibility of the Hopf-type bifurcation is observed.

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THE PERIODIC PROBLEM FOR ONE CLASS OF FIRST ORDER HYPERBOLIC SYSTEMS

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For normally hyperbolic systems of the first order, a periodic problem with respect to a spatial variable is considered. The problem posed is equivalently reduced to a system of integral equations of the Volterra type, the study of which proves the existence and uniqueness of the solution of the periodic problem. The cases when the solution is written out explicitly are also considered.

MIXED BOUNDARY VALUE PROBLEM FOR THE ANISOTROPIC HELMHOLTZ TYPE EQUATION

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We consider a new approach to investigate mixed boundary value problems for the anisotropic Helmholtz type equation

$$\sum_{k,j=1}^{3} a_{kj} \frac{\partial^2 u(x)}{\partial x_k \partial x_j} - \tau^2 u(x) = 0,$$

containing a complex parameter τ, in the case of a three-dimensional bounded or unbounded domain Ω, when the smooth boundary surface S is divided into two disjoint parts, S_D and S_N, where the Dirichlet and Neumann type boundary conditions are prescribed respectively.

With the help of the theory of pseudodifferential equations the uniqueness and existence theorems are proved for mixed boundary value problems and the solutions are represented by a linear combination of single and double layer potentials with densities supported in the Dirichlet and Neumann parts of the boundary.

This type of mixed boundary value problems are studied in scientific literature by using the potential methods (see, e.g. [1], [2]). In contrast to the existing approaches, our alternative method has two essential advantages, on the one hand, it does not require extension of the given boundary data to the whole surface and, on the other hand, the representation of a solution does not contain the Steklov-Poincare type operator, which contains the inverse operator of the single layer boundary operator, which is not available explicitly in general for an arbitrary surface. We reduce the mixed BVP under consideration to the boundary integral (pseudodifferential) equations generated by the limiting values of the single and double layer potentials and show the invertibility of the corresponding pseudodifferential operator in appropriate Bessel potential and Sobolev-Slobodetski spaces. This fact will play a crucial role in the process of construction of efficient algorithms for numerical solutions of the mixed BVPs.

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SECTION OF THEORY OF PROBABILITY AND MATHEMATICAL STATISTICS

Dedicated to the 80th birthday anniversary of Revaz Chitashvili

Chair: Elizbar Nadaraia, Omar Purtukhia

ON THE ESTIMATE OF BERNOULLI REGRESSION FUNCTION USING BERNSTEIN POLYNOMIALS FOR GROUP OBSERVATIONS

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The estimate for the Bernoulli regression function is constructed using the Bernstein polynomial for group observations. The question of its consistency and asymptotic normality is studied. Testing hypothesis is constructed in the form of the Bernoulli regression function. The question of consistency of the constructed tests is studied.

LOCAL LIMIT THEOREM FOR SUMS OF DEPENDENT RANDOM VECTORS

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Let (Ω, F, P) be a stationary sequence in the narrow sense on $\{\xi_n, Y_n\}_{n \geq 1}$ probability space. Where $\{\xi_n\}_{n\geq 1}$ is a finite regular Markov Chain. $\{Y_n\}_{n\geq 1}$ is a sequence of random vectors with chain dependences. Local limit theorems are obtained for the conditional and unconditional distributions of the sums $S_{n_1} = \frac{1}{\sqrt{n}}\sum_{j=1}^{n}[Y_j - E(Y_j|\xi_j)]$ and $S_n = \frac{1}{\sqrt{n}}\sum_{j=1}^{n}[Y_j - E(Y_1)]$, respectively.

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THE BELLMAN-CHITASHVILI EQUATION AND AN OPTIMAL EQUIVALENT CHANGE OF MEASURE

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R. Chitashvili in [1] derived the Bellman equation related to the stochastic optimal control problem in a non-Markov setting. He solved a problem of definition of controlled measures using the notion of

stochastic line integral and proved that the value process of the problem is a solution of corresponding semimartingale backward equation [1].

Using these results we study the problem of sufficiency of partial information of the problem of an optimal equivalent change of measure.

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Dedicated to the 80th Anniversary of Revaz Chitashvili

MONTE CARLO SIMULATIONS FOR SOME KNOWN PROBABILITY PROBLEMS

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Monte Carlo simulation method refers to a general class of stochastic methods and is based on computational calculations. In this paper we produce an approximate solution method for some known probability problems using Monte Carlo simulation method.

MARTINGALE REPRESENTATION OF ONE NON-SMOOTH FUNCTIONAL OF BROWNIAN MOTION

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The well-known Clark-Ocone formula (1984) allows constructing a constructive martingale representation for stochastically smooth Brownian functionals. In the case of stochastically non-smooth functionals, if the conditional mathematical expectation of the functional with respect to the natural filtration of the Brownian motion is stochastically smooth, one can use the Glonti-Purtukhia generalization (2017) of the Clark-Ocone formula. Here we study functionals to which the above results cannot be applied and derive a constructive martingale representation.

There are functionals that do not satisfy even the weakened Glonti-Purtukhia condition. It should be noted that when taking the conditional mathematical expectation of a functional of this type, a term similar to the original functional is separated from it, either as a term or as a factor. Such, for example, is the integral over the Lebesgue measure $\int_0^T u_s(\omega)ds$ of a process $u_s(\omega)$ that does not satisfy the Clark-Ocone condition, but satisfies the weakened Glonti-Purtukhia condition (here the conditional mathematical expectation gives a adapted to filtration term of the same type), and the corresponding representation for it was obtained in [1]. Another functional of this type is $I_{\{B_T^* \leq x\}}$ ($B_T^* = max_{t \in [0,T]}B_t$), which we learned about from Andrei Ionescu (PhD student at King's College London), for which we are grateful (here the conditional mathematical expectation gives a multiplier of the same type adapted to filtration).

Theorem. The following stochastic integral representation is valid

$$I_{\{B_T^* \leq x\}} = P\{B_T^* \leq x\} - 2\int_0^T I_{\{B_t^* \leq x\}}\frac{1}{\sqrt{T-t}}\varphi\big(\frac{x-B_t}{\sqrt{T-t}}\big)dB_t \quad (P-a.s.),$$

where φ is the density function of the standard normal distribution.

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THE REPRESENTATION OF THE SOLUTION OF SOME BSDE

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The representation of the solution of some Backward Stochastic Differential Equation is studied.

SOME REMARKS ON THE ABSOLUTE CONTINUITY FOR RANDOM MEASURES UNDER NONLINEAR TRANSFORMATIONS AND APPLICATIONS IN THERMODYNAMICS

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It is considered the principle ways of using stochastic analyses in some problems of thermodynamics, descriptive by equations with random parameters for variety mass and volume, for example the adiabatic equation for real gas is considered.

CONSISTENT CRITERION OF HYPOTHESIS TESTING AND CONSISTENT ESTIMATOR OF PARAMETER IN THE HILBERT SPACE OF MEASURES

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Let $\{\mu_h, h \in H\}$ be probability measures defined on the measurable space (E, S). For each $h \in H$ denote by $\overline{\mu}_h$ the completion of the measure μ_h, and denote by $dom(\overline{\mu}_h)$ the σ-algebra of all $\overline{\mu}_h$-measurable subsets of E. Let

$$S_1 = \cap_{h \in H} dom(\overline{\mu}_h).$$

Definition. We will say that the statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$ admits a consistent criterion for testing hypothesis if there exists at least one measurable mapping $\delta : (E, S_1) \longrightarrow (H, B(H))$, such that

$$\overline{\mu}_h(\{x : \delta(x) = h\}) = 1, \ \ \forall \, h \in H.$$

Theorem. Let

$$M_H = \oplus_{h \in H} H_2(\overline{\mu}_h)$$

be the Hilbert space of measures, let E be a complete metric space, whose topological weights are not measurable in a wider sense. Let S_1 be a Borel σ-algebra on E. In order for the Borel orthogonal statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$ to admit a consistent criterion for hypotheses testing and also to

admit a consistent estimators of parameters in the theory of (ZFC) & (MA) it is necessary and sufficient that the correspondence $f \longleftrightarrow \psi_f$ defined by the equality

$$\int_E f(x)\nu(dx) = (\psi_f, \nu), \quad \nu \in M_H, \quad f \in F(M_H)$$

was one-to-one, where $F = F(M_H)$ the set of real functions f for which $\int_E f(x)\overline{\mu}_h(dx)$ is defined $\forall \overline{\mu}_h \in M_H$.

SECTION OF MATHEMATICAL MODELING AND NUMERICAL ANALYSIS
Dedicated to the 85th anniversaries of Tamaz Vashakmadze

Chairs: Teimurazi Davitashvili, Jemal Rogava, Tamaz Vashakmadze
Co-chair: Archil Papukashvili

RESEARCH OF THE STRESS STATE OF FLEXIBLE MULTILAYERED CORRUGATED CYLINDRICAL SHELLS OF ROTATION ACCORDING TO A REFINED THEORY

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The problem of deformation of flexible layered corrugated cylindrical shells of rotation with orthotropic layers of constant thickness is considered. The study of the regularities of the shell deformation process is carried out on the basis of a refined nonlinear theory, which takes into account the nonhomogeneity of the lateral-shear deformations.

The obtained results are compared with the results obtained in accordance with the linear theory. Graphs and tables of solutions of the problem are given.

ON THE PROPAGATION OF AN EXPLOSIVE SHOCK WAVE IN A HOMOGENEOUS GRAVITATING THREE-AXIS GAS ELLIPSOID OF ROTATION

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To solve many problems of astrophysics, it is necessary to study the dynamics of gaseous bodies interacting with a gravitational field [1, 2]. Numerous observations show that stars rotate. The study of rotating gravitating gaseous bodies has a long history and originates in the classical works of Newton, Maclaurin, Jacobi, Liouville, Dirichlet, Dedekind, Riemann, Poincaré, Lyapunov and others [3]. In this paper, we consider a non-self-similar problem of a central explosion of a homogeneous gaseous three-axis ellipsoid located in its own gravitational field and solid-body rotating around an axis z is with a constant ω angular velocity. As initial data, the exact solution of the problem of stationary solid-state rotation of a homogeneous three-axis gas Jacobi ellipsoid is considered [4]. It is assumed that at the initial moment of time a central explosion occurs with the release of finite energy. In this case, a diverging shock wave is formed in the centre (a discontinuity surface of the first kind of the unknown functions). To solve the problem, we used the previously proposed asymptotic method of a thin shock layer [5]. Zero approximations are found for the singular asymptotic expansion of the law and the velocity of the medium behind the shock wave, as well as the thermodynamic characteristics of the gravitating gas.

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THE ALGORITHM FOR NUMERICAL REALIZATION OF MULTILAYER SEMI-DISCRETE SCHEMES OF SOLUTION OF AN ABSTRACT EVOLUTIONARY PROBLEM

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In [1], as applied to difference schemes for differential equations, a perturbation algorithm is described that allows one to reduce the solution of multilayer schemes to the solution of two-layer schemes. In [2 - 5] a purely implicit three-layer and four-layer scheme for the evolution equation is reduced to two-layer schemes using the perturbation algorithm. The explicit estimate for the error of the approximate solution was obtained under fairly general assumptions about the problem data. In the presented report, the Cauchy problem for an abstract evolutionary equation with a self-adjoint, positive-definite operator is considered. For this problem, using the perturbation algorithm, a purely implicit multilayer semi-discrete scheme is reduced to two-layer schemes. Using these schemes, an approximate solution of the original problem is constructed. An estimate for the error of the approximate solution is given. The report deals with the issues of parallelization of the considered algorithm.

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NUMERICAL MODELING OF THE DYNAMICS OF THE AIR FLOW AND THE STUDY OF SOME OF ITS ENERGY CHARACTERISTICS

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Based on a three-dimensional hydrostatic mesoscale model, an air flow over the complex relief of the South Caucasus (Georgia) is modeled under the conditions of non-stationary large-scale background processes. Numerical experiments have shown a strong influence of orographic effects on air movement in the troposphere. In particular, it is shown that when an air flow of synoptic scales moves, the vertical amplitudes of the mesoscale flow and the deviation of the air velocity vector along the Likh Ridge increase significantly. Besides, the strong wind regime and statistical characteristics of the Rioni River region were researched for the period 1960-2021. The wind speeds are divided into intervals of 5 m/s, and for

each interval the wind speed recurrence rate is studied by months. It has been determined that in terms of energy, the main range of wind speed for the Kutaisi region is 16-20 m/s. Thus, from an energy point of view, speeds of such magnitude are essential, which ensure the automatic mode of the wind farms and are an important basis for the development of wind power plants in western Georgia.

ON THE FLUID FLOW OVER THE RECTANGULAR AREAS

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We consider 2D incompressible unsteady fluid flow over the rectangle and between two similar rectangles. The velocity components of the flow satisfy the nonlinear Navier - Stokes equation? (NSE) with the suitable initial-boundary conditions [1]. We suppose that near sharp edges the velocity components are non-smooth and by the methods of mathematical physics we obtain exact solutions of NSE for the specific pressure.

The exact solutions of the steady axi-symmetric NSE were obtained in [2 - 4]. The non-smooth exact solutions for 3D incompressible unsteady fluid flow over the octahedron was obtained in [5].

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ON THE ALGORITHM OF AN APPROXIMATE SOLUTION AND NUMERICAL COMPUTATIONS FOR J. BALL NONLINEAR INTEGRO-DIFFERENTIAL EQUATION

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The presented talk is a direct continuation of the articles [1 - 2] that consider the construction of algorithms and their corresponding numerical computations for the approximate solution of nonlinear integro-differential equations of the Timoshenko type. In particular, in this work, an initial-boundary value problem is considered for the J. Ball integro-differential equation, which describes the dynamic state of a beam (see, [3]). The solution is approximated using the Galerkin method, stable symmetrical difference scheme and the Jacobi iteration method. In the articles [1 - 2] the algorithm has been approved by tests. This paper presents the approximate solution to one practical problem. Particularly, the results of numerical computations of the initial-boundary value problem for an iron beam are represented in tables and graphics.

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ON THE ACCURACY OF A PROJECTION METHOD FOR A NONLINEAR PARABOLIC EQUATION

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An initial boundary value problem for a one-dimensional diffusion model is considered. The Galerkin method for the approximation of the solution with respect to the spatial variable is used. The error of the method is estimated.

DECOMPOSITION TYPE FORMULAS FOR APPROXIMATION OF A COSINE OPERATOR FUNCTION

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Based on known trigonometric formulas, a decomposition formula is constructed for the cosine operator function when the argument is the sum of two bounded operators. The error of the n-th approximation is estimated in a Banach space. The case when the number of summands is more than two is also considered, Such an algorithm is proposed that allows us to obtain a $2p + 2$-order decomposition formula from the $2p$-order of decomposition one ($p > 1$ is a natural number).

NUMERICAL SOLUTION OF NON-LINEAR INTEGRO-DIFFERENTIAL BEAM EQUATION

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We use a projection method and the difference scheme, for solving the non-linear integro-differential dynamic beam equation. The results of computer experiment are presented.

ON NUMERICAL REALISATION OF THE HIERARCHICAL MODEL OF I. VEKUA

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The problem of the numerical realization of I. Vekua's model for the one- dimensional case theory of elasticity is considered. For constructing the model, as is well known, the Legendre polynomial system is used. The same problem is solved as well for the case when as a basis the complete system is taken which satisfies Neumann type boundary value conditions. For both cases, the algorithms are created by means of which the constructed solutions are equal.

SECTION OF MECHANICS OF CONTINUA
Dedicated to the 85th birthday anniversary of Tengiz Meunargia

Chair: George Jaiani
Co-chair: Natalia Chinchaladze

TWO-DIMENSIONAL MODELS OF THERMOELASTIC SHELLS WITHIN THE FRAMEWORK OF THE CHANDRASEKHARAIAH-TZOU THEORY

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In the present paper the Chandrasekharaiah-Tzou dynamic three-dimen- sional model of the non-classical theory of thermoelasticity is considered for a shell with variable thickness, which may vanish on a part of the lateral boundary. In the Chandrasekharaiah-Tzou model of thermoelastic solids classical Fourier's law of heat conduction is replaced by its generalization proposed by Tzou, which depends on two relaxation times, and it is a generalization of the Lord-Shulman nonclassical model for thermoelastic bodies, which depends on one relaxation time. Variational formulation of the general three-dimensional initial-boundary problem with mixed boundary conditions in corresponding spaces of vector-valued distributions with respect to the time variable with values in Sobolev spaces is considered and existence and uniqueness of solution is investigated. Applying variational analogue of the dimensional reduction method proposed by I. Vekua in the classical theory of elasticity a hierarchy of dynamic two-dimensional models approximating Chandrasekharaiah-Tzou three-dimensional model for thermoelastic shell is constructed, and the obtained two-dimensional initial-boundary value problems are investigated in suitable spaces of vector-valued distributions with values in weighted function spaces. Moreover, the relationship between the constructed two-dimensional and original three-dimensional models is studied. Namely, the pointwise with respect to the time variable convergence of the sequence of vector-functions of three space variables restored from the solutions of the constructed two-dimensional problems to the solution of the original three-dimensional problem is proved in corresponding spaces and under additional regularity conditions the rate of convergence is estimated.

SOLUTION OF SOME PROBLEMS FOR THE ELASTIC MATERIAL WITH VOIDS IN $N = 1$ APPROXIMATION OF VEKUA'S THEORY

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In this report we consider some boundary value problems for a circular plate. The plate is the elastic material with voids [1]. The state of plate equilibrium is described by the system of differential equations that is derived from three dimensional equations of equilibrium of an elastic material with voids (Cowin-Nunziato model) by Vekua's reduction method [2]. Its general solutions are represented by means of analytic functions of a complex variable and solutions of Helmholtz equations. The problems are solved analytically by the method of the theory of functions of a complex variable when the components of the displacement vector or the components of the stress tensor are given.

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ON A PROBLEM IN N=0 APPROXIMATION OF HIERARCHICAL MODELS OF INCOMPRESSIBLE FLUID

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We consider shallow incompressible barotropic fluid occupying non-Lip-schitz, in general, prismatic domains within the scheme of small displacements linearized with respect to the rest state (see [1], [2]). The well-posedness of BVPs for the fluid occupied symmetric prismatic shell like domains in the zero-th approximation of hierarchical models under the reasonable BCs in velocities at the cusped edge and given velocities at the non-cusped edge are studied.

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GOVERNING EQUATIONS FOR CONTINUITY EQUATION IN THE CASE OF PRISMATIC SHELL-LIKE DOMAINS WITHIN THE FRAMEWORK OF HIERARCHICAL MODELS

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Using I. Vekua's dimension reduction method, the title governing equations are constructed when either

$$\rho = \rho_0 + \tilde{\rho}(x_1, x_2, x_3, t), \ \ \rho_0 = const, \ \ \tilde{\rho} << \rho_0$$

or

$$\rho = \rho(x_1, x_2, t),$$

where ρ is the density, x_1, x_2, x_3, t are Eulerian variables.

ON SHALLOW SHELLS CONSISTING OF A BINARY MIXTURE WITH VOIDS

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Using the dimensional reduction method developed by Ilia Vekua [1], the main two-dimensional equilibrium equations for shallow shells consisting of a binary elastic mixture were obtained in [2]. We consider the case when the shell consists of porous mixture, and each of its components are characterized by different volume functions [3].

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THE PROBLEM OF FINDING AN EQUALLY STRONG CONTOUR FOR THE VISCOELASTIC RECTANGULAR REGION

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The problem of finding an equally strong contour inside a rectangular viscoelastic plate according to the Kelvin-Voigt model is considered. It is assumed that normal compressive forces with given principal vectors (or constant normal displacements) are applied on the sides of the rectangle by means of linear absolutely rigid punches, and the unknown part of the boundary (the equally strong contour) is free from external forces. The equal strength of the sought-for contour lies in the fact that at each point of the contour the tangential normal stress takes the constant value (generally it depends on both the point and the time). To solve the problem, methods of conformal mappings and boundary value problems of analytic functions are used, and the equation of the desired contour, as a function of point and time, is constructed efficiently (in an analytical form).

THE BENDING PROBLEM OF ELASTIC RECTANGULAR PLATE WITH RIGID INCLUSION

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The bending problem of an elastic rectangular plate with rigid inclusion is considered. The elastic plate occupies the area $D \backslash I$, where $D = \left\{ (x,y) \, | \, |x| < \frac{a}{2}, 0 \leq y \leq b \right\}$ and $I = \left\{ (x,y) \, | \, |x| < c, y = \frac{b}{2} \right\}$. Along the interval I the plate is reinforced with rigid inclusion, which load by the normal stress with intensity $\mu_0(x)$. The bending function $\omega(x,y)$ satisfies the homogeneous biharmonic equation

$$\Delta^2 \omega(x,y) = 0, \ y \neq \frac{b}{2}$$

The boundary of the plate is simply supported and the boundary conditions have the form

$$\omega = M_x = 0, \ x = \pm \frac{a}{2}$$

$$\omega = M_y = 0, \ y = 0; b$$

The Jumps of bending function, angle of rotation, bending moment and lateral force on the inclusion are presented

$$< \omega > = < \omega'_y > = < M_y > = 0 \ < N_y > = \mu(x), \ |x| < c$$

where $\mu(x)$ is a unknown function. The contact between the plate and the inclusion is realized by a thin glue layer. The contact condition has the form

$$\omega_0(x) - \omega\left(x, \frac{b}{2}\right) = n_0\mu(x), \ |x| < c$$

and bending function of rigid inclusion satisfies the following condition

$$\omega_0(x) = c_0 x + d, \ |x| < c$$

where c_0, d are constants, n_0 is a known glue parameter.

The problem with this statement is formalized in the form of an integral equation. Using the method of orthogonal polynomials this equation is reduced to the system of infinite linear algebraic equations. The quasi-regularity of obtained system of infinite linear algebraic equations in the class of bounded sequences is proved. The reduction method is justified and it is possible to find an approximate solution with any accuracy.

MAGNETOHYDRODYNAMIC FLOW OF FLUID IN A CIRCULAR PIPE WITH ACCOUNT OF THE VOLUM SOURCES AND SINK MAS

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The steady flow of viscous incompressible weakly conducting fluid in a circular pipe with volume sources sink mass is studied. In this talk we consider the unsteady flow of a viscous incompressible weakly electrically conducting fluid in annular pipe under external radial magnetic field. An exact solution of the problem in general form and its extreme case are obtained[1-4].

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THE PROBLEM OF STATICS OF THE THEORY OF ELASTIC MIXTURE OF DEFINING A HOLE OF UNIFORM STRENGTH IN A POLYGONAL PLATE

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In the present talk we consider the problem of statics of the linear theory of elastic mixture of finding a hole with a uniformly strong boundary in a finite isotropic plate, whose shaped is a convex polygon. It is assumed that projection of the displacement vector on the normal on each side of the polygon has a constant value, and projection of the stress vector on the tangent is equal to zero on the boundary hole. Also assume that a normal pressing concentrated force is applied to the middle of each side, further note that the boundary of the unknown hole is free from external stresses.

The goal of the problem is find an unknown contour under the condition that the tangential normal stress vector on it takes a constant value.

ANALYTICAL SOLUTION OF SOME BOUNDARY VALUE PROBLEMS OF ELASTICITY IN BIPOLAR COORDINATE SYSTEM

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In the bipolar system of coordinates exact solutions of two dimensional static boundary value problems of elasticity are constructed for homogeneous isotropic bodies occupying domains bounded by coordinate lines of bipolar coordinates. The boundary value problems of elastic equilibrium of eccentric circular rings, half-planes with circular holes, etc are represented. In the bipolar coordinates the equilibrium equation system and Hooke's law are written. The requirement of static equilibrium of the external load at each circular boundary of the region is not taken into account in this work. This requirement, which significantly limits the range of tasks to be solved, usually appears in works devoted to the above problems. In addition, the process of obtaining exact (analytical) solutions becomes much easier compared to the traditional approach. Exact solutions are obtained using the method of separation of variables.

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