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Foreword

The present book of abstracts contains abstracts of talks given at XXXV Enlarged sessions (April 21-23, 2021) of the Seminar of I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University.

Each Section (there are 10 ones) is presented as separate Chapter of the book. The responsibility for the contents of each Chapter lies with leaders together with speakers.

SECTION OF MATHEMATICAL LOGIC AND FOUNDATIONS

Chairs: Alexander Kharazishvili, Roland Omanadze
Co-chair: Archil Kipiani

MAZURKIEWICZ SETS WITH NO WELL-ORDERING OF THE REALS

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In 1914, S. Mazurkiewicz presented a transfinite construction of a subset A of the Euclidian plane R^2 , having the extraordinary property. More precisely, a set $M \subset R^2$ is called a Mazurkiewicz subset of R^2 if $card(M \cap l) = 2$ for every straight line in R^2 (see [3]).

As is well-known, from the existence of a well-ordering of the reals follows the existence of Mazurkiewicz sets. Mazurkiewicz sets may have a complicated descriptive set theoretical structure in ZFC there is a Mazurkiewicz set which is nowhere dense and of Lebesgue measure zero and there is also a Mazurkiewicz set which is Lebesgue nonmeasurable and does not have the Baire property (see [2]).

The current presentation adds information about Mazurkiewicz sets in models of ZF with no well-ordering of the reals. Arnold Miller has shown that there exists a model of ZF with an infinite Dedekind-finite set of reals in which there is a Mazurkiewicz set (see. [4], [5]). His model is the forcing extension of the Cohen-Halpern-Levy model obtained by adding ω_1 Cohen reals. We shall prove here that there is a Mazurkiewicz set already in the Cohen-Halpern-Levy model. (see [1]) In fact, we shall present a general sufficient criterion for a Mazurkiewicz set to exist and show that it applies in the Cohen-Halpern-Levy model.

Theorem. (ZF) Assume that there is some sequence $(A_i, r_i : i < \lambda)$ such that for all $i \leq j < \lambda$

a) $A_i \subset A_j$;
b) $R = \bigcup_{k<\lambda} A_i$;
c) r_i is a real which is not in $comp(A_i)$;
d) $comp(A_i \cup \{r_i\}) \subset A_{i+1}$.

There is the a Mazurkiewicz set.

Corollary. There is a Mazurkiewicz set in the Cohen-Halpern-Levy model.

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MINIMAL PAIRS OF Q_1-DEGREES

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Tennenbaum (see [1, p. 159]) defined the notion of Q-reducibility on sets of natural numbers as follows: a set A is *Q-reducible* to a set B (in symbols: $A \leq_Q B$) if there exists a computable function f such that for every $x \epsilon \omega$ (where ω denotes the set of natural numbers),

$$x \epsilon A \Leftrightarrow W_{f(x)} \subseteq B.$$

We say in this case that $A \leq_Q B$ via f. If $A \leq_Q B$ via a computable function f and for all x, y,

$$x \neq y \Rightarrow W_{f(x)} \cap W_{f(y)} = \emptyset,$$

then we say that A is *Q_1-reducible* to B, and denoted $A \leq_{Q_1} B$.

Our notations and terminology are standard and can be found in [1] and [2].

Theorem 1. Let A be a Σ_2^0 set, let C be a Π_2^0 set, let B be an arbitrary set and let A$\leq_{Q_1}$ B$\leq_{Q_1}$ C. Then there exists a computable function h such that for all x, y,

$$x \epsilon A \Leftrightarrow W_{h(x)} \subseteq B,$$

$$x \neq y \Rightarrow W_{h(x)} \cap W_{h(y)} = \emptyset,$$

$$W_{h(x)} \text{ is finite.}$$

Theorem 2. If *a* and *b* are computably enumerable (c.e.) Q_1-degrees, then for every nonzero Σ_2^0 Q_1-degree *c* such that $c \leq_{Q_1} a, b$, there exists a c.e. Q_1-degree *d* such that $c \leq_{Q_1} d \leq_{Q_1} a, b$.

Theorem 3. If c.e. Q_1-degrees *a* and *b* form a minimal pair in the c.e. Q_1-degrees, then *a* and *b* form a minimal pair in the Σ_2^0 Q_1-degrees.

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ON SOME ASPECTS OF EQUIDECOMPOSABILITY OF ELEMENTARY FIGURES

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The report is concerned with some aspects of the theory of equidecomposability of elementary figures from the measure-theoretical standpoint. Two polyhedrons in $\mathbf{R}^3$ are equidecomposable if the first of them can be cut into finitely many polyhedrons which can be reassembled to yield the second one. Obviously, any two equidecomposable polyhedrons have the same volume. For every polyhedron P in $\mathbf{R}^3$ Dehn introduced some kinds of functionals, now widely known as the Dehn invariants, which are defined as follows:

$$D_f(P)=\sum l(e)f(\alpha),$$

where $l(e)$ is the length of an edge e of P, α is the value of the dihedral angle of P between the two faces meeting at e, and f is a solution of the Cauchy functional equation such that $f(\pi) = 0$. In other words, a function f is any solution of the classical Cauchy functional equation.

Theorem. Among the solution of the Cauchy functional equations one can meet those ones which are nonmeasurable with respect to every translation invariant measure on the real line $\mathbf{R}$, extending the Lebesgue measure.

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INVARIANT σ-FINITE MEASURES ON POLISH LINEAR SPACES

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In this report we show that on every infinite-dimensional Polish space E there exists a σ-finite measures which is invariant with respect to a dense linear subspace and this measure cannot be unique.

Next statements are valid.

Theorem 1. On every infinite-dimensional Polish linear space E there is a non-zero σ-finite Borel measure which is invariant with respect to some dense linear subspace.

Theorem 2. Let be an infinite-dimensional Polish space, let $K\subset E$ be compact, let $L\subset E$ be a linear subspace and let μ be an L-invariant σ-finite Borel measure on E with $\mu(K)=1$. Then there exists an L-invariant σ-finite Borel measure μ' on E with $\mu'(K)=1$ and such that μ and μ' are not equivalent.

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SECTION OF APPLIED LOGICS AND PROGRAMMING

Chair: Matthias Baaz (Austria)
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NRANKED FUZZY REASONING

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One of the main tools in knowledge representation is ontology, which is a collection of logic-based formal language sentences. These sentences are used by automated reasoning programs to extract new knowledge and answer the given questions.

Although ontology languages are standardized by W3C, there are still many problems remaining. One of the most important problems is related to, so called, fuzzy ontologies. These are ontologies, where information is vague and imprecise. Fuzzy ontologies are obtained by integrating fuzzy logic with ontologies. Such kind of ontologies have applications in many different fields, such as medicine, biology, e-commerce and the like.

In this talk, we develop an unranked fuzzy logic and a tableau method for reasoning over such logic. The novelty of our approach is that we will extend many-valued logics with sequence variables and flexible-arity function and predicate symbols. The unranked fuzzy language and corresponding reasoning method will broaden the knowledge engineering capabilities in different fields.

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IMPLEMENTATION OF UNRANKED FUZZY LOGIC

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Fuzzy logic is a logic of imprecision that aims at modeling the imprecise modes of reasoning. It attempts to assist the human ability to make decisions in an environment that is imprecise and uncertain.

Unranked fuzzy logic is a combination of first-order unranked predicate logic and many-sorted first-order logic. An unranked fuzzy tableaux is developed as a reasoning method for it.

In this talk we present an ongoing work on implementation of unranked fuzzy logic. Our target is to integrate unranked fuzzy theory into a framework called GAPT (General Architecture for Proof Theory). GAPT focuses on transforming and analyzing formal proofs and provides corresponding data structures, algorithms and user interfaces. It contains multiple number of algorithms like transformations between naive and structural first-order clause sets, interpolation in first-order proofs and a built-in tableaux prover for propositional logic. GAPT system provides a graphical user interface called PROOFTOOL which is a convenient tool for proof visualization to display large proofs. PROOFTOOL also supports the variety of structural proof theory algorithms that are implemented in GAPT.

MEASURING THE HEART RATE OF AN INDIVIDUAL USING MOBILE DEVICES

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Heart rate variability can be a huge threat for individuals, especially for those who are diagnosed with any kind of heart diseases or recovered from a previous cardiovascular condition.
In this talk we aim at measuring the heart rate of a patient during exercise or normal daily activities using mobile devices. This measurement is beneficial in order to detect any sudden abnormality in the patient's heart and to be able to perform any prompt intervention if required.
Technological innovations play its role in nowadays ambulatory monitoring devices, like smartwatches, wristbands, etc. Continuous monitoring of cardiac diseases such as arrhythmia with these devices is becoming very popular, since they can record the patient's data and send it directly to the hospital for consideration.

The Health Level Seven (HL7) protocol is used to transmit patient's medical data between medical devices (like laboratory analyzers) and hospital information systems (HIS). It is a default standard. HL7 is a two-way communication protocol that can send and receive messages. We use this protocol to transmit heart rate data from the measurement device to our system.

UNRANKED PROBABILISTIC LOGIC

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Since the early days of Artificial Intelligence (AI) logical and probabilistic methods have been used independently to solve tasks that require some sorts of 'intelligence'. Probability theory deals with the challenges posed by uncertainty, while logic is more used for reasoning with perfect knowledge. Considerable efforts have been devoted to combining logical and probabilistic methods in a single framework, which influenced the development of several formalisms and programming tools. In this talk we discuss probabilistic extension of unranked logic theories.

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SOME NOTES ON GEORGIAN CONTROLLED LANGUAGE

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Controlled natural languages are used as representative languages of a high level of knowledge. These languages occupy a very important place in the field of computer linguistics because of their two very interesting properties: first, like natural languages, they have a non-formal structure, therefore, they are simpler to use than formal languages. Second, they are precisely defined as subgroups of natural languages and can be translated automatically into a formal target language and processed after that. They can balance the disadvantages of natural languages and formal languages for the most accurate representation of knowledge and can help domain specialists write specifications in a controlled natural language.

Controlled Georgian Language for crisis management will be discussed in this talk.

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ON THE LONG SHORT-TERM MEMORY IN THE RECURRENT NEURAL NETWORKS

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Up to now, the recurrent neural networks have been trained by means of their unfolding in time and using the modified backpropagation through time method. It is turned out that in the teaching process with the availability of sufficiently long input sequences a network forgets information on remoted objects. In some cases, it becomes necessary for the network to “remember” information on the objects being in the beginning of the sequence.

The article deals with the examples of problems, which confirm the necessity of memory presence for the network.

DECONVOLUTIONAL NEURAL NETWORKS

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Deconvolutional Neural Networks (*DNN*) were originally proposed as a convolutional variety of those rarefied autocoders which are used to visualize maps of convolutional neural networks. Later, the idea of deconvolutional networks was widely used in solving semantic segmentation problems, since the mentioned networks allow to obtain such a map at the exit that is comparable in accordance with size to the incoming image.

CHALLENGES OF DISTANCE LEARNING IN SCHOOL

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The past year has faced many challenges and problems in all areas, including education, but the pandemic also created new opportunities. The switching to the distance learning format of educational institutions has accelerated the enhancement of teachers and students competencies in digital technologies.

During this period, I did research on my own pedagogical practice, the topic was how well the schools solve these problems, how changed this process the quality of learning and motivation of students. This talk is about research results and about ways how to solve these problems.

SECTION OF NUMBER THEORY, ALGEBRA AND GEOMETRY

Chairs: Mikhail Amaghlobeli, George Khimshiashvili, Teimuraz Vepkhvadze, Malkhaz Bakuradze
Co-chair: Ketevan Shavgulidze

CT_1-GROUPS AND ALGEBRAIC GEOMETRY OF FREE 2-NILPOTENT GROUPS

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The notion of a group *H* discriminated by a group *G* (more generally, by a class of groups *K*) is central in modern group theory. It occurs in various branches of algebra under different names: in varieties of groups (as groups discriminated by *K*), in combinatorial group theory (as residually *K* groups), in geometric group theory via Gromov-Hausdorff spaces (as limit groups of *K*), etc. Lately this notion played an important part in model theory of groups (to characterize groups universally equivalent to *G*) and algebraic geometry of groups (to describe groups which are the coordinate groups of irreducible varieties over *G*).

In 1967 B. Baumslag [1] proved that a group *H* is discriminated by a free non-abelian group *F* if and only if it is residually *F* and commutative transitive. Recall that *H* is commutative transitive, or *CT*-group, if the centralizers of non-trivial elements in *H* are abelian. It turned out that a similar result holds for many other groups *G* (for example, torsion-free hyperbolic ones).

Non-abelian nilpotent groups are never *CT*-groups, since they have a non-trivial center, so the technique above does not apply in this case. However, one can generalize the notion of a *CT*-group so it fits better to nilpotent groups (see [2, 3]). Namely, a group H is called commutative transitive of level k (or CT_k-group) if the centralizer of any element not in $Z_k(H)$ is abelian, here $Z_k(H)$ is the *k*-th term of the upper central series of *G*, in particular, $Z_0(H) = 1$ and $Z_1(H)$ is the center of *H*. Clearly, *CT*-groups are CT_0-groups, and free 2-nilpotent groups are CT_1-groups. We prove the following result.

Theorem. Let *H* be a finitely generated G-group G-separated by a non-abelian free 2-nilpotent group *G* of finite rank. Then *H* is G-discriminated by *G* if and only if *H* is a CT_1-group.

This gives a pure algebraic characterization of finitely generated groups universally equivalent to the group *G*, as well as the coordinate groups of irreducible algebraic varieties over *G*.

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ON THE NUMBER OF REPRESENTATIONS OF POSITIVE INTEGERS BY THE DIAGONAL QUADRATIC FORMS IN NINE VARIABLES WITH COEFFICIENTS THAT ARE ONES AND FOURS

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The modular properties of generalized theta-functions with characteristics and spherical functions are used to build cusp forms of half-integral weight. It gives the opportunity of obtaining formulas for the numbers of representations of positive integers by all diagonal quadratic forms with coefficients that are ones and fours.

ANALYTIC REPRESENTATION OF THE VV AND SV CUTTING SURFACES FOR GENERALIZED MOBIUS-LISTING'S BODIES

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The report will present analytical representation of the corresponding cutting surface of any VV (vertex-vertex) and VS (vertex-side), for Generalized Möbius-listing's bodies GML_m^n [1,2], which radial cross section is a regular convex polygon with m-angle.

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ON FACTORIZATION OF MONOIDS

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Given a monoid M, a *congruence* on a left M-set X (or, a (*left*) M*-congruence on*X), is an equivalence relation $\rho \subseteq X \times X$ on X such that $(x, x') \in \rho$ implies $(mx, mx') \in \rho$for all $x, x' \in X$ and $m \in M$. (Similarly, one defines a *right*ρ*-congruence* on a right M-set Y). The ρ-equivalence class of an element $x \in X$ is denoted by $[x]_\rho$. In the special case of the left (resp. right) M-set (M, m_M), we write $C^l(M)$ (resp.$C^r(M)$) for the corresponding set of left (resp. right) congruences on it. A *transversal* of a congruence ρon a left (resp. right) M-set X is a set $T \subseteq X$, such that T consists of exactly one representative of every equivalence class of ρ.

A monoid M is said to be *factorizable* if it contains two submonoids M_1 and M_2 such that the multiplication map $M_1 \times M_2 \to M, (m_1, m_2) \mapsto m_1 m_2$, is bijective. The couple (M_1, M_2) is called a *factorization* of M.

The goal of our talk is to prove the following result.

Theorem. For any monoid M, the assignment

$(\alpha, \beta) \longmapsto ([1_{\mathrm{M}}]_\alpha, [1_{\mathrm{M}}]_\beta)$

yields a bijection between the set of those pairs $(\alpha, \beta) \in \mathrm{C}^{\mathrm{l}}(\mathrm{M}) \times \mathrm{C}^{\mathrm{r}}(\mathrm{M})$ such that

(1) $\alpha \cap \beta = \triangle_{\mathrm{M}}$,

(2) $[1_{\mathrm{M}}]_\beta$ is a transversal of α,

(3) $[1_{\mathrm{M}}]_\alpha$ is a transversal of β

and the set of factorizations of M.

HILBERT'S THEOREM 90 FOR COALGEBRA-GALOIS EXTENSIONS

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We prove a version of Hilbert's theorem 90 for coalgebra-Galois extensions.

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ON THE UPPER BOUNDS OF THE DIMENSIONS OF SOME SPACES OF GENERALIZED THETA-SERIES WITH QUADRATIC FORMS OF FIVE VARIABLES

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The upper bounds of the dimensions of the spaces of generalized theta-series with some diagonal quadratic forms of five variables are obtained [1, 2]. The basis of the spaces of generalized theta-series is constructed.

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SECTION OF REAL ANALYSIS

Chairs: Ushangi Goginava, Leri Gogoladze
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NOTE ON CLASSES OF FUNCTIONS OF GENERALIZED BOUNDED VARIATION

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Classes of functions of generalized bounded variation are studied. For continuous functions from these classes uniform convergence of their trigonometric Fourier series is proved.

ON TWO DEFINITIONS OF QUASI-ORTHOGONALITY

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We plan to compare the definitions of quasi-orthogonalities appeared in Menchoff D. A., Sur les series des functiones orthogonales, *Fund. Math.*, **10** (1927), 375-420 and in M. Kac, R. Salem and A. Zygmund, A gap theorem, *Transactions of the American Mathematical Society*, **63** (1948), 235-248.

ON SOME LOCAL PROPERTIES OF THE CONJUGATE FUNCTION AND THE MODULUS OF SMOOTHNESS OF FRACTIONAL ORDER

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Moduli of smoothness play a basic role in approximation theory, Fourier analysis and their applications. For a given function f, they essentially measure the structure or smoothness of the function via the difference of some order. In fact, for the functions f belonging to the Lebesgue space L^p or the space of continuous functions C, the modulus of smoothness has turned out to be a rather good measure for determining the rate of convergence of best approximation.

In the present talk we study the behavior of the smoothness of fractional order of the conjugate functions of many variables at fixed point in the space C if the global smoothness as well as the behavior at this point of the original functions are known. The direct estimates are obtained and exactness of these estimates are established by proper examples.

UNCONDITIONAL CONVERGENCE OF GENERAL FOURIER SERIES FOR LIPSCHITZ CLASS FUNCTIONS

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It is well known that Fourier series are not, in general, convergent, even in the case when the function is differentiable. Thus if we want the Fourier series of the differentiable function to be convergent, it is necessary that the ONS functions of the given orthonormal system satisfy some condition. In the present paper the conditions are found to be satisfied by the given ONS functions so that the Fourier series of Lipschitz class functions are unconditionally convergent. It is shown that the obtained conditions are best possible in a certain sense. The behave our of the given ONS subsystems is studied.

ON THE MIXED-TYPE $(C,-1<\alpha<0,\ \beta=0)$ CESARO SUMMABILITY OF DOUBLE SERIES WITH RESPECT TO BLOCK-ORTHONORMAL SYSTEMS

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Below a question related to the almost everywhere summability by $(C,-1<\alpha<0,\ \beta=0)$ methods of double series with respect to block-orthonormal systems are considered. Let $\{M_k\}$ and $\{N_k\}$ be increasing sequences of natural numbers and $\Delta_{p,q}=\left(M_p,M_{p+1}\right]\times\left(N_q,N_{q+1}\right],\ (p,q\geq 1)$. Let $\{\varphi_{mn}\}$ be a system of functions from $L^2\left(0,1\right)^2$. The system $\{\varphi_{mn}\}$ will be called a $\Delta_{p,q}$-orthonormal system if $\|\varphi_{mn}\|_2=1$, $m=1,2,...,\ n=1,2,...$ and $\left(\varphi_{ij},\varphi_{kl}\right)=0$, for $(i,j),(k,l)\in\Delta_{p,q}$, $(i,j)\neq(k,l)$, $(p,q\geq 1)$. The notion of block-orthonormal system was introduced by Móricz. In [1] we obtained a two-dimensional analogue of Kaczmarz' and Móricz' theorems on the a.e. summability by the methods $(C,1,1)$ ($(C,1,0)$ or $(C,0,1)$) of double series with respect to block-orthonormal systems. Statements connected with the $(C,-1<\alpha<0,\ \beta=0)$ almost everywhere summability of series $\sum_{m,n=1}^{\infty}a_{mn}\varphi_{mn}(x,y)$ with respect to $\Delta_{p,q}$-orthonormal system $\{\varphi_{mn}\}$ are given. It is stated the conditions on the sequences $\{M_k\}$ and $\{N_k\}$, when the condition

$\sum_{m,n=1}^{\infty} a_{mn}^2 m^{-2\alpha} (\ln(n+1))^2 < \infty$ guarantees the $(C, -1 < \alpha < 0, 0)$ almost everywhere summability of corresponding block-orthogonal series.

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SECTION OF COMPLEX ANALYSIS AND APPLICATIONS

Chair: Grigor Giorgadze
Co-chair: George Akhalaia

ON RELATIVISTIC FORMULATION OF BRACHISTOCHRONE PROBLEM

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The trajectory joining two points x_1 and x_2, which minimizes the transit time for a particle, initially at rest, to fall in a uniform gravitational field from x_1 to x_2, is called the brachistochrone. The brachistochrone problem is the most famous problem in the calculus of variations.

In the report, we consider some modification of this classical problem. In particular, we discuss the relativistic version of this problem [1]. It is known that there is no global time concept in the general theory of relativity, therefore there is no universal rule that measures to move to the particle x_1 point in x_2. The variational principle we stated in terms of geodesics in a suitable Riemannian and sub-Riemannian structure on manifold. We consider in more details the problem in Heisenberg space [2] using the methods of geometric control theory [3].

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ON J-SPECTRAL FACTORIZATION OF SYMMETRIC MATRICES

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A symmetric matrix function $G = G^*$defined on the unit circle T in the complex plane is called of *constant signature* if $G(t)$ has the same number of positive and negative eigenvalues for a. a.$t \in T$. J- spectral factorization of non-degenerated matrix G of constant signature is called the representation

$$G(t) = G_+(t)\, \Lambda G_+^*(t), \qquad t \in T, \tag{1}$$

where G_+can be extended analytically inside T together with its inverse,G_+^* is Hermitian conjugate of G_+, and$\Lambda$is a constant diagonal matrix with 1-s and -1-s on the diagonal (according to the number of positive and negative eigenvalues). $\Lambda$is called the *signature matrix.* If G is positive definite on T, the representation (1) is called the (standard) spectral factorization. In this case, $\Lambda$is the unit matrix.

J-spectral factorization plays important role in Control Theory, particularly in H_∞ control [2].Therefore, it is important to develop a numerical algorithm for approximate J-factorization of a given matrix function G. There are different algorithms of this type in the literature (see, e.g. [4]). However, none of them are direct extensions of spectral factorization algorithms.

Matrix spectral factorization method [1], [3] became popular in the last years. We generalize this method for such symmetric matrices which have constant signatures for all leading principle submatrices.We expect to extend the algorithm to all matrices with a constant signature by removing the restriction on submatrices.

Acknowledgment. The first author was partially supported by the Shota Rustaveli National Science Foundation of Georgia (DI-18-118)

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GEOMETRY OF STABLE LINEAR DEFORMATIONS OF HARMONIC POLYNOMIALS

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Recall that harmonic polynomials of bi-degree (n, m) are defined as functions of the form $f(z) = p_n(z) + \overline{q_m(z)}$, where $p_n(z)$ and $q_m(z)$ are polynomials of complex variable having the degrees n and m, respectively, and symbol "bar" means the complex conjugation. In this talk, we discuss the geometry of harmonic polynomials of bi-degree $(n, 1)$ interpreted as deformations of complex polynomial $p_n(z)$. It is known that, for a generic complex number a, the linear deformation $f_a(z) = p_n(z) + a\bar{z}$ defines a stable self-mapping of the complex plane. By Whitney's theorem the singularities of such a mapping consist of fold curves and certain amount $c(a)$ of cusp points on these curves. Another natural geometric invariant of harmonic polynomial $f_a(z)$ is its valence $v(a)$ defined as the maximal cardinality of fibres $f_a^{-1}(w)$.

The valence of harmonic polynomials was studied by many authors. In particular, A.Wilmshurst [1] proved that the valence of harmonic polynomial of bi-degree (n, m) with $n > m$, is either infinite or belongs to the integer segment $[n, n^2]$. For harmonic polynomials of bi-degree $(n, 1)$ it was shown in [2] that the valence does no exceed $3n - 2$ and this estimate is exact. We complement these results by estimating the number of cusps for linear deformations as above.

Theorem. For any complex polynomial $p(z)$ of degree n, there exists a finite set of real numbers $a_1, \dots, a_n$ such that, for any real number a different from all a_j, the linear deformation $p(z) + a\bar{z}$ is stable and the number of its cusps $c(a)$ belongs to the integer segment $[n+1, 3n-3]$.

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SPLIT OCTONIONIC REPRESENTATION OF SO(4,4) VECTOR AND SPINORS AND TRIALITY SYMMETRY

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Vectorial and spinorial pseudo orthogonal groups of space with (4,4) metric are represented with split octonion algebra. A peculiar property of eight dimensional space, called triality, that manifests as equivalence of spinors and vector, is expressed in terms of these split octonionic objects. It is shown that this description respects triality symmetry, unlike standard Clifford algebraic approach.

Acknowledgements. This work was supported by the joint grant of Volkswagen Foundation and Shota Rustaveli National Science Foundation of Georgia (Ref. 93 562 & #04/48).

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ON THE ALGORITHM OF COMPUTATION OF MODULI OF QUADRILATERALS

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In the talk we consider the conformal moduli of special type domainof that is sometimes called a polygonal quadrilateral [1]. Some examples related to numerical methods and Schwarz-Christoffel formula for such domains can be found in the modern literature (see [2]). We present Schwarz-Christoffel formula by power series and give realization of the algorithm [3].

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ON ONE CLASS OF THE DEGENERATE SYSTEM OF DIFFERENTIAL EQUATIONS

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In this presentationwe will discuss the system of differential equations with the pointwise high-degeneracy on a complex plane. The regular pair of real numbers is explicitly constructed and the uniqueness theorem is obtained. The existence theorem for the nonhomogeneous polyanalytic system is obtained as well.

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SPACE DIMENSION RENORMDYNAMICS AND CONFINING POTENTIALSFROM HADRONS TO GALAXIES AND PERIODIC STRUCTURE OFUNIVERSE

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Scale dependent space dimension and potential for the quarkonium model of hadrons [1] extended to the modified Newton potential for galaxies. With this potential, the periodic structure of the Universe [2,3] explained. For the quarkonium in the quark-gluon matter the string strength parameter dependence on the temperature obtained.

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ON THE INTEGRAL EQUATIONS INDUCED FROM BERS-CARLEMAN-VEKUA IRREGULAR EQUATIONS

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In the report the integral equation of the third kindwhich are associated with the irregular Bers-Carleman-Vekua equations when the coefficients of equations belong to a wide class of functions will be discussed. It is shown that the irregular Bers-Carleman-Vekua equation [1]and the corresponding integrated equations from this class have only a trivial solution in the class of analytic functions of two variables. The existence of a wide class of integral equations is proved, where the non-trivial solution does not existand it is shown that for such equations the Fredholm alternative does not hold.

Acknowledgment. The work is supported by the Shota Rustaveli National Science Foundation (SRNSF grant # FR 17-96).

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SECTION OF ORDINARY DIFFERENTIAL EQUATIONS AND OPTIMAL CONTROL

Chairs: Roman Koplatadze, Tamaz Tadumadze
Co-chair: Tea Shavadze

ON THE EXISTENCE OF AN OPTIMAL ELEMENT FOR THE NEUTRAL OPTIMAL PROBLEM WITH DISCRETE DELAY IN CONTROLS

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For the neutral optimal problem

$$\dot{x}(t)=A(t)\dot{x}(t-\sigma)+f(t,x(t),x(t-\tau),u(t))+g(t,x(t),x(t-\tau),u(t-\theta)),t\in[t_0,t_1],$$

$$x(t)=\varphi(t),\ t<t_0,\ x(t_0)=x_0,\ q(t_0,t_1,\sigma,\tau,x_0,x(t_1))=0,$$

$$q^0(t_0,t_1,\sigma,\tau,x_0,x(t_1))\to \min$$

the existence of an optimal element $(t_0,t_1,\sigma,\tau,x_0,u(t))$ is proved by the scheme given in [1].

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ABOUT THEOREMS OF HILLE AND NEHARY

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For the second order linear differential equation

$$u''+p(t)u=0,\ p\in C(R_+;R_+)$$

sufficient conditions of the existence oscillatory solutions are established. Besides, necessary and sufficient conditions of the existence oscillatory solutions are established on a certain subset $C(R_+;R_+)$.

THE LINEAR PROBLEM OF OPTIMIZATION OF DELAY PARAMETERS WITH THE MIXED INITIAL CONDITION

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For the optimization problem

$$\dot{x}(t) = (\dot{p}(t), \dot{q}(t)) = A(t)x(t) + B(t)p(t-\tau) + C(t)q(t-\sigma) + D(t)u(t) + E(t)u(t-\theta), \ t \in [t_0, t_1]$$

$$x(t) = (\varphi(t), g(t)), \ t < t_0, \ \ x(t_0) = (p_0, g(t_0)),$$

$$z^i(\tau, \sigma, \theta, x(t_1)) = 0, \ \overline{1, l},$$

$$z^0(\tau, \sigma, \theta, x(t_1)) \to \min,$$

necessary conditions of optimality are obtained for delay parameters τ, σ, θ and control $u(t)$ on the basis of variation formula [1] and by the scheme given in [2].

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ON THE CRITERION OF THE WELL-POSEDNESS FOR THE GENERAL BOUNDARYVALUE PROBLEMS FOR THE SYSTEMS OF ORDINARYLINEAR DIFFERENTIAL EQUATIONS

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Consider the general boundary value problem

$$\frac{dx}{dt} = P(t)x + q(t),$$

$$l(x) = c_0,$$

where $P : [a,b] \to R^{n \times n}$ and $q : [a,b] \to R^n$ are the integrable matrix-and vector-functions, respectively, l is a linear bounded operator from the space of continuous vector-functions into R^n, and $c_0 \in R^n$.

Let x_0 be the unique solution of the problem. Along with the problem consider the sequence of the boundary value problems

$$\frac{dx}{dt} = P_m(t)x + q_m(t),$$

$$l_m(x) = c_m$$

$(m = 1,2,\ldots)$, where $P_m : [a,b] \to R^{n\times n}$ and $q_m : [a,b] \to R^n$ are the integrable matrix-and vector-functions, respectively, $l_m$is a linear bounded operator from the space of continuous vector-functions into R^n, and $c_m \in R^n$.

We present the necessary and sufficient conditions, guaranteeing the unique solvability of the perturbed problems and convergence of that to x_0 uniformly on $[a,b]$ as $m \to \infty$. Corresponding propositions are proved, for example, in [1].

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THE PROBLEM OF MODIFYING MODELS FOR PREDICTING THE SPREAD OF CORONAVIRUS (COVID-19)

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The problem of modification of the coronavirus (Covid-19) prediction models we have developed is discussed, with the aim of increasing the prediction horizon. For this purpose, the possibilities related to the transition from the forecast of days to months to the average daily increase in the number of infected people are predicted**.**

Over the past year, we have been discussing the coronavirus (Kovid-19) outbreak prediction (in terms of days) both on the example of Georgia and the world, using models such as logistic, trendy and auto-regression and moving models.

For the sake of clarity, it should be noted that in terms of forecasting, we considered such key indicators of coronavirus spread as the number of common cases of infection and the number of active cases at the moment.

For the sake of clarity, it should be noted that in terms of forecasting, we considered such key indicators of coronavirus spread as the number of common cases of infection and the number of active cases at the moment.

It has been shown that these models show a sufficiently high accuracy for a maximum of a month run (then their accuracy drops!). On the other hand, given that the virus is "not going to stop" in the near future, the problem of increasing the forecast horizon is on the agenda. Therefore, it may make sense to consider a new indicator such as e.g. "Average daily increase in the number of infected people during the month."

This will allow us to make a prediction of this indicator for a horizon containing several months, especially since according to the central limit theorem of probability theory, the distribution of this indicator should be close to normal, which somewhat simplifies the task of making reliable predictions for it. Clearly, however, this in itself implies that the accuracy of such predictions should increase with the accumulation of relevant (over the months) information. The present paper is dedicated to the study of these possibilities.

THE REPRESENTATION FORMULA OF SOLUTION FOR THE PERTURBED CONTROLLED DIFFERENTIAL EQUATION WITH DISCRETE AND DISTRIBUTE DELAY IN CONTROLS

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For the controlled differential equation

$$\dot{x}(t) = f\left(t, x(t), u_0(t) + \delta u(t), u_0(t-\theta) + \delta u(t-\theta), v_0(t) + \delta v(t), \int_{-\sigma}^{-\tau} [v_0(t+s) + \delta v(t+s)]ds\right),$$

$$x(t_0) = x_{00} + \delta x_0$$

the representation formula of solution is proved by the method given in [1], when the solution of the initial equation

$$\dot{x}(t) = f\left(t, x(t), u_0(t), u_0(t-\theta), v_0(t), \int_{-\sigma}^{-\tau} v_0(t+s)ds\right), \; x(t_0) = x_{00}$$

is known. Here $\delta u(t)$, $\delta v(t)$ and δx_0 denote perturbations of $u_0(t)$, $v_0(t)$ and x_{00}, respectively. The obtained formula is concretized for the linear differential equation and for a market relation differential model.

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ON GENELARIZED BICRITERIA ASSINGMENT PROBLEM

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For generalized bicriteria assignment problem the issue of ϵ - Constraint method is investigated. It is shown that the solution obtained by this method is Sleiter Optimal.

EXISTENCE THEOREMS OF AN OPTIMAL ELEMENT FOR THE TWO-STAGE VARIATIONAL AND OPTIMAL PROBLEMS WITH DISCRETE DELAYS

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For the two-stage variational problem

$$\int_{t_0}^{\theta} f(t,x(t),x(t-\tau),\dot{x}(t),\dot{x}(t-\tau))dt+\int_{\theta}^{t_1} g(t,y(t),y(t-\sigma),\dot{y}(t),\dot{y}(t-\sigma))dt\rightarrow \min,$$

$$x(t)=a(t),t\in[t_0-\tau,t_0];\quad y(s)=G(s,x(s)),s\in[\theta-\sigma,\theta]\quad y(t_1)=b$$

and for the two-stage optimal problem

$$\begin{cases}\dot{p}(t)=\psi(t,p(t),p(t-\omega),u(t),u(t-\omega)),\ t\in(h_0,\vartheta),\\ \dot{q}(t)=\chi(t,q(t),q(t-\rho),v(t),v(t-\rho)),\ t\in(\vartheta,h_1)\end{cases}$$

$$p(t)=\alpha(t),t\in[h_0-\omega,h_0];\quad q(s)=Z(s,p(s)),s\in[\vartheta-\rho,\vartheta],\quad q(h_1)=\beta,$$

$$\int_{h_0}^{\vartheta}\psi^0(t,p(t),p(t-\omega),u(t),u(t-\omega))dt+\int_{\vartheta}^{h_1}\chi^0(t,q(t),q(t-\rho),v(t),v(t-\rho))\rightarrow \min$$

the existence of optimal elements $(\theta,x(\cdot),y(\cdot))$ and $(\vartheta,u(\cdot),v(\cdot))$ is proved by the scheme given in [1].

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SECTION OF PARTIAL DIFFERENTIAL EQUATIONS

Chairs: Temur Jangveladze, Sergo Kharibegashvili, David Natroshvili
Co-chair: Zurab Kiguradze

ON THE INVESTIGATION OF AN ANALYTICAL SOLUTION OF A CERTAIN DIRICHLET GENERALIZED HARMONIC PROBLEM

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The presentation is devoted to the analysis of an explicit analytic solution of the Dirichlet generalized harmonic problem for a right cylindrical region with a base being a concentric circular ring. In the literature (see, for example, [1-4]), for construction of analytical solutions of these type problems, the following methods are applied mainly: separation of variables, particular solutions and heuristic method. Since the heuristic method does not guarantee finding the best solutions, moreover, in some cases it may give an incorrect answer, it is necessary to check whether they satisfy all conditions of the problem under consideration (see, for example, [1]). For problem domain, we consider the case when the boundary condition is constant on the outer lateral surface of the cylinder, and equals zero on the rest part of it. The solution of such problem in the form of series is given in [4]. We intend to use it for testing. Because of this, properties of the series were investigated. Namely, 1) uniform convergence and harmonicity, 2) uniform convergence of the series on the outer lateral surface (it is zero on the rest part). Performed calculations showed that the above-mentioned analytic solution has accuracy, which is enough for a wide group of practical problems. In addition, the results of calculations for inner control points are in a good accordance with the real physical situations. Finally, we note that the problem considered from our viewpoint, can be used in the role of a test with the help of the above-mentioned analytic solution.

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ON NON-CLASSICAL SOLUTIONS FOR SOME NONLOCAL BITSADZE-SAMARSKII BOUNDARY VALUE PROBLEM

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On a rectangular area, the Bitsadze-Samarskii nonlocal boundary problem is considered for the following equation $-\Delta u(x,y)+\lambda u(x,y)=f(x,y),\ \lambda\geq 0$. On the basis of the variational approach, the concept of a classical solution is generalized. Examples of relevant generalized solutions for some discontinuous $f(x,y)$ functions are given.

ON THE CONVERGENCE OF THE SYMMETRICAL, SEMI-DISCRETE SCHEME FOR THE NON-LINEAR CHARNEY-OBUKHOV DIFFERENTIAL EQUATION

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For the non-linear Charney–Obukhov differential equation, the initial-boundary problem (with the periodic boundary conditions) in the rectangular domain is considered. For the stated problem, a symmetrical, semi-discrete scheme of an approximate solution is designed, which is locally linear. The order of an approximation of this scheme is $O\left(\tau^2\right)$ (τ is the step for the temporal variable). The error estimate of the approximate solution is obtained in terms of L_2-norm for vortex, while for the stream function the error is estimated in terms of both W_2^1- norm and C- norm.

CORRECTLY POSED BOUNDARY VALUE PROBLEMS FOR MAXWELL'S SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS

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Boundary value problems for Maxwell's system of partial differential equations are considered (see, for example, [1]). Necessary and sufficient conditions, imposed on the boundary coefficients that ensure the correctness of the problem are found (see, for example, [2-6]). It is shown what type of violation of the correctness of the problem occurs when these conditions are not fulfilled. It is also shown what changes in the initial conditions should be made to make the problem correct. In the case of a correctly posed problem, the solution is written explicitly.

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ON THE PROBLEM WITH NONLOCAL BOUNDARY CONDITIONS FOR ONE NONLINEAR INTEGRO-DIFFERENTIAL EQUATION

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Initial-boundary value problem with nonlocal boundary conditions [1, 2] for one nonlinear integro-differential equation is considered. Integro-differential models of this type are based on the system of Maxwell equations and are studied in many works (see, for example, [3] and references therein).

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SECTION OF THEORY OF PROBABILITY AND MATHEMATICAL STATISTICS

Chairs: Elizbar Nadaraia, Omar Purtukhia

ON THE ONE NONPARAMETRIC ESTIMATOR OF THE POISSON REGRESSION FUNCTION

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In the paper, the limiting distribution is established for an integral square deviation of one nonparametric kernel type estimate of the Poisson regression function. The test for the hypothesis on the Poisson regression function is constructed. The question of consistency of the constructed test is studied and for some close alternatives, the asymptotic behavior of the test power of the constructed test is investigated.

THE LAW OF LARGE NUMBERS FOR WEAKLY CORRELATED RANDOM ELEMENTS IN $l_p, 1 \le p < \infty$, SPACES

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In the presentation, Khinchine's result [1] for weakly correlated random variables is generalized for the case of random elements with values in $l_p, 1 \le p < \infty$, spaces.

Let $\xi_1, \xi_2, \dots, \xi_n, \dots$ be a sequence of weak second order random elements with values in the space $l_p, 1 \le p < \infty$. Let $R_n = R_{\xi_n} : l_p^* \to l_p$ be the covariance operator of ξ_n and V_{nm} be the coefficient of correlation of ξ_n and ξ_m.

Consider the following condition:

$$\sigma_n^s \equiv \sum_{k=1}^{\infty} \langle e_k, R_n e_k \rangle^{s/2} < \infty, \qquad n = 1,2,\cdots \tag{1}$$

where $s = min\{p, 2\}$ and (e_k) is a sequence of unit vectors $e_k = \left(\overbrace{0, \cdots, 0}^{k-1}, 1, 0, \cdots \right), k = 1,2,\cdots$, in the dual space l_p^*.

The following theorem is proved:

Theorem. Let $\xi_1, \xi_2, \dots, \xi_n, \dots$ be a sequence of weak second order random elements with values in $l_p, 1 \le p < \infty$, and let (1) hold. Then the following assertions are valid:

(i) $\mathbb{E}\left\|\xi_n\right\|_{l_p}^2 < \infty, \quad n = 1,2,\cdots.$

(ii) If, besides there exists a non-negative real function g, defined on the set of non-negative integers, such that

$$\|V_{nm}\| \leq g(|n-m|), \qquad n,m = 1,2,\ldots.$$

and

$$\lim_{n\to\infty} \frac{1}{n^2}\left(\sum_{i=0}^{n-1} g(i)\right)\left(\sum_{i=1}^{n} \sigma_i^s\right)^{2/s} = 0,$$

then

$$\lim_{n\to\infty} \mathbb{E}\left\|\frac{1}{n}\sum_{i=1}^{n}\xi_i\right\|_{l_p}^{s} = 0.$$

In particular, the sequence $\xi_1, \xi_2, \ldots, \xi_n, \ldots$ satisfies the law of large numbers.

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EXISTING APPROACHES AND DEVELOPMENT PERSPECTIVES FOR TESTING SSTATISTICAL HYPOTHESES

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Existing approaches to testing statistical hypotheses are discussed ([1]-[5]). Their characterization and comparison with each other are given. The ideas that underlie existing approaches and basic methods of hypothesis testing are discussed. Disadvantages of existing methods are shown, related to the perspective of using these methods at the modern level, caused by the unprecedented increase in data volume and increased demands on the reliability of the decision made.

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UTILITY MAXIMIZATION PROBLEM UNDER BINOMIAL MODEL UNCERTAINTY

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We study a robust wealth optimization problem under model uncertainty for an investor with power utility.

REMARK ON RIGHT CONTINUOUS EXPONENTIAL MARTINGALES

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Let us introduce a basic probability space $(\Omega,\mathscr{F},P)$ and a right continuous filtration $(\mathscr{F}t)0\le t\le\infty$. Let $\mathscr{F}\infty$ be the smallest $\sigma-$algebra containing all $\mathscr{F}t$ and let Mt be a local martingale on the stochastic interval $[\![0;]\!]$ where T is a stopping time. Denote by $\Delta Mt=Mt-Mt-$ jumps of M and by $\mathscr{E}(M)$ the stochastic exponential of M:

$$\mathscr{E}(M)=eMt-12\langle Mc\rangle t\Pi(1+\Delta Ms)e-\Delta Ms0<s\le t,$$

where Mc denotes continuous local martingale part of M. Notice that $M=Mc+Md$ where Md is a purely discontinuous local martingale part, which means that Md is orthogonal to any continuous local martingale. With this we know that $Md=\int\int(\mu-\nu)\infty-1t0$, where $\mu(\omega,t,x)$ is jump measure of M and $\nu(\omega,t,x)$ is its compensator.

It is well known that $\mathscr{E}(M)$ is a nonnegative local martingale and if it is uniformly integrable, then we can define new probability measure Q using Radon-Nikodim derivative: $dQ=\mathscr{E}T(M)dP$. It is clear that $Q\ll P$ and if $\{\mathscr{E}(M)>0\}=1$, then Q and P will be equiva-lent probability measures $(Q\sim P)$. To know whether $Q\sim P$ or not we must study the set $\{\mathscr{E}(M)=0\}$. In case when $M=Mc$ it was shown in Kazamaki [2] that $\{\mathscr{E}(Mc)=0\}=\{\langle Mc\rangle T=\infty\}$. For general M J. Jacod [1] proved that $\{\mathscr{E}\infty(M)>0\}=\{\langle Mc\rangle\infty+\int\int x21+|x|d\nu\infty-1\infty0+\int dBs\mathscr{E}s-(M)\infty0<\infty\}$, where Bs is the predictable nondecreasing process from the Doob-Meyer decomposition of $\mathscr{E}(M)$. In 2019 M. Larsson and J. Ruf [3] gave us set inclusion $\{\lim t\uparrow\tau\mathscr{E}t(M)=0\}\subset\{\lim t\uparrow\tau Mt=-\infty\}\cup\{[M]\tau=\infty\}\cup\{\Delta Mt=-1,t\in[0;\tau)\}$ for any predictable stopping time τ. With this they proved, that if in addition $\Delta M\ge-1$ and $\lim t\uparrow\tau Mt<\infty$, then the reverse set inclusion also holds.

The aim of this paper is to characterize the set $\{\mathscr{E}(M)=0\}$ using $\langle Mc\rangle$, $\mu(\omega,t,x)$ and $\nu(\omega,t,x)$, for any stopping time T:

Theorem. Let M be a local martingale with $\Delta M\ge-1$. Then the set equalities hold true P a. s.:

(i) $\{\mathscr{E}(M)=0\}=\{\langle Mc\rangle T+\int\int x21+xd\mu1-1T0+\int x21+xd\nu\infty1=\infty\}$;

(ii) If $E11+\Delta M\sigma<\infty$ for any $\sigma<\infty$, then $\{\mathscr{E}(M)=0\}=\{\langle Mc\rangle T+\int x21+xd\nu\infty-1=\infty\}$;

(iii) If $E\Delta M\sigma<\infty$ for any $\sigma<\infty$, then $\{\mathscr{E}(M)=0\}=\{\langle Mc\rangle T+\int x21+xd\mu\infty-1=\infty\}$.

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ON MULTIPLE ITO INTEGRALS IN A BANACH SPACE

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We consider the multiple Ito integrals ([1]) in an arbitrary separable Banach space and develop the problem of existence of these integrals. The multiple Ito integral is the important tool to study functionals of the Wiener process. We give the sufficient condition of existence of the multiple Ito integrals in terms of p-absolutely summing operators.

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STOCHASTIC INTEGRAL REPRESENTATION OF PATH-DEPENDENT NONSMOOTH BROWNIAN FUNCTIONALS

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The first proof of the stochastic integral representation theorem was implicitly provided by Ito ([1]) himself. Many years later, Dellacherie (1974) gave a simple new proof of Ito's theorem using Hilbert space techniques. Many other articles were written afterward on this problem and its applications but one of the pioneer works on explicit descriptions of the integrand is certainly the one by Clark (1970). Those of Haussmann (1979), Ocone (1984), Ocone and Karatzas (1991) and Karatzas, Ocone and Li (1991) were also particularly significant. When the Brownian functional F belongs to the Hilbert space $D_{1,2}$ (where $D_{1,2}$ denotes the space of square integrable functionals having the square integrable first order stochastic derivative) Ocone ([2]) proved that the integrand in the Clark stochastic integral representation is $E[D_tF \mid \Im_t^B]$ -- the optional projection of Malliavin's derivative D_tF of F (later this formula was called the Clark-Ocone formula).

Here we will explore the stochastically nonsmooth Brownian functionals and study the issues of their stochastic integral representation, which is known to play a significant role in the hedging problem of European Options. It has turned out that the requirement of smoothness of functional can be weakened by the requirement of smoothness only of its conditional mathematical expectation. Glonti and Purtukhia (see [3]) generalized the Clark-Ocone formula in case, when functional is not stochastically smooth, but its conditional mathematical expectation is stochastically differentiable and established the method of finding the integrand in an explicit form. Here we will consider functionals which don't satisfy even these weakened conditions. To such functionals belong, for example, Lebesgue integral (with respect to time variable) from stochastically nonsmooth square integrable processes. The class of functionals under consideration includes such nonsmooth functionals to which not only the well-known Clark-Ocone formula ([2]), but also its generalization Glonti-Purtukhia ([3]), is inapplicable. Moreover, for smooth functionals from the proved result, one can easily obtain the Clarke-Ocone formula.

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AMERICAN OPTION PRICING IN THE MULTIDIMENSIONAL FINANCIAL MARKET

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Financial (B, S) market in discrete time ([1]-[3]) with k number of bonds and one risky asset is considered. Interest rate introduced, which is the combination of interest rates $r_1, r_2, \ldots, r_k$ related to bonds. In this scheme the fair price of the American option is derived, also representation for the optimal stopping moment is obtained.

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SOME ISSUES ON STATISTICAL ASSESSMENT

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This given paper presents problems on asymptote behavior of non-parametric statistical assessment of some classes. Some qualities of non-parametric assessment of multidimensional unknown distribution of the density and average square integral deviation of Laplace transformation of asymptote behavior are examined.

The suggested method enables us to extract unknown coefficient before end behavior apparently.

ESTIMATION OF SOLUTION OF FIRST ORDER NONLINEAR DIFFERENTIAL EQUATIONS WITH RANDOM MEASURES PARAMETERS

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Random measures and connected with them questions of absolute continuity under nonlinear transformations in abstract Hilbert space are considered. Estimation of solution of first order differential equations with random measures parameters are given.

THE CONSISTENT ESTIMATORS FOR SHARLLE'S STATISTICAL STRUCTURES

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We investigate the problems of consistent estimators ([1]) for Sharlle's statistical structures and give necessary and sufficient conditions for the existence of such estimators.

Let (E,S) be a measurable space and there given the family of probability measures $\{\mu_i, i \in I\}$ defined on S.

Definition 1. The set of objects $\{E,S,\mu_i, i \in I\}$ is called a statistical structure.

Definition 2. A statistical structure $\{E,S,\mu_i, i \in I\}$ is called orthogonal (singular) if μ_i and μ_j are orthogonal for each $i \neq j$, $i \in I$, $j \in I$.

Definition 3. We will say that the statistical structure $\{E,S,\mu_i, i\in I\}$ admits a consistent estimator of parameters $i\in I$, if there exists at least one measurable mapping $\delta:(E,S)\to(I,B(I))$, such that $\mu_i(\{x:\delta(x)=i\})=1, \quad \forall i\in I$.

Definition 4. The statistical structure $\{E,S,\mu_i, i\in I\}$, in which $\{\mu_i, i\in I\}$ is the probability measures of Charlier, will be called the statistical structure of Charlier.

Definition 5. We will say that the orthogonal charlier statistical structure $\{E,S_1,\overline{\mu}_i, i\in I\}$ admits a consistent estimator of parameters if there exists at least one measurable mapping $\delta:(E,S_1)\to(I,B(I))$, such that $\overline{\mu}_i(\{x:\delta(x)=i\})=1, \quad \forall i\in I$.

Theorem 1. In order that the Charlier orthogonal statistical structure $\{E,S_1,\overline{\mu}_i, i\in I\}$ admitted a consistent estimator of parameters in the theory of (ZFC)&(MA) it is necessary and sufficient that the correspondence $f\to l_f$ defined by the equality

$$\int_E f(x)\overline{\mu}_i(dx)=l_f(\overline{\mu}_i),\quad \overline{\mu}_i\in M_B$$

was one-to-one.

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AN EXAMPLE OF A CONDITIONALLY m-DEPENDENT VECTOR SEQUENCE

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Using the Rademacher functions, that are defined on the interval $[0,1[$, we construct a regular finite Markov chain ([1]).

On the square $[0,1[\times[0,1[$ we have the sequence $\{T_n\}_{n\geq 1}$ of conditionally m-dependent vectors, that are controllable by this Markov chain ([2]). Thus, the limiting distribution of the sum

$$S_n=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}[T_i-ET_i]$$

is determined.

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SECTION OF MATHEMATICAL MODELING AND NUMERICAL ANALYSIS

Chairs: Teimuraz Davitashvili, Jemal Rogava, Tamaz Vashakmadze
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EXPERIMENTAL-ANALYTICAL METHOD BASED ON THE GENERAL PRINCIPLE OF MECHANICS TO STUDY THE BEHAVIOR OF EXISTING BUILDING DURING AN EARTHQUAKE

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In recent years, high-rise constructions are intensively built in seismically active areasworldwide (US, Canada, Japan, Chili, Europe, etc.) accompanied by the development of the respective regulatory framework. According to the authoritative experts, it is not appropriate for high-rise buildings to use current regulatory requirements based on the summarized experience of the seismic behavior of low-rise buildings during seismic loading. The principles of seismic safety ofhigh-rise buildings have not been conclusively formulated yet. For that it is necessary to make newbuilding regulations. Consquently there is a big experimental testing ground over a vast territory of the world and high-rise buildings constructed there wait for experimental testing during realearthquakes. The necessity of testing of the seismic stability of high-rise buildings experimentally without awaiting for testing by real earthquake comes to the fore in the described situation.

In the paper, an algorithm is developed based on the relevant experimental data using the principle of work reciprocity known in mechanics to determine the behavior of the object during the passage on his base a seismic wave of any nature.

ON ONE ECOLOGICAL NUMERICAL MODEL OF THE ATMOSPHERE

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A numerical model of the full cycle of cloud and fog genesis in the mesoboundary layer of atmosphere has been created.

A numerical model of the distribution of aerosol from an instantaneous point source into the mesoboundary layer of the atmosphere has been created.

The formation of smog is simulated based on the synthesis and "overlay" of the two above models.

SPLITTING OF THE FOUR-LAYER SEMI-DISCRETE SCHEMES OF SOLVING THE EVOLUTIONARY EQUATIONWITH VARIABLE OPERATOR ON TWO-LEVEL SCHEMES

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Let us consider the following evolutionary problem in the Hilbert space H :

$$\frac{du(t)}{dt} + Au(t) = f(t), \quad t \in]0,T], \tag{1}$$

$$u(0) = u_0, \tag{2}$$

where $A(t)$ is a self-adjoint positive-definite operator with the definition domain $D(A)$, which is everywhere dense in H; $f(t)$ is a continuously defferentiable function with values in H; u_0 is a given vector from H; $u(t)$ is a sought function.

On $[0,T]$ we introduce a grid $t_k = k\tau$, $k = 0,1,\ldots,n$, with step $\tau = T/n$. Let us aproximate the first derivative by purely implicit four level semi-discrete schemes. Then by the perturbation algorithm [1], we obtain the following system of equations:

$$\frac{u_k^{(0)} - u_{k-1}^{(0)}}{\tau} + A_k u_k^{(0)} = f_k, \qquad f_k = f(t_k), \quad u_0^{(0)} = u_0, \quad k = 1,\ldots,n,$$

$$\frac{u_k^{(1)} - u_{k-1}^{(1)}}{\tau} + A_k u_k^{(1)} = -\frac{1}{2}\frac{\Delta^2 u_{k-2}^{(0)}}{\tau^2}, \qquad k = 2,\ldots,n,$$

$$\frac{u_k^{(2)} - u_{k-1}^{(2)}}{\tau} + A_k u_k^{(2)} = -\frac{1}{2}\frac{\Delta^2 u_{k-2}^{(1)}}{\tau^2} - \frac{1}{3}\frac{\Delta^3 u_{k-3}^{(0)}}{\tau^3}, \qquad k = 3,\ldots,n,$$

where $A_k = A(t_k)$, $\Delta_k^{(0)} = u_{k+1}^{(0)} - u_k^{(0)}$.

We introduce the notation $v_k = u_k^{(0)} + \tau u_k^{(1)} + \tau^2 u_k^{(2)}$, $k = 3,\ldots,n$. Let the vector v_k be an approximate value of the exact solution of problem (1), (2) for $t = t_k$, $v(t_k) \approx v_k$.

The following theorem is valid. **Theorem.** Let $A(t)$ be a self-adjoint, positive-definite operator with the definition domain $D(A)$ which is everywhere dense in H, and besides, the following condition is fulfilled

$$\left\| (A(t') - A(t''))A^{-1}(s) \right\| \le c|t' - t''|, \qquad \forall s, t', t'' \in [0,T], \quad c = const > 0.$$

Let the solution of problem (1),(2) be a smooth enough function and $\|u(t_k) - v_k\| = O(\tau^3)$, $k = 1,2$. Then the estimation $\|u(t_k) - v_k\| = O(\tau^3)$, $k = 3,\ldots,n$, holds true.

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PARALLELIZATION OF THE PERTURBATION ALGORITHM FOR REALISATION OF SEMI-DISCRETE SOLUTION SCHEMES OF THE EVOLUTIONARY PROBLEM

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Let us consider the following evolutionary problem

$$\frac{du(t)}{dt} + Au(t) = f(t), \quad t \in]0,T], \qquad u(0) = u_0, \quad (1)$$

where operator A and other data of this problem guarantee the existence and uniqueness of solving the problem.

On $[0,T]$ we introduce a grid $t_k = k\tau$, $k = 0,1,\dots,n$, $\tau = T/n$. Let us approximate the first derivative by purely implicit three-level semi-discrete schemes. Then by perturbation algorithm [1], we obtain the approximate solution of problem (1): $v_k = u_k^{(0)} + \frac{\tau}{2} u_k^{(1)}$, $k = 2,\dots,n$, where $u_k^{(0)}$ and $u_k^{(1)}$ are solutions of the following system of equations:

$$\frac{u_k^{(0)} - u_{k-1}^{(0)}}{\tau} + Au_k^{(0)} = f_k, \qquad f_k = f(t_k) \quad (2)$$

$$\frac{u_k^{(1)} - u_{k-1}^{(1)}}{\tau} + Au_k^{(1)} = -\left(\frac{u_k^{(0)} - 2u_{k-1}^{(0)} + u_{k-2}^{(0)}}{\tau^2} \right) \quad (3)$$

and the estimation $\| u(t_k) - v_k \| = O(\tau^2)$ holds thrue. From (2) we can see that $u_2^{(1)} = g(u_1^{(1)}, u_0^{(0)}, u_1^{(0)}, u_2^{(0)})$, $u_3^{(1)} = g(u_2^{(1)}, u_1^{(0)}, u_2^{(0)}, u_3^{(0)})$, ..., $u_n^{(1)} = g(u_{n-1}^{(1)}, u_{n-2}^{(0)}, u_{n-1}^{(0)}, u_n^{(0)})$. So for calculating $u_2^{(1)}$ we need only $u_1^{(1)}, u_0^{(0)}, u_1^{(0)}, u_2^{(0)}$ (not all $u_1^{(0)}, u_2^{(0)}, u_3^{(0)}, \dots, u_n^{(0)}$). Respectively for calculating $u_3^{(1)}$ we need only $u_2^{(1)}, u_1^{(0)}, u_2^{(0)}, u_3^{(0)}$ and so on. As to calculating v_k, it's obvious that we can calculate v_2 after determining $u_2^{(0)}$ and $u_2^{(1)}$. Similarly we can colculate v_3 after determining $u_3^{(0)}$ and $u_3^{(1)}$ and so on.

From the above reasoning it follows that we can make a parallel calculation algorithm for realisation three-level schemes (parallelly calculating equations (2) and (3)).

Similarly, we can make a parallel calculation algorithm for realisation of four-level schemes.

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MODELLING OF REGIONAL CLIMATIC EXTREMES BASED ON STATISTICAL AND DYNAMICAL DOWNSCALING TECHNIQUES

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Characterizing present climate conditions and providing future climate projections at a regional scale is an extremely difficult task as it involves additional uncertainties while reducing, a spatial scale of Global Climate Models (GCMs) simulated climate parameters. Decreasing in spatial accuracy of GCMs simulated climate variables occurs from continental to local scale using statistical downscaling (SD) or dynamical downscaling (DD) techniques [1]. There is a gap in the most studies, specifically focused on estimating the uncertainty of downscaling results due to different statistical methods, as well as in creating ensembles from different GCM and SD methods at several sites in Georgia [2].

In this article, a climate change parameter such as temperature has been investigated by SD and DD methods with an emphasis on SD. Namely monthly extremes of air temperature from three GCMs of CMIP5 database has been statistically downscaled using RCMES package, with four different methods for the territory of Georgia. The extreme values downscaling methods have been trained for the period of1961-1985 and validated for the period of 1986-2010. Downscaling model, driven by the validation study was used for future extremes time series construction for the 2021-2070 period under RCP4.5 and RCP8.5 scenarios. Validations of statistical downscaling methods show that all of the methods have some advantages and disadvantages on the temporal and spatial scale.

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ON SOME NUMERICAL SCHEMES FOR THE BIHARMONIC OPERATOR

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We consider the problems of creation of the convergence difference schemes and numerical realization for the estimate of order of arithmetic operations needed for finding an approximate solution of the Dirichlet problem for the biharmonic equation in multidimensional ($n \geq 2$) cube.

ON THE EXACT SOLUTION OF THE ZAKHAROV-KUZNETSOV TYPE NONLINEAR PARTIAL DIFFERENTIAL EQUATION

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Using the special exp-function method [1] traveling wave exact solutions of the (2+1)D nonlinear Zakharov-Kuznetsov type partial differential equation are obtained.

It is shown that such solutions can be expressed through hyperbolic, trigonometric, exponential, and rational functions and have spatially isolated structural (soliton-like) forms. Revision of previously obtained solutions is discussed.

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ON ONE METHOD FOR SOLVING OF A STATIC BEAM PROBLEM

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In the present work, the variational-iterative method is used to solve a boundary value problem

$$u''''(x) - a\left(\int_0^L u'^2(x)\,dx\right)u''(x) = f(x), \quad 0 < x < L,$$
$$a(\lambda) \geq \text{const} > 0, \quad 0 \leq \lambda < \infty,$$
$$u(0) = u(L) = 0, \quad u''(0) = u''(L) = 0,$$

that describes the static state of a beam. This equation is an equation of the Kirchhoff type [1]. Some computational aspects of this equation and its various modifications are investigated in many works [2-6].

In the presented work, the error of the proposed method is estimated and its effectiveness is checked by an example.

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AN APPROXIMATE SOLUTION OF THE ANTI-PLANE PROBLEMS OF ELASTICITY THEORY FOR ISOTROPIC COMPOSITE PLANE WEAKENED BY CRACK USING A METHOD OF DISCRETE SINGULARITY

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The anti-plane problem of the elasticity theory for a composite (piecewise homogeneous) orthotropic (in particular, isotropic) plane weakened by a crack, when the crack intersects an interface or reaches this one with the right angle, is studied by the integral equation method. When the crack reaches the interface, the problem is changed with a singular integral equation containing a fixed-singularity, but when the crack intersects the dividing border of interface – system (pair) of singular integral equations containing a fixed-singularity concerning characteristic functions of disclosure of crack. The behaviour of the solutions is studied (see [1], [2]). The present paper develops new computational algorithms for the approximate solution of the above-mentioned problems by the collocation (in particular by a discrete singularity) method ([3]). The algorithms are carried out in various specific practical problems. Numerical results are presented. In the case of loads of different quantities on the crack, the stress intensity factors at the ends of the crack are calculated, which allows us to make a hypothetical prediction about the crack spread.

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ON THE APPLICATION OF DIRECT COMPUTATIONAL METHODS TO NUMERICAL SOLUTION OF SINGULAR INTEGRAL EQUATIONS WITH CAUCHY KERNEL

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A number of quadrature processes connected with approximation of Cauchy type singular integrals are considered in relation with approximate solution of boundary problems of certain type. Namely, significant attention is payed to accuracy and simplicity of approximating scheme related with boundary integral problems based on corresponding approximation. Results given in [1], [2] are improved from the viewpoint of convergence order and structure. High accuracy quadrature formulas with simple structure and with constant and equal coefficients are constructed for singular integrals with weight function, which improves the results given in [3], [4].

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MATHEMATICAL MODELING OF EXPLOSIVE PROCESSES IN NONHOMOGENEOUS GRAVITATING GAS BODIES

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Mathematical modeling of explosive processes in gravitating gas bodies is one of the current problems of astrophysics [1 - 6].

The work considers a nonautomodel problem about the centralexplosion of nonhomogeneous gas body (star) bordering vacuum, which is in equilibrium with its own gravitational field. Asymptotic method of thin impact layer is used to solve the problem. The solution of the problem in the vicinity behind the shock wave (the fracturing surface of the first kind) is sought in the form of a singular asymptotic decomposition by a small parameter. Analytically, the main (zero) approximation for the law of motion and the thermodynamic characteristics of the medium was accurately found. The Cauchy problem for zero approximation of the law of motion of the shock wave is solved exactly, in the form of elliptic integrals of the first and second general. Corresponding asymptotics of solutions are calculated.

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ON A PROBLEM OF INTEGER VALUED OPTIMIZATION

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Let $L > n > 1$ be natural numbers and let $x_i, k_i, s_i,\ i = 1,\dots,n$ be non-negative integers. Find the maximum value of the product

$$\prod_1^n (x_i + s_i)$$

under the following restrictions:

$$\sum_1^n x_i = L, \qquad x_i \ge k_i,\ i = 1,2,\dots,n.$$

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ON 3D STOKES FLOW IN THE CYLINDRICAL AND PRISMATIC PIPES

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In the paper the 3D problem for the non-stationary Stokes flow in the infinite cylindrical and prismatic areas is studied. We admit that the pressure can be controlled and depends on time exponentially. The linear Stokes system is considered with the appropriate initial-boundary conditions. By means of the integral equation method the system is equivalently reduced to the system of integral equations with the weakly singular kernel. The existence and uniqueness of solution is obtained, if the power at the exponent satisfies certain conditions. The exact solutions are obtained by means of the stepwise approximation method. Several examples are given.

Hence the solution of the Stokes non-stationary system is obtained in a cylindrical and prismatic pipes when their cross section is an arbitrary simply-connected region bounded with the piecewise smooth line.

The axisymmetric case was considered in [1-4].

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ON THE CRITERION OF THE CONVERGENCE OF THE DIFFERENCE SCHEMES FOR THE GENERAL BOUNDARY VALUE PROBLEMS FOR THE SYSTEMS OF ORDINARY LINEAR DIFFERENTIAL EQUATIONS

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Consider the general boundary value problem

$$\frac{dx}{dt} = P(t)x + q(t), \quad t \in [a,b],$$

$$l(x) = c_o,$$

where $P:[a,b] \to R^{n\times n}$ and $q:[a,b]\to R^n$ are the integrable matrix and vector-functions, respectively, l is a linear bounded operator from the space of continuous vector-functions into R^n, and $c_0 \in R^n$.

Let x_0 be the unique solution of the problem.

Along with the problem consider the sequence of the difference boundary value problems

$$\Delta y(k-1) = G_{1m}(k)y(k) + G_{2m}(k-1)y(k-1) + g_{1m}(k) + g_{2m}(k-1)(k=1,\dots,m),$$

$L_m(y) = \gamma_m$,

$(m = 1,2,\dots)$, G_{1m}, G_{2m} and g_{1m}, g_{2m} are, respectively, the matrix-and vector-functions of discrete variables, L_m is a linear bounded operator, and $\gamma_m \in R^n$.

We present the necessary and sufficient conditions guaranteeing the unique solvability of the discrete problems and convergence of that to x_0 as $m \to \infty$.

Moreover, we give the necessary and sufficient conditions guaranteeing the stability of the difference scheme.

Corresponding propositions are proved for example in [1].

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SECTION OF MECHANICS OF CONTINUA

Chairs: Tengiz Meunargia, George Jaiani
Co-chair: Natalia Chinchaladze

NUMERICAL ANALYSIS OF LINEAR DEFORMATION OF CORRUGATED LAYERED CYLINDRICAL SHELL BY SURFACE FORCE AND TEMPERATURE FIELD INFLUENCE, BASED ON DISTINCT THEORIES

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We consider numerical analysis of non linear deformation of corrugated layered cylindrical shell produced by acting of surface force and temperature field based on distinct theories. The considered theories are constructed on the basis of the broken lines hypothesis in either case of linear or non-linear deformations. A particular example of such a deformation is given. In this example it is considered the deformation of corrugated three layered cylindrical shell with fixed ends produced by acting on it of normal surface force and temperature field.

On the basis of distinct theories the numerical realization of this example is given. The comparison of obtained results gives the possibility to estimate the process the deformation.

ONE-DIMENSIONAL MODELS OF THERMOELASTIC BARS WITHIN THE FRAMEWORK OF LORD-SHULMAN THEORY

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In the present paper Lord-Shulman dynamical three-dimensional model [1] of the nonclassical theory of thermoelasticity is considered for the bar with variable rectangular cross-section with thickness or width which may vanish on one of the ends of the bar. The butt end of the bar with positive area is clamped and temperature vanishes on it, and along the remaining part of the boundary the density of surface force and the normal component of heat flux is given. Applying variational approach, the three-dimensional model is reduced to a hierarchy of one-dimensional ones. The initial-boundary value problems, corresponding to the obtained one-dimensional models are investigated in suitable spaces of vector-valued distributions with values in weighted function spaces. Moreover, the pointwise with respect to the time variable convergence of the sequence of vector-functions of three space variables, restored from the solutions of the reduced one-dimensional problems to the solution of the original three-dimensional problem is proved and under additional conditions the rate of convergence is estimated.

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EXPLICIT SOLUTION OF THE DIRICHLET TYPE BOUNDARY VALUE PROBLEM OF ELASTICITY FOR POROUS SPHERICAL LAYER

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Let us assume that D is a spherical layer, $\{D: R_1 < |\mathbf{x}| < R_2\}$ centered at point $O(0,0,0)$, $S_i \ (i=1,2)$ is a spherical surface of radius $R_i \ (i=1,2)$. Let us assume that the domain D is filled with an isotropic porous materials, $\mathbf{x}(x_1, x_2, x_3) \in D$.

The basic system of equations of motion in the coupled linear quasi-static theory of elasticity for porous elastic materials, expressed in terms of the displacement vector $\mathbf{u}(\mathbf{x})$, the changes of volume fraction of pores $\varphi(\mathbf{x})$ and the change of fluid pressure in pore network $p(\mathbf{x})$ has the following form [1]

$$\begin{aligned} &\mu\Delta\mathbf{u} + (\lambda+\mu)\operatorname{grad}(\operatorname{div}\mathbf{u}) + \operatorname{grad}(b\varphi - \beta p) = 0, \\ &(\alpha\Delta - \alpha_1)\varphi - b\operatorname{div}\mathbf{u} + mp = 0, \\ &(k\Delta + i\omega a)p + i\omega\beta\operatorname{div}\mathbf{u} + i\omega m\varphi = 0, \end{aligned} \tag{1}$$

The analytical solution $\mathbf{U} = (\mathbf{u}, \varphi, p)$ of system (1) is given by means of the elementary (harmonic, bi-harmonic and meta-harmonic) functions.

We consider the Dirichlet type boundary value problem in the coupled linear quasi-static theory of elasticity, when on the boundary $S_i \ (i=1,2)$ the displacement vector, changes of volume fraction of pores and the change of fluid pressure in pore network are given.

The explicit solution of the Dirichlet boundary value problem is given as absolutely and uniformly convergent series.

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SOME PROBLEMS FOR MATERIALS WITH VOIDS

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In the present paper the static two-dimensional problems for an elastic material with voids are consider. The corresponding system of differential equations is written in a complex form and its general solution is presented with the use of two analytic functions of a complex variable and a solution of the Helmholtz equation. The boundary value problems are solved for a circle and a circular ring when on the boundary the displacement vector or the stress tensor and changes of volume fraction of pores given.

ON THE SATISFACTION OF BOUNDARY CONDITIONS ON FACE OF SURFACES FOR PLATES

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This talk is dedicated to the question ([1],[2]) of satisfaction of boundary conditions on the face surfaces of the plates in the refined theories.

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SOME COMMENTS TO HIERARCHICAL MODELS

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The main aim of the present comments is by quotations, brought from competent publications, to emphasize the importance of hierarchical models, their purpose and requirements for them.

Let’s start by quoting from the Encyclopedia of Computational Mechanics [1] (see pp. 2-10):

„1.4 Families of Problems

We will address two types of problems on our thin domain Ω^d : (i) find the displacement u solution to the equilibrium equation $\operatorname{div}\sigma(u) = f$ for a given load f, (ii) Find the (smallest) vibration eigen-modes (Λ, u) of the structure. For simplicity of exposition, we assume in general that the structure is clamped (this condition is also called 'condition of place') along its lateral boundary Γ^d and will comment on other choices for lateral boundary conditions. On the remaining part of the boundary $\partial\Omega^d \setminus \Gamma^d$ ('top' and 'bottom') traction free condition is assumed."

„1.5 Computational obstacles

With the twofold aim of improving the precision of the models and their approximability by finite elements, the idea of hierarchical models becomes natural: Roughly, it consists of an Ansatz of polynomial behavior in the thickness variable, with bounds on the degrees of the three components of the 3-D displacement. The introduction of such models in variational form is due to Vogelius and Babuška (1981) and Szabo and Sahrmann (1988). Earlier beginnings in that direction can be found in Vekua (1955, 1965). The hierarchy (increasing the transverse degrees) of models obtained in that way can be discretized by the p -version of the elements."

3.1 The concepts of the hierarchical models

„The idea of hierarchical models is a natural and efficient extension to that of limiting models and dimension reduction. In the finite element framework, it has been firstly formulated in Szabo and Sahmann (1988) for isotropic domains, mathematically investigated in Babuška and Li (1991, 1992), and generalized to laminated composites in Babuška, Szabo and Actis (1992) and Actis, Szabo and Schwab (1999)."

„Any model that belongs to the hierarchical family has to satisfy three requirements; see Szabo and Babuška (1991) Chap. 14.5:

(a) Approximability. At any fixed thickness $\varepsilon > 0$:
$$\lim_{q\to\infty}\left\|u^{\varepsilon} - u^{\varepsilon,q}\right\|_{E(\Omega^d)} = 0$$
(b) Asymptotic consistency. For any fixed degree triple q:
$$\lim_{\varepsilon\to 0}\frac{\left\|u^{\varepsilon} - u^{\varepsilon,q}\right\|_{E(\Omega^d)}}{\left\|u^{\varepsilon}\right\|_{E(\Omega^d)}} = 0$$
(c) Optimality of the convergence rate. There exists a sequence of positive exponents $\gamma(q)$ with the growth property $\gamma(q) < \gamma(q')$ if $q \prec q'$, such that 'in the absence of boundary layers and edge singularities'
$$\left\|u^{\varepsilon} - u^{\varepsilon,q}\right\|_{E(\Omega^d)} \le C\varepsilon^{\gamma(q)}\left\|u^{\varepsilon}\right\|_{E(\Omega^d)}$$

$\|\cdots\|_{E(\Omega^d)}$ means the strain energy norm".

As we see, here the question is the convergence of the approximate solutions (models) to the corresponding 3D solutions (models), asymptotic consistency of the approximate models, and optimality of the convergence rate. There is no question of satisfying boundary conditions on the face surfaces (which could have caused violation of the hierarchicality models) of the plate, just as in the Kirchhoff-Love model it is not required to satisfy the boundary conditions on face surfaces of the plate, but here the effect of the load is significant (it is taken into account in the wright-hand side of the governing equation) along with satisfaction (which is approximate as well in the above sense) of boundary conditions at the boundary of the plate (i.e., on the lateral boundary).

The accuracy of the approximation was the interest of I. Vekua in [2] (see §11), that is why he raised the question in such a form:

“Теперь, очевидно, встает вопрос, **<u>в какой мере</u>** приближения вида

$$U_N(x^1,x^2,x^3)=\sum_{k=1}^{N}\overset{(k)}{U}(x^1,x^2)P_k\left(\frac{x^3-\bar{h}}{h}\right), \tag{11.1}$$

и

$$P^i(U_N)=\sum_{k=1}^{N}\overset{(k)}{P^i}(x^1,x^2)P_k\left(\frac{x^3-\bar{h}}{h}\right), \tag{11.2}$$

<u>удовлетворяют</u> краевым условям на лицевых поверхностях S^+ и S^-, где мы считаем заданными напряжения $\overset{(+)}{P}$ и $\overset{(-)}{P}$ соответственно”, i.e., "in which measure" approximations (1) and (2) satisfy boundary conditions on the face surfaces S^+ and S^-, where we assume prescribed tractions $\overset{(+)}{P}$ and $\overset{(-)}{P}$, respectively.

In other words I. Vekua was interested in the accuracy of the approximations (11.1) and (11.2) constructed by I. Vekua in §11, in what measure he met the boundary condition of the face surfaces, to which he himself responded, but was not satisfied and presented an article of D. Gordeziani [3], which was dedicated to the accuracy of the approximate solutions constructed by Vekua, for publication in the Journal “Reports of the Academy of Sciences of the Soviet Union”. This article is sited in [1] as well, there is also cited there the paper of the M. Avalishvili and D. Gordeziani [4], which is dedicated to the same problem. There is also cited an article of Schwab [5] in which, Vekua hierarchical models for the plates of constant thickness are investigated in accordance with the requirements mentioned at the beginning of the present comments. Here should be also mentioned the following articles of D. Gordeziani, G. Avalishvili, and M. Avalishvili [6,7].

<u>Remark.</u> It should be noted that by means traditional methods (e.g., based on Korn’s inequality) used for investigation of hierarchical models is not possible to reveal completely peculiarities of setting the boundary conditions on cusped edges (ends). It is caused by that, that in this case governing equations are singular differential equations and peculiarities of setting the boundary conditions besides the principal part of equations besides the principal part of equations depend on values of coefficients of less order derivatives on the boundary belonging to the line of degeneration of equations (see [8]) or on behaviour of that in a neighbourhood (see [9]) of the line of degeneration of equations.

In the second part of the talk in the spirit of the present paper we shortly discuss a question of satisfaction of boundary conditions on so called face surfaces within the framework of 2D models. Let us start with the simplest model of plane deformation in a finite cylinder. In this case in order to maintain the plane deformation we are forced to apply $+\sigma_{33}$ and $-\sigma_{33}$, which are calculated after solving the 2D BVP under BCs on the lateral surface of the cylinder, at top and bottom bases, which play a role of the face surfaces, of the cylinder, respectively. Therefore, they cannot be prescribed arbitrarily. Note that the main part of the governing system for in-plane displacements of I. Vekua’s hierarchical models is similar (see [2]) to the system of plane deformation, which coinsides with the Vekua’s N=0 approximation governing system for in-plane displacements in the case of the plate of constant thickness.

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THE CONTACT PROBLEM FOR THE PIECEWISE-HOMOGENEOUS VISCOELASTIC PLATE WITH INCLUSION

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In this paper we consider the plate, which includes $z = x + iy$ complex plane and consists of two half-planes $S_1 = \{z \mid \mathrm{Re}z > 0, \quad z \notin l_1\}$, $\quad S_2 = \{z \mid \mathrm{Re}z < 0\}$, with different isotropic visco-elastic properties.

The plate is reinforced along the **ox** axis with a semi-infinite $(l_1 = (0,\infty))$ or finite $(l_1 = (0,1))$ rigid inclusion, which is loaded by the normal force with intensity $p_0(x,t)$.

At the interface of the two materials we have the following continuity conditions

$$\sigma_x^{(1)} = \sigma_x^{(2)}, \quad \tau_{xy}^{(1)} = \tau_{xy}^{(2)}, \qquad \frac{\partial u_1}{\partial y} = \frac{\partial u_2}{\partial y}, \quad \frac{\partial v_1}{\partial y} = \frac{\partial v_2}{\partial y}. \tag{1}$$

On the boundary of interaction of rigid inclusion and half-plane (along the line l_1) the conditions of jumps of stresses and displacements, of rigid contact and the equilibrium conditions are given

$$\sigma_y^{(1)+} - \sigma_y^{(1)-} = p(x,t), \quad \tau_{xy}^{(1)+} - \tau_{xy}^{(1)-} = 0, \quad u_1^+ - u_1^- = 0, \quad v_1^+ = v_1^- \equiv v(x,t), \tag{2}$$

$$\frac{dv_0(x,t)}{dx} = \frac{dv(x,t)}{dx} = 0, \qquad \int_{l_1} [p(x,t) - p_0(\mathrm{x,t})]\,\mathrm{dt} = 0. \tag{3}$$

The contact problem consists of determining the jump of normal contact stresses $p(x,t)$ along the contact line and of establishing their behavior in the neighborhood of the ends of the inclusion. Using analogs of Kolosov-Muskhelishvili formulas for viscoelasticity theory the solution of problems of linear conjugation the complex potentials are represented in the form

$$\Phi_1(z,t)=\frac{1}{2\pi(æ_1+1)i}\int_0^\infty\frac{p(x,t)dt}{x-z}+W_1(z,t)\equiv A_1(z,t)+W_1(z,t),$$

$$\Psi_1(z,t)=\frac{æ_1-1}{2\pi(æ_1+1)i}\int_0^\infty\frac{p(x,t)dx}{x-z}-\frac{1}{2\pi(æ_1+1)i}\int\frac{xp'(x,t)dx}{x-z}+Q_1(z,t)\equiv B_1(z,t)+Q_1(z,t)$$

where $W_1(z,t), Q_1(z,t)$ are unknown analytical functions in the half-plane S_1, which are defined from the conditions on the interface (1). Using conditions (2) and (3) the problem is reduced to an integral equation, the solution of which is presented explicitly. Appropriate asymptotic estimates are obtained.

THE PUNCH PROBLEM OF THE VISCOUS HALF-PLANE WITH A FRICTION

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The punch problem of the viscous half-plane with a friction is considered. However, it is based of the Kelvin-Voigt model. Using the methods of complex analysis which in the plane theory of elasticity are developed by acad. N. Muskhelishvili and his followers, the unknown complex potentials, that describe the equilibrium of a half-plane, are constructed effectively (analytically). Two specific examples of the outline of a punch base are considered when it represents an arc of a parabola with a radius of great curvature, or an ellipse arc whose half-axis is small in the direction of the Oy axis. In these cases, using the theory of residuals, the integral in the solution is constructed explicitly.

THE BENDING PROBLEM OF INFINITY ANISOTROPIC PLATE WITH A CIRCULAR HOLE AND CUTS

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The bending problem of infinity anisotropic plate with a hole and defects is considered. Using the methods of the theory of analytic functions the solution of this problem is presented in an explicit form.

Infinity anisotropic plate is weakened by a unit circle hole and two symmetrical cuts, which are positioned along $y=0$, $b<x<c$, $-c<x<-b$ $(b>1)$ intervals. On the boundary of the cuts the jumps of bending function, angle of rotation, bending moment and lateral force are known:

$<W>=<W_y'>=<M_y>=0$; $<N_y>=\mu(x)$

The boundary of a hole is rigidly fixed and the boundary conditions have the form

$W=0$; $\dfrac{\partial W}{\partial n}=0$

At infinity the unbounded plate is under the action of the bending moment. In the plate the bending function, bending moments, torque and cutting force have to be determined.

Using complex parameters of bending, S_1 and S_2 areas are produced from area of plate S by affine transformation. Accordingly the bending function is presented by $W_1(z_1)$ and $W_2(z_2)$ complex potentials, which are determined, respectively in S_1 and S_2 areas:

$$W = W_0 + 2\,\mathrm{Re}\left[W_1\left(z_1\right) + W_2\left(z_2\right)\right]$$

where W_0 is the solution of the nonhomogeneous bending equation. Using the boundary value problems of the theory of analytic functions (in particular, boundary value problems of linear conjugation) and the theory of Cauchy-type integral the unknown analytic functions $W_1(z_1)$ and $W_2(z_2)$ are presented effectively. Accordingly sought functions are determined.

THE MAIN CONTACT PROBLEM FOR AN ELASTIC PLANE WITH VOIDS

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Let an elastic plane with voids by a closed line L be divided into two regions: internal D_1 and external D_2. Assumes D_1 and D_2 are filled with different elastic materials. The system of differential equations for materials with voids has the following form

$$\mu_k \Delta \overset{k}{u}(\mathbf{x}) + (\lambda+\mu) grad\, div \overset{k}{u}(\mathbf{x}) + \beta_k\, grad \overset{k}{\varphi}(x) = 0,$$
$$(\alpha_k \Delta - \xi_k)\overset{k}{\varphi}(\mathbf{x}) - \beta_k\, div \overset{k}{u}(\mathbf{x}) = 0, \quad \mathbf{x}\in D_k,\ k=1,2, \tag{1}$$

where $\overset{k}{u}(\mathbf{x})$ is the displacement vector, $\overset{k}{\varphi}(x)$ is change in the relative pore area. Let us formulate the following contact problem. To find a regular vector $\overset{k}{U}(\mathbf{x}) = (\overset{k}{u}(\mathbf{x}), \overset{k}{\varphi}(\mathbf{x}))$ in D_k that satisfies system (1) and the following contact conditions on the boundary L:

$$\overset{1}{u}(\mathbf{z}) - \overset{2}{u}(\mathbf{z}) = f(\mathbf{z}), \qquad \overset{1}{P}(\partial_z, \mathbf{n})\overset{1}{U}(z) - \overset{2}{P}(\partial_z, \mathbf{n})\overset{2}{U}(z) = F(\mathbf{z}),$$
$$\overset{1}{\varphi}(\mathbf{z}) - \overset{2}{\varphi}(\mathbf{z}) = \Psi_1(\mathbf{z}), \qquad q_1 \frac{\partial}{\partial n}\overset{1}{\varphi}(\mathbf{z}) - q_2 \frac{\partial}{\partial n}\overset{2}{\varphi}(\mathbf{z}) = \Psi_2(\mathbf{z}), \qquad z\in L,$$

and at infinity - the regularity conditions:

$$\overset{2}{U}(x) = O(1),\ r^2 \frac{\partial \overset{2}{U}(x)}{\partial x_i} = O(1),\ i=1,2,$$

where $\overset{k}{P}(\partial_x, \mathbf{n})\overset{k}{U}(x)$ is the stress vector in elasticity theory for porous bodies with voids.

This report solves a static two-dimensional problem for an elastic plane with voids, in which an elastic circle from a different material with voids is inserted. Special representations of a general solution of a system of differential equations (1) are constructed via elementary (harmonic, biharmonic, and metaharmonic) functions which make it possible to reduce the initial system of equations to equations of simple structure and facilitate the solution of the initial problems. Solutions are written explicitly in the form of absolutely and uniformly converging series.

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ON EXACT SOLUTION OF ONE TASK OF MAGNETIC HYDRODYNAMICS

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The subject of this work is the study of the unsteady motion of an electrically conductive viscous incompressible fluid between two plane-parallel plates placed in an external uniform magnetic field perpendicular to the pipe axis. It is believed that the motion is created by constant longitudinal pressure drop applied at the initial moment of time, although it is not difficult to generalize the problem both to the case of the presence of an initial velocity distribution and to the case of moving walls.

The developed flows of a conducting medium in channels currently have been studied in great detail in the works [1-4], and the possibilities of obtaining new analytical exact solutions seem to be very limited. However, such opportunities still exist and it is possible, as will be shown below, to find even simple new solutions, which, at the same time, have rather interesting qualitative features.

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EXPLICIT SOLUTIONS OF SOME BOUNDARY VALUE PROBLEMS FOR AN INCOMPRESSIBLE CONFOCAL ELLIPTIC RING

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It is known that many elastic materials under finite deformations are deformed without noticeable changes in volume. Such materials are incompressible elastic materials. An example of a homogeneous isotropic incompressible body is a rubber body. The boundary value problems in the present work are considered for an incompressible (rubber) confocal elliptic ring in an elliptic coordinate system. For incompressible bodies, equilibrium equations and Hook's Law are written in the elliptic coordinate system, boundary value problems are set and explicit (analytical) solutions are presented with two harmonic functions, which are obtained by a method of separation of variables. The present work considers the boundary value problems for a confocal elliptic semi-ring

$\overline{\Omega} = \{\vartheta_0 \le \vartheta \le \vartheta_1, 0 \le \alpha \le \pi\}$, when the conditions of discontinuous continuation (of symmetry or asymmetry) of solutions are given on boundaries $\alpha = 0$ and $\alpha = \pi$. Following the discontinuous continuation of solutions, the solutions of the boundary value problems for a whole (closed) confocal elliptic ring $(\vartheta_0 \le \vartheta \le \vartheta_1,\ 0 \le \alpha < 2\pi)$ are obtained. The boundary value problems for a confocal elliptic semi-ring are given with the superposition of the internal and external problems of a semi-ellipse. The numerical results of the concrete problems are obtained and corresponding graphs are constructed.

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