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GEOMETRIC FIGURES WHICH APPEAR AFTER **VS**-CUTTING IN THE RADIAL CROSS SECTION OF THE GML BODIES *

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Abstract. In the previous works, we were able to calculate all possible, and various options that appear after VV, VS or SS cuts of the GML bodies with the help of the so-called straight chordal knives [1-2]. Then we did not specify how many and what traces of flat figures appear on the radial cross section of the GML^n_m body, depending on: m-number of polygon vertices, n-number of twist and a parameter showing which vertices (sides) are connected by this knife!? In this article, a regularity is given with the help of which it is possible to calculate the number and nature of flat figures appearing after an arbitrary VS in arbitrary regular m-gon. It should be obligatorily noted that at present this regularity has been discovered and tested on many examples of parameters, but by this time there is no complete mathematical proof. Therefore, we call this regularity hypothetical regularity.

Keywords and phrases: Analytic representation, Möbius-Listing's bodies.

AMS subject classification (2010): 53A05, 51B10, 51E12.

1 Introduction. In this article, we use all the traditional definitions and notation introduced in previous works [1-3], so we will not repeat them, but add only a few new parameters that turned out to be decisive for these results.

• 1: V_1S_i - is the "trace of corresponding knife" connecting the first numbered vertex and the i-numbered side of the regular m-polygon in the radial cross section of the GML body. It is known from previous works that it suffices to consider $i = 2, ..., [m/2] + 1$ if m is odd number and $i = 2, ..., [m/2]$ if m is even number, where $[m/2]$-is an integer part of a fraction.

• 2: $\kappa \equiv i - 1$- is the parameter describing the given "trace of corresponding knife".

• 3: $j \equiv gcd(m, n)$- is a parameter characterizing this GML^n_m body, $j = 0$-means $n \equiv 0$ body without twisting.

General Remarks: In what follows, a polygon with three vertices is called an 3-gon, a polygons with 16 vertices is called a 16-gon etc.. Further, it should be noted that the methodology uses planar sections, whereas Generalized Möbius-Listing bodies are three dimensional. This means that the resulting ν-gons with the same color and shape form a single body after cutting.

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We have identified three cases in which different regularities of appearance of different flat geometric figures in the radial cross section of the GML_m^n bodies are obtained.

First Case. (I. Full rotation $j = 0, m$ and arbitrary κ . II. For arbitrary j cut line includes the center of symmetry of the polygon)

- Case 1A. If $j = m = 2\omega$ or $j = 0$, and for arbitrary $\kappa = 1, ..., \omega - 1$, then one plane $(\kappa + 2)$- gon and one $[m - (\kappa - 1)]$-gon appears after such VS-cutting!
- Case 1B. If $j = m = 2 + 1$ or $j = 0$, for arbitrary $\kappa = 1, ..., \omega$, then one plane $(\kappa + 2)$- gon and one $[m - (\kappa - 1)]$-gon appears after cutting. But when $\kappa = \omega$, then 2 different$(\kappa + 2)$-gons appear after cutting, and if the cut line includes the center of symmetry of the polygon, then mirror symmetrical two $(\kappa + 2)$-gons appear after such VS-cutting!
- Case 1C. If $m = 2\omega + 1$ and $\kappa = \omega$ and center of symmetry of the m-polygon is located on this cutting line, then for arbitrary $j = 2\beta + 1 < m, \beta = 0, 1, ..$ two different groups consisting of mirror symmetrical plane $(\kappa + 3)$- gons appear after such VS-cutting.

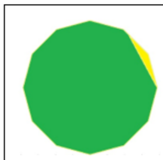
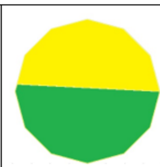
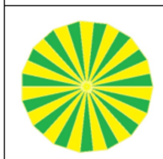
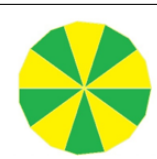
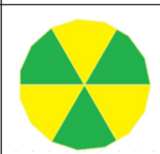
			
Case 1A. m=12, $\kappa = 1$, j=0 or 12. One 3-gon and one 12-gon	Case1A. m=12 $\kappa = 3$, j=0 or 12. One 5-gon and one 10-gon	Case1A. m=12 $\kappa = 5$, j=0 or 12. One 7-gon and one 8-gon	Case1B. m=15 $\kappa = 7$, j=0 or 15. Two mirror symmetrical 9-gons
			
Case1C. m=15,$\kappa = 7$, j=1. 2 groups (15 similar 3-gons)	Case1C. m=15,$\kappa = 7$,j=3. 2 groups (5 similar 4-gons)	Case1C.m=15,$\kappa = 7$, j=5. 2 groups (3 similar 5-gons)	Case1C. m=21,$\kappa = 10$, j=7. 2 groups (3 similar 6-gons)

Table 1: Examples for the first cases with different parameter values

Second Case. For arbitrary m when $\kappa < j$, then (m/j)- pieces $(\kappa + 2)$-gons and one piece of $[m - (\kappa - 1) \cdot (m/j)]$-gon appears after such VS-cutting!

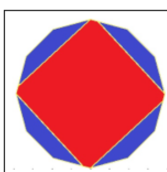
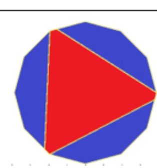
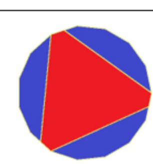
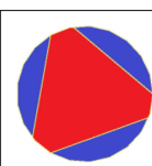
			
m=12, $\kappa = 2 < j = 3$ 4 similar 4-gon and one 8-gon	m=12, $\kappa = 3 < j = 4$ 3 similar 5-gon and one 6-gon	m=15, $\kappa = 4 < j = 5$ 3 similar 5-gon and one 6-gon	m=21, $\kappa = 6 < j = 7$ 3 similar 6-gon and one 9-gon

Table 2: Examples for the second case with different parameter values

Third case For arbitrary m when $\kappa > j$: This turned out to be the most difficult case to study, which has many branches and shows a strong connection with the structure of numbers and geometric shapes.

Case 3.I. (**This subcase is considered separately, since for any values of m and n (even when these numbers are coprime) it is realized.**) For arbitrary m and $\kappa = 2, ..., m/2$. when $j = 1$. then two different groups, each of which consists of $[m/j]$-pieces 3- gons ,$(\kappa - 2)$ different groups each of which consists of $[m/j]$- pieces 4-gons, one group of $[m/j]$- pieces 5-gons and one piece of $[m/j]$ -gon appears after such-VS cutting!

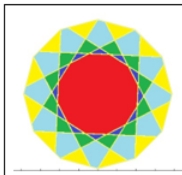
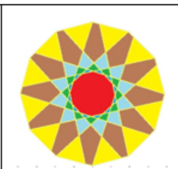
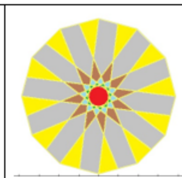
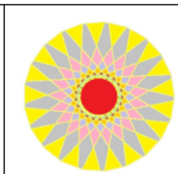
			
m=12, $\kappa = 3 > j = 1$; 2-group (12 similar 3-gons), 1-group (12 similar 4-gons),1 group (12 similar 5-gons) and one 12-gon	m=12, $\kappa = 4 > j = 1$; 2-group (12 similar 3-gons), 2 group (12 similar 4-gons),1 group (12 similar 5-gons) and one 12-gon	m=12, $\kappa = 5 > j = 1$ 2-group (15 similar 3-gons), 3 group (12 similar 4-gons),1 group (12 similar 5-gons) and one 12-gon	m=21, $\kappa = 8 \geq j = 1$ 2-group (21 similar 3-gons), 6 group (21 similar 4-gons),1 group (12 similar 5-gons) and one 21-gon

Table 3: Examples for the case 3.1 with different parameter values, but $j = 1$

- Case 3GA. For arbitrary m and $\kappa = j \cdot \beta + l$, where $\beta = 2, 3, ...$ and $l = 0, 1...j - 1$ then: - one group consisting of $[m/j]$ pieces 3-gons, $(\beta - 2)$ different groups each of which consists of $[m/j]$- pieces 4-gons, one group consisting of $[m/j]$ pieces $[j - (l - 4)]$-gons,one group consisting of $[m/j]$ pieces $(l + 3)$-gons and one piece of $[m/j]$ -gon appears after such VS-cutting!

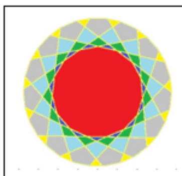
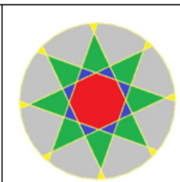
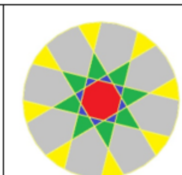
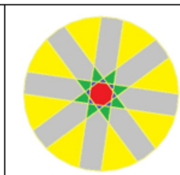
			
m=30, $\kappa = 8 \geq j = 2$ 1-group (15- similar 3-gons), 2-group (15 similar 4-gons), 1-group (15 similar 6-gons) and 1-group (15 similar 3-gons) and one 15-gon	m=32, $\kappa = 12 > j = 4$ 1-group (8 similar 3-gons), 1-group (8 similar 4-gons),1-group (8 similar 8-gons), 1-group (8 similar 3-gons) and one 8-gon	m=32, $\kappa = 13 > j = 4$ 1-group (8 similar 3-gons), 1-group (8 similar 4-gons),1-group (8 similar 7-gons), 1-group (8 similar 4-gons) and one 8-gon	m=32, $\kappa = 14 > j = 4$ 1-group (8 similar 3-gons), 1-group (8 similar 4-gons),1-group (8 similar 6-gons), 1-group (8 similar 5-gons) and one 8-gon

Table 4: Examples for the case GA with different parameter values

- Case 3GA^*. For arbitrary m and $\kappa = j \cdot \beta + l$, where $\beta \equiv 1$ and $l = 0, 1...j - 1$ then one group consisting of $[m/j]$ pieces $[j - (l - 3)]$-gons,- one group consisting of $[m/j]$-pieces $(l + 3)$-gons and one piece of $[m/j]$-gon appears after such VS-cutting!

Remark 1. The case 3GA^* contains a very important subcase when $\kappa = j$!

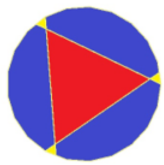
			
m=21, $\kappa = 7 = j$ 1-group (3- similar 3-gons), 1 group (3 similar 10-gon) and one 3-gon	m=21, $\kappa = 8 > j = 7$ 1-group (3 similar 9-gons), 1-group (3 similar 4-gons), and one 3-gon	m=21, $\kappa = 9 > j = 7$ 1-group (3- similar 8-gons), 1-group (3 similar 5-gons), and one 3-gon	m=25, $\kappa = 9 > j = 5$ 1-group (5 similar 4-gons), 1-group (5 similar 7-gons), and one 5-gon

Table 5: Examples for the case 3GA^* with different parameter values.

Remark 2. This work is another step towards resolving the question - is it possible to unequivocally restore the GML_m^nbody knowing the information about the traces left on the radial cross section. In order to answer this question, at least we need to know how many and what kind, and also how many layers of flat figures consist of the corresponding cut marks left on the radial section. Unambiguity during recovery is excluded because parameter j is one and the same parameters for different values of m and n.

Remark 3. After the VS cut, the classical Möbius phenomenon never takes place. It occurs sometimes only with VV and SS cuts.

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