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RENORMSTATISTICS IN MULTIPARTICLE PRODUCTION PROCESSES

Nugzar Makhaldiani

Abstract. Motion equations of the Renormdynamics (RD) for the higher energy multiparticle production processes formulated. Perturbative RD motion equations need nonperturbative modifications at low energy hadronization phase. Modifications by methods of statistical physics - Renormstatistics with some applications given.

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We say that we find **New Physics** (NP) when either we find a phenomenon which is forbidden by SM in principal - this is the qualitative level of NP - or we find a significant deviation between precision calculations in SM of an observable quantity and a corresponding experimental value. Resent indication on NP is CDF collaboration new measured value of the W-boson mass [1]

$$m_W = 80.4335 \pm 0.0094 GeV$$

which is in excess of the SM prediction [2]

$$m_{SMW} = 80.375 \pm 0.006 GeV$$

at 7σ level. An example of NP on the qualitative level will be e.g. observation of the three gamma decay of the neutral pion.

All standard models of physics are local type, are described by differential structures (Lagrangian and motion equations) and permit quantitative analysis by parallel algorithms and corresponding programs. In the 80s of the previous century as a supercomputer were Cray computers with conveyer architectures and extended software including operations with linear algebra. Today multiprocessor supercomputers (e.g. Govorun, JINR Dubna) permit adaptation of the architecture to the algorithm of solution of a physical problem (e.g. for QCD hydrodynamic motion equations or lattice formulation of (non) equlibrium dynamics under NICA project) with corresponding optimal control problem. Probably future supercomputers will contain also quantum processors with their intrinsic parallel processes.

In fractal calculus (FC, [3]), the classical derivatives and integrals of integer order are generalized to derivatives and integrals of arbitrary (real, complex or p-adic) order α,

$$_0I_x^\alpha f = \frac{1}{\Gamma(\alpha)} \int_0^x |x-t|^{\alpha-1} f(t) dt = |x|^\alpha \frac{\Gamma(1+x\partial_x)}{\Gamma(1+x\partial_x+\alpha)} f,$$

$$
{}_0I_x^{-1}f = \frac{1}{x}x\partial_x f = f'(x),\ {}_0I_x^1 f = x\frac{1}{\partial_x x}f = xx^{-1}\partial_x^{-1}f = \int_0^x f(t)dt.
$$

Fractal derivatives have attracted increasing attention because they universally appear as empirical descriptions of complex social and physical phenomena. Fractal calculus operators have memory and are more flexible in describing the dynamic behaviour of phenomena and systems using fractal differential equations.

Black-body radiation has been widely used to measure the temperature of a body. The spectral radiance for arbitrary dimension D can be expressed as [4]

$$
f(x,\nu,D) = C_D\frac{x^D}{e^x - 1},\ x = \frac{h\nu}{kT}, \tag{1}
$$

where h is the Planck constant, k is the Boltzmann constant and explicit form of C_D is irrelevant for our purpose.

Fractal dimention as a function of the maximum frequency which is the most probable photon frequency can be found from the derivative of Eq. (1),

$$
D(x) = \frac{x}{1 - e^{-x}},\ x = \frac{h\nu}{kT},
$$

where x represents the dimensionless photon energy in which the photon energy is divided by thermal energy and D is the arbitrary fractal dimention of the body. The dimension of a body D can be found by measuring the maximum frequency factor x, (see Fig 1). To different values of the external (control) parameter T, correspond different values of D.

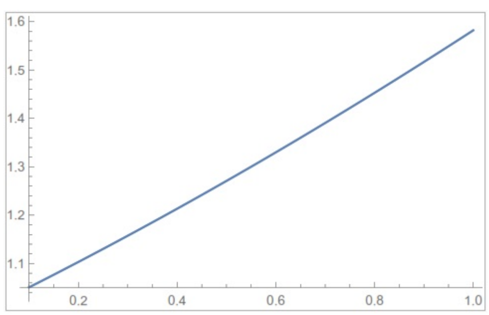

Figure 1: Fractal dimension of the black-body as a function of the maximal frequency of radiation

For h-deformed distributions we take the h-deformed exponent and corresponding Bose and Fermi distributions $f_\mp$,

$$
f_\mp = \frac{1}{e_h(x) \mp 1},\ e_h(x) = (1 + hx)^{\frac{1}{h}} = (1 + \frac{x}{k})^k,\ k = 1/h,
$$

$$
e_0(x) = e^x,\ e_1(x) = 1 + x,
$$

$$
f(x,\nu,D) = C_D\frac{x^D}{e_h(x) \mp 1},\ x = \frac{h\nu}{kT}.
$$

For grassmann valued $x=\theta,\ \theta^2=0,\ e_0(x)=e_1(x)=1+x$.

Deformed and classical exponents are connected as following

$$(1+hx)^{-1/h}=\frac{1}{\Gamma(1/h)}\int_0^\infty dtt^{1/h-1}e^{-(1+hx)t}$$

$$=\sum_{n\geq 0}\frac{\Gamma(n+1/h)}{\Gamma(1/h)}\frac{(-hx)^n}{n!}=\frac{\Gamma(\delta+1/h)}{\Gamma(1/h)}h^\delta e^{-x},$$

$$h^{-\delta}\frac{\Gamma(1/h)}{\Gamma(\delta+1/h)}(1+hx)^{-1/h}=e^{-x},\ \delta=x\partial_x.$$

Let us define corresponding deformed calculus based on the deformed derivative operator ∂_h,

$$\partial e^{-x}=-e^{-x},\ e^{-x}=A(1+hx)^{-1/h},\ A=h^{-\delta}\frac{\Gamma(1/h)}{\Gamma(\delta+1/h)},$$

$$\partial_h e_{-h}(-x)=-e_{-h}(-x),\ \partial_h=A^{-1}\partial A,\ e_{-h}(-x)=(1+hx)^{-1/h}.$$

Note that $e_{-h}(-x)$ coincides with the generating function of the negative binomial distribution (NBD). Indeed, NBD for normed topological cross sections is [5]

$$\frac{\sigma_n}{\sigma}=P(n)=\frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}\Big(\frac{k}{<n>}\Big)^k\Big(1+\frac{k}{<n>}\Big)^{-(n+k)}$$

The generating function for NBD is

$$F(h)=\Big(1+\frac{<n>}{k}(1-h)\Big)^{-k}=\Big(1+\frac{<n>}{k}\Big)^{-k}(1-ah))^{-k},a=\frac{<n>}{<n>+k}.$$

Indeed,

$$(1-ah))^{-k}=\frac{1}{\Gamma(k)}\int_0^\infty dtt^{k-1}e^{-t(1-ah)}$$

$$=\frac{1}{\Gamma(k)}\int_0^\infty dtt^{k-1}e^{-t}\sum_0^\infty\frac{(tah)^n}{n!}=\sum_0^\infty\frac{\Gamma(n+k)a^n}{\Gamma(k)n!}h^n,$$

where the parameter k counts the number of independently radiating primordial black holes and can be measured using experimental data.

NBD provides the best description of the high energy multiparticle production processes, has very clear physical interpretation and corresponds to the independently radiating primordial black holes (PBH) intermadiate states.

QCD possesses a compact gauge group, and this implies a non-trivial topological structure of the vacuum. QCD perturbation theory ignores the compact nature of $SU(3)$ gauge group that gives rise to the periodic θ-vacuum of the theory. After a modification of the gluon propagator to reconcile perturbation theory with the anomalous Ward identities for the topological current in the θ-vacuum [6], the gluon couples to the Veneziano ghost describing the tunneling transitions between different Chern-Simons sectors of the vacuum. The emerging gluon dressed by ghost loops has a propagator of the form

$$G(p)=(p^2+M^4/p^2)^{-1},\ M^4=\chi_t,$$

where χ_t is the Yang-Mills topological susceptibility related to the η' mass by Witten-Veneziano relation. This propagator describes confinement of gluons at distances $M^{-1} \simeq 1$ fm. The same functional form of the propagator was originally proposed by Gribov as a solution to the gauge copies problem that plagues perturbation theory. We can define the minimal form of the renormdynamic (RD) equation

$$\dot{A} = \beta_2 A^2 + \beta_3 A^3,$$

with the following solution

$$\begin{aligned} &\frac{dA}{\beta_2 A^3(1/A + \beta_3/\beta_2)} = dt \Rightarrow \frac{d(1/A)1/A}{1/A + \beta_3/\beta_2} = -\beta_2 dt \Downarrow \\ &x - a\ln(x+a) = -\beta_2 t + c,\ x = 1/A,\ a = \beta_3/\beta_2. \end{aligned} \tag{2}$$

Nonperturbative extension with confining, running mass: $m^2(p) = M^4/p^2$, means the following change

$$t = \ln\frac{p^2}{\Lambda^2} \to t_m = \ln\frac{p^2 + m^2(p)}{\Lambda^2},\ \frac{dt_m}{dt} = \frac{p^2 - m^2(p)}{p^2 + m^2(p)},\ m^2(p) = M^4/p^2,$$

in the solution (2). Corresponding RD motion equation is

$$\begin{aligned} &\dot{A} = (\beta_2 A^2 + \beta_3 A^3)\frac{p^2 - M^4/p^2}{p^2 + M^4/p^2} = \begin{cases} \beta_{pert}, & p^2 \gg M^2, \\ 0, & p^2 = M^2, \end{cases} \\ &\frac{p^2/M^2 - M^2/p^2}{p^2/M^2 + M^2/p^2} = \sqrt{1-4k},\ k = c\frac{M^2}{\Lambda^2}e^{x/\beta_2}/(x+a)^{a/\beta_2}. \end{aligned}$$

REFERENCES

1. Aaltonen, T. et al. (CDF). *Science*, **376** (2022), 170.
2. Patrignani, G. et al. (Particle Data Group). *Chin. Phys. C*, **40** (2016), 100001.
3. Makhaldiani, N. Fractal Calculus (H) and some Applications. *Physics of Particles and Nuclei Letters*, **8** (2011), 325.
4. Landsberg, P.T. De Vos, A. The Stefan-Boltzmann constant in n-dimensional space. *J. Phys. A: Math. Gen.*, **22** (1989), 1073.
5. Makhaldiani, N.V. Renormdynamics, Multiparticle Production, Negative Binomial Distribution and Riemann Zeta Function. *Physics of Atomic Nuclei*, **76** (2013), 1169.
6. Kharzeev, D.E., Levin, E.M. Color confinement and screening in the θ-vacuum. *Phys. Rev. Lett.*, **114** (2015), 242001.

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Author(s) address(es):

Nugzar Makhaldiani
Joint Institute for Nuclear Research
Joliot-Curie str. 6, 141980 Dubna Moscow region, Russia
E-mail: mnv@jinr.ru