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MODELING THE DYNAMICS OF A MIXTURE OF NATURAL GAS AND HYDROGEN IN A PIPELINE *

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Abstract. This work is devoted to a brief review of the possible transportation of hydrogen through the territory of Georgia and one mathematical model describing the flow of a mixture of natural gas and hydrogen substances in a pipeline. A quasi-nonlinear system of two-dimensional partial differential equations is considered, which describes the unsteady flow of a mixture of natural gas and hydrogen substances in a pipeline. The distribution of pressure through a branched gas pipeline has been studied.

Keywords and phrases: Modelling, hydrogen, gas flow, branched pipeline.

AMS subject classification (2010): 62P12, 68U20.

1 Introduction. In order to reduce greenhouse gas emissions to mitigate climate change, it becomes necessary to reduce the use of fossil fuels in electricity generation. On the other hand, the development of economies is aimed at increasing interest in renewable energy sources (RES) as an energy carrier of the future [1]. However, before renewable energy can be effectively used, it is necessary to create a new economy based on the production, storage, and transportation of renewable energy. Among other renewable energy sources, wind energy is not used to its potential yet, and according to ESIEA, wind energy is expected to become the leading source of electricity after 2030 (http://windeurope.org; World Energy Outlook 2017). Since 2021, the EU and Georgia have taken the initiative to lay an underwater cable across the Black Sea to connect Azerbaijan (via Georgia) and other European countries for the export of Azerbaijani wind energy. After the Ukrainian-Russian war, the Black Sea Submarine Cable Project has received increased attention as an area of both energy and communications infrastructure. According to the European Commission, in 2023 the European Investment Bank plans to apply for an investment grant of 20 million euros to support this project. Wind power is known to cause problems with uneven power generation due to variable atmospheric conditions. Due to the uneven and unstable energy supply, it is required to convert wind energy into gaseous hydrogen, which, can store a large amount of energy and this enables silk road countries to use existing routes to transport hydrogen to EU countries. At present, the problems of efficient production, storage, and transportation of gaseous hydrogen are the main focus of many researchers around the world since hydrogen is considered one of the most promising fuels of the future. It is expected to be used in a wide variety of applications

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such as the generation and storage of electricity, automotive fuels, reactive devices, various industries, and even our domestic energy needs [3]. At present, the use of a natural gas pipeline network to transfer hydrogen mixed with natural gas seems to be a good economic solution, since burst tests have shown that mixing hydrogen with natural gas does not affect the mechanical strength of API X52 steel, which is the most used steel in existing gas pipelines [4]. The study of the behavior of a mixture of natural gas and hydrogen substances during flow in pipelines has become an urgent task of our time and has attracted the attention of a number of scientists [1]-[4].

2 Hydrogen and natural gas mixture equation. To determine the density of a mixture of hydrogen with natural gas, assume that the flow of a homogeneous fluid mixture of hydrogen and natural gas in the pipeline is one-dimensional and this mass ratio of the fluid mixture is permanent during the study process and suppose that the density of hydrogen and natural gas varies according to the polytropic laws. Then the mass ratio of hydrogen and natural gas can be expressed by the following expression [1]

$$\rho = \left(\frac{\eta}{\rho_{h_0}} \left(\frac{p_0}{p} \right)^{\frac{1}{n_1}} + \frac{1-\eta}{\rho_{g_0}} \left(\frac{p_0}{p} \right)^{\frac{1}{n_2}} \right)^{-1}, \tag{1}$$

where $\eta = \frac{m_h}{m_h+m_g}$ and m_h m_g represent the masses of hydrogen and natural gas respectively; ρ_{h_0} is a the density of hydrogen at the initial conditions (kg/m^3); p_0 is the permanent regime pressure (N/m^2); n_1 and n_2 are the speicfic heat ratios for hydrogen and natural gas repsectively, which are defined as the ratio of the specific heat at constant pressure $C\rho$ and the specific heat at constant volume Cv for each gas.

As a rule, when transporting gas through a pipeline, the initial value of pressure and temperature is known, so the initial value of density can be easily determined from the following equation of state:

$$\rho_{g_0} = \frac{p_0 M_g}{Z R_g T_0}, \qquad \rho_{h_0} = \frac{p_0 M_h}{Z R_h T_0}, \tag{2}$$

where M_g, M_h, are the molecular mass of natural gas and hydrogen respectively, Z is variable compressibility, $R_h = R_u/M_h$, $R_g = R_u/M_g$ are hydrogen and natural gas constants respectively, and R_u is universal gas constant. Thus, the density of the mixture of hydrogen and natural gas in (1) is a function of the mass ratio η and pressure $p(x,t)$ of the stream, and the determination of those two parameters can define the basic properties of the stream.

Hence, the analytical study of this process is important. Mathematically, gas flow in a pipe can be modeled by the non-linear system of partial differential equations with the suitable initial-boundary conditions. The purpose of our study is to determine the pressure distribution of the gas mixture along an inclined and branched pipeline.

3 Gas flow in the inclined and branched pipeline. The gas flow inside a pipeline is governed by the dynamic laws of the conservation of mass, moment and energy which in case of the one-dimensional gas flow dynamic through branched and inclined pipeline is describing by the following set of PDEs [1],[3],[5].

$$\frac{\partial \rho}{\partial t} + \frac{1}{S}\frac{\partial q}{\partial x} + q^*\delta(x - x^*) = 0, \tag{3}$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(Sp + \frac{q^2}{S\rho}\right) + \frac{2fq|q|}{DS\rho} + \rho Sg\sin\theta = 0, \tag{4}$$

$$T_2 = T_{out} + (T_2 - T_{out})e^{\frac{-L\pi k_T D}{qc_p}} + \frac{q}{\pi k_T D}\left(c_p\mu_{JT}\frac{\partial p}{\partial x}\right)\left(1 - e^{\frac{-L\pi k_T D}{qc_p}}\right) + qg\sin\theta, \tag{5}$$

$$p(x,0) = p_0(x), \qquad q(x,0) = q_0(x),$$
$$p(0,t) = p_1(t), \qquad q(L,t) = q_1(t),$$
$$0 \le x \le L, \quad t \ge 0,$$

where $\rho(x,t)$ is density, $p(x,t)$ is pressure, q is flow rate ($q = pvS = \rho Q$); $v(x,t)$ is speed, T is temperature, S is the cross-sectional area of gas duct, D is diameter, δ is Dirac function and x^* is placement of an offshoot in the pipeline, q^* is volumetric gas consumption in a branchline ($q^* = BV$, V is gas volumetric discharge consumption in branch-line in the offshoot), θ is inclination andgle, f is Fanning factor, k_T is the heat transfer coefficient.

The nonlinear system of equations (2)-(5) with corresponding initial and boundary conditions can only be solved using appropriate numerical methods. For the initial analysis of the nonlinear system of equations (2)-(5) and obtaining the pressure distribution of the mixture of hydrogen and gas in a branched and inclined pipeline, it is necessary to make some simplifications. Using the relation $p = c^2\rho$, which is valid for an isothermal process and taking into account the value of each term in the system of equations (2)-(5), we obtain [5]:

$$\frac{\partial p}{\partial t} = a\frac{\partial^2 p}{\partial x^2} + b\frac{\partial p}{\partial x} + \Phi(x), \tag{6}$$
$$p(x,0) = p_0(x),$$

where

$$a = \frac{-DS}{2f\lambda}, \quad b = \frac{-DSg\sin\theta}{2f\lambda c^2},$$

and

$$\Phi(x) = \frac{DSc^2q^*}{2f\lambda}\delta(x - x^*) \quad or \quad \Phi(x) = \frac{DSc^2q^*}{2f\lambda}\frac{\alpha}{\pi(1 + \alpha^2(x - x^*)^2)}.$$

And finally exact solution of the considered second order non-homogeneous heat (diffusion) linear parabolic partial differential equation (6) has the following form:

$$p(x,t) = e^{\beta t + \mu x}\left(\int_{-\infty}^{+\infty} U_0(\xi)G(x,\xi,t)d\xi + \int_0^t\int_{-\infty}^{+\infty}\Phi_0(\xi,\tau)G(x,\xi,\tau)d\xi d\tau\right),$$

where

$$\Phi_0 = e^{-\beta t - \mu x}\Phi(x), \quad \beta = -\frac{b^2}{4a}, \quad \mu = -\frac{b}{2a}, \quad U_0 = p_0(x)e^{-\mu x},$$

$$G(x,\xi,t) = \frac{1}{2\sqrt{\pi a t}} \sum_{n=-\infty}^{\infty} e^{-\frac{(x-\xi+2nL)^2}{4at}} + e^{-\frac{(x+\xi+2nL)^2}{4at}},$$

$$U(x,t) = \int_{-\infty}^{\infty} U_0(\xi)G(x,\xi,t)d\xi + \int_{-\infty}^{\infty} \Phi(\xi,\tau)G(x,\xi,\tau)d\xi d\tau.$$

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