Reports of Enlarged Sessions of the
Seminar of I. Vekua Institute
of Applied Mathematics
Volume 36, 2022

ON THE ALGORITHM OF AN APPROXIMATE SOLUTION AND NUMERICAL COMPUTATIONS FOR J. BALL NONLINEAR INTEGRO-DIFFERENTIAL EQUATION

Archil Papukashvili Giorgi Geladze Zurab Vashakidze Meri Sharikadze

Abstract. An initial-boundary value problem is posed for the J. Ball integro-differential equation, which describes the dynamic state of a beam. The solution is approximated utilizing the Galerkin method, stable symmetrical difference scheme and the Jacobi iteration method. This paper presents the approximate solution to one practical problem. Particularly, the results of numerical computations of the initial-boundary value problem for an iron beam are represented in the tables.

Keywords and phrases: Nonlinear dynamic beam equation, J. Ball equation, Galerkin method, Implicit symmetric difference scheme, Jacobi iterative method, Iron beam, Numerical realization.

AMS subject classification (2010): 65M60, 65M06, 65Q10, 65M15.

1 Statement of the problem.

Let us consider the nonlinear equation

$$\begin{gathered} u_{tt}(x,t)+\delta u_t(x,t)+\gamma u_{xxxxt}(x,t)+\alpha u_{xxxx}(x,t) \\ =-\left(\beta+\kappa\int_0^L u_x^2(x,t)\,dx\right)u_{xx}(x,t)-\sigma\left(\int_0^L u_x(x,t)\,u_{xt}(x,t)\,dx\right) \\ \times u_{xx}(x,t)=f(x,t), \quad 0<x<L, \quad 0<t\le T, \end{gathered} \tag{1}$$

with the initial boundary conditions

$$\begin{gathered} u(x,0)=u^0(x), \quad u_t(x,0)=u^1(x), \\ u(0,t)=u(L,t)=0, \quad u_{xx}(0,t)=u_{xx}(L,t)=0. \end{gathered} \tag{2}$$

Here $\alpha,\gamma,\kappa,\sigma,\beta$ and δ are given constants, among which the first four are positive numbers, while $u^0(x)\in W_2^2(0,L)$ and $u^1(x)\in L_2(0,L)$ are given functions such that $u^0(0)=u^1(0)=u^0(L)=u^1(L)=0$. It will be assumed that the inequality $|\delta|<\gamma\left(\frac{\pi}{L}\right)^4$ is fulfilled when $\delta<0$ and $\alpha\left(\frac{\pi}{L}\right)^2>|\beta|$ holds when $\beta<0$. The equation (1) obtained by J. Ball [1] using the Timoshenko theory describes the vibration of a beam. The right-hand side $f(x,t)\in L_2((0,L)\times(0,T))$. We suppose that there exits a solution $u(x,t)\in W_2^2((0,L)\times(0,T))$ of problem (1)-(2).

The presented paper is a direct continuation of papers [2], [3], in which to find a solution for J.Ball's equation [1] the algorithm from work [5] is used and approved by tests. Here the solution of the problem (1), (2) was approximated using the Galerkin method, stable symmetrical difference scheme and the Jacobi iteration method. This paper presents the approximate solution to one practical problem. Particularly, the results of numerical computations of the initial-boundary value problem for an iron beam are represented in the tables.

A physical model that J. Ball uses in the article [1] is taken from the handbook of Engineering Mechanics written by E. Mettler (see [4]). For this model he wrote the corresponding initial-boundary value problem for the integro-differential equation of beam (1). Here $\alpha, \gamma, \kappa, \sigma, \beta$ and δ are given constants from which the following five have the form

$$\alpha = \frac{E \cdot I}{\rho}, \quad \beta = \frac{E \cdot A \cdot \Delta}{L \cdot \rho}, \quad \gamma = \frac{\eta \cdot I}{\rho}, \quad \kappa = \frac{E \cdot A}{2L \cdot \rho}, \quad \sigma = \frac{A\eta}{L \cdot \rho},$$

where E is Young' modulus, A is the cross-section area, η is the effective viscosity, I is the cross-sectional second moment of area, ρ is the mass per unit length the reference configuration, L is beam length, Δ is beam length change (extension) and δ is the coefficient of external damping.

2 The numerical realization. For approximate solving initial-boundary value problem (1), (2) several programs are composed in Maple, several numerical experiments are carried out. This paper presents the approximate solution to the one practical problem. Particularly, the results of numerical computations of the initial-boundary value problem for an iron beam are represented in the tables.

Issues of the initial-boundary value problem of the iron beam are studied for the following meanings of parameters: spatial, temporal, mathematical algorithm and physical nature of the beam.

$L = 1$ meters, $T = 1$ seconds, the grid length of a spatial variable $H = 5$, the grid length of a temporal variable $M = 5$, the amount of coordinate functions in the Galerkin method $n = 5$; number of iterations $n_{iter} = 5$; $E = 1.9 \cdot 10^6 \frac{kg}{sm^2}$; $\Delta = 7.874 \frac{g}{sm^3}$; $\Delta = 0.01m$; $A = 0.01m^2$; $I = datv = 1000Pa$; $\eta = 0$; 1.9; $1.9 \cdot 10^1$; $1.9 \cdot 10^2; \cdots; 1.9 \cdot 10^6$; $1.9 \cdot 10^7$;

$\alpha = 0.24613 \cdot 10^6 \cdot datv$; $\beta = 241.3$; $\gamma = 0.12954 \cdot datv \cdot \eta$; $\kappa = 12065$; $\sigma = 0.0127 \cdot \eta$; $\delta = 0$. The initial functions $u^0(x) = \sin(\frac{\pi x}{L})$, $u^1(x) = 0$, the right-hand side function $f(x,t) = 0$.

The numerical computations of bending function of the beam $u(x,t)$ is presented in Table 1 for the several meanings of the following effective viscosity η: Case 1 ($\eta = 0$), Case 2 ($\eta = 1.9 \cdot 10^4$), Case 3 ($\eta = 1.9 \cdot 10^5$).

$t \setminus x$	Case	$x = 0$	$x = 0.2$	$x = 0.4$	$x = 0.6$	$x = 0.8$	$x = 1$
t=0	1	0	0.587785	0.951057	0.951057	0.587785	0
t=0	2	0	0.587785	0.951057	0.951057	0.587785	0
t=0	3	0	0.587785	0.951057	0.951057	0.587785	0
t=0.2	1	0	0.587785	0.951057	0.951057	0.587785	0
t=0.2	2	0	0.587785	0.951057	0.951057	0.587785	0
t=0.2	3	0	0.587785	0.951057	0.951057	0.587785	0
t=0.4	1	0	1.763356	2.853170	2.853170	1.763356	0
t=0.4	2	0	1.549623	2.507342	2.507342	0.587785	0
t=0.4	3	0	0.587795	0.951072	0.951072	0.587795	0
t=0.6	1	0	2.938926	4.755283	4.755283	2.938926	0
t=0.6	2	0	2.336585	3.780675	3.780675	2.336585	0
t=0.6	3	0	0.587795	0.951072	0.951072	0.587795	0
t=0.8	1	0	4.114497	6.657340	6.657340	4.114497	0
t=0.8	2	0	2.980467	4.822497	4.822497	2.980467	0
t=0.8	3	0	0.587795	0.951072	0.951072	0.587795	0
t=1	1	0	5.290067	8.559551	8.559551	5.290067	0
t=1	2	0	3.507282	5.674901	5.674901	3.507282	0
t=1	3	0	0.587795	0.951072	0.951072	0.587795	0

Table 1.

Conclusion

As numerical experiments demonstrate, once the coefficients of effective viscosity η increase, concomitantly, the numerical values of displacement functions (curvature) $u(x,t)$ decrease for specific values of x and t. However, for every specific values of η, the numerical values of the displacement functions for specific x increase with the increasement of time t. The numerical values of the displacement functions with respect to temporal variable t are symmetrical to the midpoint of the beam at $x = L/2$.

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Received 15.05.2022; revised 22.07.2022; accepted 16.09.2022.

Author(s) address(es):

Archil Papukashvili
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University
School of Science and Technology the University of Georgia
University str. 2, 0186 Tbilisi, Georgia
E-mail: archil.papukashvili@tsu.ge, apapukashvili@rambler.ru

Giorgi Geladze
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University
University str. 2, 0186 Tbilisi, Georgia
E-mail: giorgi.geladze@tsu.ge

Zurab Vashakidze
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University,
School of Science and Technology the University of Georgia
University str. 2, 0186 Tbilisi, Georgia
E-mail: zurab.vashakidze@gmail.com

Meri Sharikadze
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University
University str. 2, 0186 Tbilisi, Georgia
E-mail: meri.sharikadze@tsu.ge