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ANALYTIC REPRESENTATION OF THE VV, SV AND SSB CUTTING SURFACES FOR GENERALIZED MÖBIUS-LISTING'S BODIES *

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Abstract. We will present the method and scheme of analytical representation of the corresponding cutting surfaces of any VV (vertex-vertex), VS (vertex-side) and SSB (side-side-center of symmetry), for Generalized Möbius-Listing's bodies GML_m^n which radial cross section is a regular convex polygon with m-angle.

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• **Notations and abbreviations.** In this article we use the following notations:

- X, Y, Z, t is the ordinary notation for space and time coordinates;
- $\widetilde{x}, \widetilde{z}, \theta$ are local coordinates in a prism with a regular polygon $(m-gon)$ section, where:

$$\begin{aligned} &1.\ (\widetilde{x}, \widetilde{z}) \in [m-gon] \textit{ and are classic Deckart coordinates}, \\ &2.\ \theta \in [0, 2\pi h] \text{ where } \ h \in \mathbf{R}\ (Real); \end{aligned} \tag{1}$$

Further we will use the notation accepted in earlier works [1-3]. In particular, relying on formulas (2) and (6*) in [1], the analytical representation of the GML_m^n body is given below by the following formula:

$$\begin{aligned} X(\widetilde{x}, \widetilde{z}, \theta) &= [R + p(\widetilde{x}, \widetilde{z}, \theta)\cos(\tfrac{n\theta}{m})]\cos(\theta) \\ Y(\widetilde{x}, \widetilde{z}, \theta) &= [R + p(\widetilde{x}, \widetilde{z}, \theta)\cos(\tfrac{n\theta}{m})]\sin(\theta) \\ Z(\widetilde{x}, \widetilde{z}, \theta) &= Q(\theta) + p(\widetilde{x}, \widetilde{z}, \theta)\cos(\tfrac{n\theta}{m}) \end{aligned} \tag{2}$$

In this article, the contour of a polygon, or the radial cross section of a GML_m^n body, is defined by a system of straight lines for all natural m, where according to [1-2] m number of vertexes and $n \in \mathbf{Z}$ is a number of rotation around of $\widetilde{o}$-trace os basic line before gluing the ends of prism (1):

$$\begin{gathered} p(\widetilde{x}, \widetilde{z}, \theta) = \widetilde{x}_i \cos(\alpha + \tfrac{2i\pi}{m}) + \widetilde{z}\sin(\alpha + \tfrac{2i\pi}{m}) - \tfrac{\mu}{2\sin(\frac{\pi}{m})}\cos(\alpha + \tfrac{2i\pi}{m}) \\ i = 0, 1, ..., m-1 \end{gathered} \tag{3}$$

where $\alpha = \pi - \widetilde{\alpha}$ and $\widetilde{\alpha}$ is a real angle between axis $\widetilde{x}$ and straight line, side of the polygon, whose center of symmetry coincides with the origin of the local coordinate system $\widetilde{x}, \widetilde{z}, \widetilde{o}$. In this article, without loss of generality, $Q(\theta) \equiv 0$.

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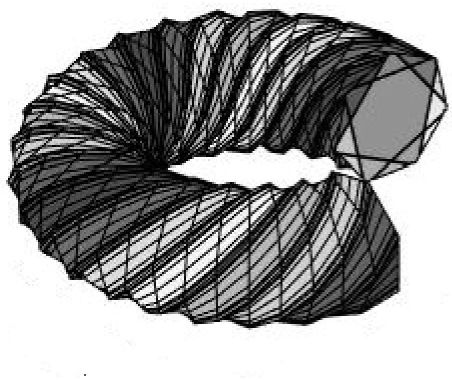

Figure 1: Example of the GML_m^n body with V_1V_3 cutting, where $m = 7$ and $n = 16$

Remark 1. if we consider equation (2) with substitution (3) or (4) we get the surface of the GML_m^n body with a radial cross section - a regular polygon (Side - see Definition 4 in [4]). Its baseline (see Definition 2 in [4]) always, i.e. for all meaning of argument θ coincides with the center of symmetry of that polygon.

According to (Definition 8) in [4], the process of cutting the GML_m^n body occurs along the surfaces of the radial cross section of which are also straight lines. And this enables us to describe the analytical "chordal" or "diametral" knives [3]. These knives can be viewed as different lines on a given polygon, this is the essence of the method. The scheme of the method implementation implies: classification and then establishment of the characteristic parameters of these lines

Remark 2. Given the symmetry of the polygon, these lines are six different types:

1. VV (vertex by vertex, different diagonals);
2. VVB (vertex by vertex and center of symmetry, one diagonal when m is even number);
3. VS (vertex by side);
4. VSB (vertex by side and center of symmetry, orthogonal from the vertex to the opposite side, exist only when m is odd number);
5. SS (Side by side);
6. SSB (Side by side and center of symmetry).

Considering the difference between flat polygons with even and odd vertices, for the sake of completeness, we must describe 12 different cases.

Remark 3. To describe these lines, it is enough to calculate the points intersections with the local axis and set the angles (or the range in which these angles change). This article provides answers to 10 out of 12 cases.

For simplicity, without limiting generality, we will assume that the vertex of polygon number one V_1 lies on the local $\widetilde{x}$ axis (see Fig 2).

Case 1a. VV**-cuts** when $m = 2k+1$ is an odd number. In this case we have $k-1$ different diagonals of polygon V_1V_i and $i = 3, ..., k+1$ and real angles between $\widetilde{ox} \equiv \widetilde{o}V_1$ and cutting line V_1V_i equals to the $\beta_i \equiv \frac{(m-2i+2)\pi}{2m}$. The point of intersection of these lines with the local $\widetilde{ox}$ axis always coincides with the vertex number one i.e. V_1 of the polygon and has coordinates $(\widetilde{x}, \widetilde{z}) = (\frac{\mu}{2\sin(\frac{\pi}{m})}, 0)$, where μ is length of the side of polygon.(see examples red lines in Fig.2 a.)

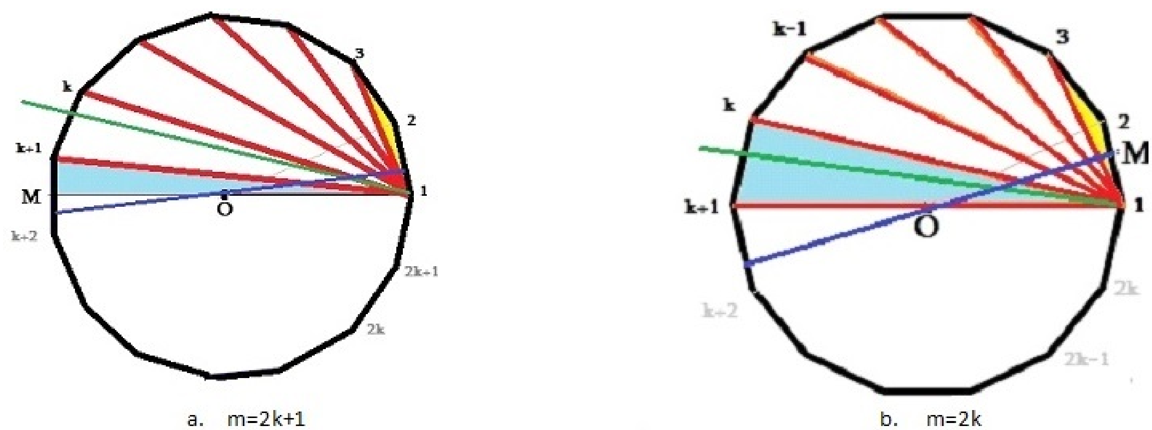

Figure 2: Examples of polygons with different number of the vertexes. Red lines are different V_1V_i diagonals or traces of VV knives; Green lines are examples of the V_1S_k or traces of "chordal nife" VS and Blue lines are some examples of the $S_1S_{k+1}B$ or traces of "diametral knives" SSB.

Case 1b. VV**-cuts** when $m = 2k$ is even number. In this case we have $k-1$ different diagonals of polygon V_1V_i and $i = 3, ..., k+1$ and real angles between $\widetilde{ox} \equiv \widetilde{o}V_1 \equiv V_1V_{k+1}$ and cutting line V_1V_i equals to the $\gamma_i \equiv \frac{(k+1-i)\pi}{m}$. The point of intersection of these lines with the local $\widetilde{ox}$ axis always coincides with the vertex number one i.e. V_1 of the polygon and has coordinates $(\widetilde{x}, \widetilde{z}) = (\frac{\mu}{2\sin(\frac{\pi}{m})}, 0)$,where μ is length of the side of polygon.(see examples red lines in Fig.2 b.)

Case 2a. VVB**-cuts** when $m = 2k+1$ is an odd number. Such a diagonal does not exist!

Case 2b. VVB**-cuts** when $m = 2k$ is an even number. In this case there is only one diagonal V_1V_{r+1} and, accordingly, the angle equal to 0. (see line V_1V_{k+1} in Fig.2 b.).

Case 3a. VS**-cuts** when $m = 2k+1$ is an odd number. In this case we have k fundamentally different chordal knives V_1S_i where side, i.e. straight line, of the polygon $S_i \equiv [V_iV_{i+1}]$ for $i = 2, ..., k+1$. Real angles between $\widetilde{ox} \equiv \widetilde{o}V_1$ and cutting line V_1S_i is $\beta_i \equiv \widetilde{\beta} + \frac{(m-2i)\pi}{2m}$ where $\widetilde{\beta}$ varies in the interval $(0, \frac{\pi}{m})$, for all $i = 2, ..., k$. When we have chordal knife $V_1S_k + 1$ the real angles varies - $0 < \beta_{k+1} < \frac{\pi}{2m}$. The point of intersection of these lines with the local $\widetilde{ox}$ axis always coincides with the vertex number one i.e. V_1 of the polygon and has coordinates $(\widetilde{x}, \widetilde{z}) = (\frac{\mu}{2\sin(\frac{\pi}{m})}, 0)$, where μ is length of the side of polygon. (see example green line in Fig.2 a.)

Case 3b. VS-**cuts** when $m = 2k$ is an even number. In this case we have $k-1$ fundamentally different chordal knives V_1S_i where side $S_i \equiv [V_iV_{i+1}]$ and $i = 2, ..., k$ and real angles between $\widetilde{ox} \equiv \widetilde{o}V_1$ and cutting line V_1S_i is $\gamma_i \equiv \widetilde{\gamma} + \frac{(k-i)\pi}{m}$ where $\widetilde{\gamma}$ varies in the interval $(0, \frac{\pi}{m})$. The point of intersection of these lines with the local $\widetilde{ox}$ axis always coincides with the vertex number one i.e. V_1 of the polygon and has coordinates $(\widetilde{x}, \widetilde{z}) = (\frac{\mu}{2\sin(\frac{\pi}{m})}, 0)$,where μ is length of the side of polygon.$(\widetilde{x}, \widetilde{z}) = (\frac{\mu}{2\sin(\frac{\pi}{m})}, 0)$,where μ is length of the side of polygon.(see example green line in Fig.2 b.)

Case 4a. VSB-**cuts** when $m = 2k+1$ is an odd number. in this case these exist only one line V_1S_{k+1} and $\beta_{k+1} \equiv 0$ (see line V_1M where M is middle point of S_{k+1} Fig.2 a.).

Case 4b. VSB-**cuts** when $m = 2k$ is an even number. Such line does not exist!

Case 6a. and b. SSB-**cuts** for all m there are many lines, but they all pass through the center of symmetry of the polygon and S_1, i.e. through the origin of local coordinates $\widetilde{o}$. The rest of the lines are obtained by a simple turn around $\widetilde{o}$, therefore, for our purposes, they are not carriers of distinctive information. So the angle of inclination changes in the interval $(0, \frac{2\pi}{m}$.(see examples blue lines in Figures 2.)

Unfortunately, until now, it has not been possible to describe the analytic, all different cases of the SS cuts. We will try to analytically describe all sorts of different S_1S_i lines for the next seminar.

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