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SPACE DIMENSION RENORMDYNAMICS AND CONFINING POTENTIALS FROM HADRONS TO GALAXIES AND PERIODIC STRUCTURE OF THE UNIVERSE

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Abstract. The fundamental Planck's [1] and Stoney's [2] constants are derived and compared. Scale dependent space dimension and potential for the quarkonium model of hadrons [3] are extended to the modified Newton potential for galaxies. With this potential, the periodic structure of the Universe [4, 5] is explained. For the quarkonium in the quark-gluon matter the string strength parameter dependence on the temperature is obtained.

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We say that we find **New Physics** (NP) when either we find a phenomenon which is forbidden by SM in principal - this is the qualitative level of NP - or we find a significant deviation between precision calculations in SM of an observable quantity and a corresponding experimental value. In the Universe, matter has manly two geometric structures, homogeneous, and hierarchical. The homogeneous structures are naturally described by real numbers with an infinite number of digits in the fractional part and usual archimedean metrics. The hierarchical structures are described with p-adic numbers with an infinite number of digits in the integer part and non-archimedean metrics. A discrete, finite, regularized, version of the homogenous structures are homogeneous lattices with constant steps and distance rising as arithmetic progression. The discrete version of the hierarchical structures is hierarchical lattice-tree with scale rising in geometric progression.

In the 1870's G.J. Stoney [2], the physicist who coined the term "electron" and measured the value of elementary charge e, introduced as universal units of Nature for L, T, M :

$$l_s = \frac{e}{c^2}\sqrt{G} = 1.381 \times 10^{-36} m,\ t_s = \frac{e}{c^3}\sqrt{G} = 4.605 \times 10^{-45} s,\ m_s = \frac{e}{\sqrt{G}} = 1.860 \times 10^{-9} kg \quad (1)$$

When M. Planck discovered in 1899 h he introduced [1] as universal units of Nature for L, T, M:

$$l_p = \frac{h}{cm_p} = \frac{l_s}{\sqrt{\alpha}} = 11.7 l_s,\ t_p = \frac{l_p}{c} = \frac{t_s}{\sqrt{\alpha}},\ m_p = \sqrt{\frac{hc}{G}} = \frac{m_s}{\sqrt{\alpha}}, \quad (2)$$

where α is the fine-structure constant. In Stoney's units, the numerical value of the reduced Planck's constant is $\hbar = h/(2\pi) = \alpha^{-1} = 137.036$

Let us derive Stoney's units using Newton and Coulomb laws and Einstein's formula

$$V_n = G\frac{m_s^2}{l_s} = V_c = \frac{e^2}{l_s} \Rightarrow m_s = \frac{e}{\sqrt{G}},$$
$$m_s c^2 = \frac{e^2}{l_s} \Rightarrow l_s = \frac{e^2}{m_s c^2} = \frac{e}{c^2}\sqrt{G},\ t_s = \frac{l_s}{c} = \frac{e}{c^3}\sqrt{G} \quad (3)$$

Using Planck's formula $E = h\nu$ we derive Planck's units,

$$V_n = G\frac{m_p^2}{l_p} = \frac{h}{t_p} = \frac{hc}{l_p} \Rightarrow m_p = \sqrt{\frac{hc}{G}},$$
$$m_p c^2 = \frac{hc}{l_p} \Rightarrow l_p = \frac{h}{cm_p} = \sqrt{\frac{hG}{c^3}},\ t_p = \frac{l_p}{c} = \sqrt{\frac{hG}{c^5}} \quad (4)$$

Note that

$$m_p c^2 - \frac{e^2}{l_s} = 0;\ G\frac{m_s^2}{l_s} - \frac{e^2}{l_s} = 0,$$
$$m_p^2 = 137 m_s^2,\ l_p^2 = 137 l_s^2,\ t_p^2 = 137 t_s^2 \quad (5)$$

So, planbrane=137stonbrane; Planck's constant is derivable from elementary charge and light velocity:

$$h = \frac{e^2}{c\alpha} \quad (6)$$

Stoney's fundamental constants are more fundamental just because they are less than Planck's constants :) Due to the value of $\alpha^{-1} = 137$, we can consider relativity theory and quantum mechanics as deformations of the classical mechanics when deformation parameter $c = 137$ (in units $e = 1, \hbar = 1$) and $\hbar = 137$ (in units $e = 1, c = 1$), correspondingly. These deformations have an analytic sense of p-adic convergent series. The number 137 has a very interesting geometric sense,

$$137 = 11^2 + 4^2, \quad (7)$$

so, $\sqrt{137}$ is the hypotenuse length of a triangle with other sides of lengths 11 and 4.

The Babylonians used a base 60 number system which is still used for measuring time - 60 seconds in a minute, 60 minutes in an hour - and for measuring angle - 360 degrees in a full turn. The base 60 number system has its origin in the ration of the Sumerian mina (m) and Akkadian shekel (s), $m/s \simeq 60 = 3 \cdot 4 \cdot 5$. We also can consider base 137 system for fundamental theories. For the nuclear physics strong coupling phenomena description we may take as a base $p = 13$. For the hadronic physics, valence scale QCD, and graphen strong coupling phenomena description we may take as a base $p = 2$. For the weak coupling physics SM m_Z scale and MSSM unification scale phenomena description we may take as a base $p = 29$.

There are different opinions about the number of fundamental constants [6]. According to Okun, there are three fundamental dimensionful constants in Nature: Planck's constant, $\hbar$; the velocity of light, c; and Newton's constant, G. According to Veneziano, there are only two: the string length L_s and c. According to Duff, there are not fundamental constants at all.

Usually $L_s = l_p$, so, the fundamental area is $L_s^2 = 137 l_s^2 = |4l_s + i11l_s|^2$. If $z_1 = 4 + i11,\ z_2 = 11+i4,\ |z_1 - z_2| = \sqrt{7^2+7^2} = \sqrt{98} = \sqrt{100-2} = 10(1-1/100+O(10^{-4})) = 10-1/10+O(10^{-3})$. The vertices $z_n = \pm 4 \pm i11$ and $\pm 11 \pm i4$ on the complex plane form an octagon with sides of length 8 and almost 10. If we cover the surface with such octagon we obtain figures of size 10 before the correction becomes of size $1l_s$. Note that $l_p = 11.7l_s$ This hints about microscopic origin - structure of quantum theory. The value $s_s = l_s^2 -$ Stoney area - stonbrane, is more like on a fundamental area :)

Strongly interacting matter of sufficiently high density undergo a transition to a state of deconfined quarks and gluons. Deconfinement occurs when color screening shields a given quark from the binding potential of any other quarks or antiquarks. Bound states of very heavy quarks have radii which are much smaller than those of the usual mesons and nucleons; hence they can survive in a deconfined medium until the temperature or density becomes so high that screening also prevents their tighter binding. Color screening and deconfinement for heavy quark resonances are therefore crucial for the experimental investigation of quark plasma formation.

For the study of the deconfinement of heavy quarks at some transition temperature, the following potential was considered [7]

$$\begin{aligned} V(r,T) &= \frac{\sigma_0}{\mu}(1-\exp(-\mu r)) - \frac{\alpha}{r}\exp(-\mu r) \\ &= \sigma r - \frac{\alpha}{r} + ...,\ \sigma = \sigma_0 - \alpha\mu^2/2. \end{aligned} \tag{8}$$

where $\mu = 1/\lambda(T)$ is the inverse of the Debye screening length λ, and the parameter α stands for the effective running coupling.

To apply these considerations to actual physical situations, we need to know the specific dependence of $\mu(T)$ on T. If nuclear collisions produce strongly interacting matter, then it is the temperature, not $\mu(T)$, which can be empirically determined. At $T = 0$, we have $\mu(0)$=0 only in a world without light quarks. In the presence of light quarks, the binding of any quark-antiquark system is broken when its binding energy exceeds that needed for the spontaneous creation of a $q\bar{q}$ state out of the vacuum. Hence $\mu(0) \neq 0$. The corresponding vacuum screening length is of the order of one fermi. At the critical temperature T_c of deconfinement string tension $\sigma(T_c) = 0$. Knowing $T_c,\ \alpha(T)$ and $\mu(T)$ we can find

$$\sigma_0 = \alpha(T_c)\mu(T_c)^2/2. \tag{9}$$

We may consider the string tension as deconfinement phase transition order parameter.

The Schwarzschild line element for realistic (small) values of the cosmological constant is [8]

$$ds^2 = \Big(1 + \Lambda r^3/3 - \frac{2G_N M}{r}\Big)dt^2 - \Big(1 + \Lambda r^3/3 - \frac{2G_N M}{r}\Big)^{-1} dr^2 + r^2 d\Omega^2 \tag{10}$$

we see that Newton's potential is modified by confining potential of the same form as in the QCD case [4]. As in QCD, we may take as a modified gravitational potential

$$\begin{aligned} V &= -\frac{G_N M \cos\mu r \cosh\mu r}{r} = -\frac{G_N M\mu \cos x \cosh x}{x} \\ &= G_N M\mu\left(-\frac{1}{x} + \frac{x^3}{6} + ...\right) = -\frac{G_N M}{r} + \Lambda r^3/6 + ...,\ \Lambda = G_N M \mu^4 \end{aligned} \tag{11}$$

Observations of the large scale structure of the universe suggest inhomogeneities on scales between $100h^{-1}$ and $150h^{-1}$ Mpc (where $h = 0.5 - 1$ is the Hubble constant in units of 100 $kms^{-1}Mpc^{-1}; 1pc = 3.09 \times 10^{16} m$). A deep redshift survey with a"pencil-beam"geometry of galaxies at the galactic poles indicated strong clustering,with a provocative regularity at $128h^{-1}$ Mpc [4].

In our potential period is $l = 2\pi/\mu$. If this period is equal to the large scale period, we define $\mu = 2\pi/l = 0.32heV$. We estimated the number of periods in the visible Universe as 24 [5] which coincides with the effective number of degrees of freedom of the fundamental bosonic string. This hints on that the large scale periodic structure is the relict of early time when the string model acts.

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