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NUMERICAL ANALYSIS OF NON LINEAR DEFORMATION OF CORRUGATED LAYERED CYLINDRICAL SHELL BY SURFACE FORCE AND TEMPERATURE FIELD INFLUENCE BASED ON DISTINCT THEORIES *

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Abstract. We consider numerical analysis of non linear deformation of corrugated layered cylindrical shell produced by acting of surface force and temperature field based on distinct theories. The considered theories are constructed on the basis of the broken lines hypothesis in either cases of linear or non-linear deformations. A particular example of such a deformation is given. In this example we consider the deformation of the corrugated three layered cylindrical shell with fixed ends produced by acting on it of normal surface force and temperature field. On the basis of distinct theories the numerical realization of this example is given. The comparison of obtained results give the possibility to estimate the process of the deformation.

Keywords and phrases: Layered shell, non-linear deformation, non-uniformity of in-plane shear deformation.

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In this work we consider the class of layered shells that are composed from the layers having substantially different mechanical properties. The study of mode of deformation of such class shells is desirable by application of one variant of the adjusted theory that is constructed on the base of the broken line hypothesis. The essence of the broken line hypothesis is as follows: the element of the shell arranged on the normal surface of the coordinate surface after deformation passes to the line, which gives the possibility of taking into account the heterogeneity of deformation of in-plane shear along the layered shell thickness.

Based on the proposed improved theory, the mode of deformation of orthotropic layered shell is considered in the case of impact of acting on it surface forces and temperature filed. It is assumed that the deformation of the shells is elastic, i.e. connections between deformations and forces in each I layer are described by Hooke's law, based on the Duhamel-Neumann hypothesis [1], which has the following form in the α, β, γ curvilinear coordinate system:

$$\sigma_\alpha^i = B_{11}^i \varepsilon_{\alpha\alpha}^{(\gamma)} + B_{12}^i \varepsilon_{\beta\beta}^{(\gamma)} - \beta_1^i T,$$

$$\sigma_\beta^i = B_{21}^i \varepsilon_{\alpha\alpha}^{(\gamma)} + B_{22}^i \varepsilon_{\beta\beta}^{(\gamma)} - \beta_2^i T,$$

$$\tau_{\beta\gamma}^i = B_{44}^i \varepsilon_{\beta\gamma}^{(\gamma)}, \quad \tau_{\alpha\gamma}^i = B_{55}^i \varepsilon_{\alpha\gamma}^{(\gamma)}, \quad \tau_{\alpha\beta}^i = B_{66}^i \varepsilon_{\alpha\beta}^{(\gamma)},$$

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where $T(\alpha, \beta, \gamma)$ is the temperature field.

Let us present in the case of the improved theory the basic equations and ratios of orthotropic layered shells. The tangential displacements have the following form [2,3]:

$$\begin{aligned} u_\alpha^{(i)} &= u + a_1^{(i)}\gamma_\alpha^{(0)} + \gamma(\psi_\alpha - a_2^{(i)}\gamma_\alpha^{(0)}), \\ u_\beta^{(i)} &= v + b_1^{(i)}\gamma_\beta^{(0)} + \gamma(\psi_\beta - b_2^{(i)}\gamma_\beta^{(0)}), \end{aligned} \tag{1}$$

where u, v are the tangential displacement of coordinate surface, ψ_α, ψ_β are the angles of full rotation on normal of coordinate surface, $\gamma_\alpha^{(0)}$, $\gamma_\beta^{(0)}$ represent the in-plane deformations the shear, that passes the coordinate surface. The formulas for calculation of coefficients a_1^i, a_2^i, b_1^i, b_2^i are given [3]. Taking into account the tangential displacements (2) the components of deformation will be presented as follows:.

$$\begin{aligned} &\varepsilon_{\alpha\alpha}^{(\gamma)} = \varepsilon_{\alpha\alpha}^{(i)} + \gamma\varkappa_{\alpha\alpha}^{(i)}, \quad \varepsilon_{\alpha\beta}^{(\gamma)} = \varepsilon_{\alpha\beta}^{(i)} + 2\gamma\varkappa_{\alpha\beta}^{(i)}, \\ &\varepsilon_{\beta\beta}^{(\gamma)} = \varepsilon_{\beta\beta}^{(i)} + \gamma\varkappa_{\beta\beta}^{(i)}, \quad \varepsilon_{\alpha\gamma}^{(\gamma)} = \gamma_\alpha^{(i)}, \\ &\varepsilon_{\gamma\gamma}^{(\gamma)} = 0, \qquad\qquad\quad\;\; \varepsilon_{\beta\gamma}^{(\gamma)} = \gamma_\beta^{(i)}. \end{aligned} \tag{2}$$

Expressions for $\varepsilon_{\alpha\alpha}^{(i)}$, $\varepsilon_{\beta\beta}^{(i)}$,..., $\varkappa_{\beta\beta}^{(i)}$ from (2) can be found in the works [2, 3, 4]. The equilibrium equations of element of the layered shell are as follows:

$$\begin{aligned} &\frac{\partial BN_\alpha}{\partial\alpha} + \frac{\partial AN_{\beta\alpha}}{\partial\beta} + \frac{\partial A}{\partial\beta}N_{\alpha\beta} - \frac{\partial B}{\partial\alpha}N_\beta + ABk_1Q_\alpha^* + ABq_1 = 0, \\ &\frac{\partial AN_\beta}{\partial\beta} + \frac{\partial BN_{\alpha\beta}}{\partial\alpha} + \frac{\partial B}{\partial\alpha}N_{\beta\alpha} - \frac{\partial A}{\partial\beta}N_\alpha + ABk_2Q_\beta^* + ABq_2 = 0, \\ &\frac{\partial BQ_\alpha^*}{\partial\alpha} + \frac{\partial AQ_\beta^*}{\partial\beta} - ABk_1N_\alpha - ABk_2N_\beta + ABq_3 = 0, \\ &\frac{\partial BM_\alpha}{\partial\alpha} + \frac{\partial AM_{\beta\alpha}}{\partial\beta} + \frac{\partial A}{\partial\beta}M_{\alpha\beta} - \frac{\partial B}{\partial\alpha}M_\beta - ABQ_\alpha = 0, \\ &\frac{\partial AM_\beta}{\partial\beta} + \frac{\partial AM_{\alpha\beta}}{\partial\alpha} + \frac{\partial B}{\partial\beta}M_{\beta\alpha} - \frac{\partial A}{\partial\beta}M_\alpha - ABQ_\beta = 0, \end{aligned}$$

where

$$\begin{aligned} Q_\alpha^* &= Q_\alpha - (N_\alpha + k_1M_\alpha)\theta_\alpha - (N_{\alpha\beta} + k_1M_{\alpha\beta})\theta_\beta, \\ Q_\beta^* &= Q_\beta - (N_{\beta\alpha} + k_2M_{\beta\alpha})\theta_\alpha - (N_\beta + k_2M_\beta)\theta_\beta. \end{aligned}$$

Furthermore, the tasks of axis symmetric deformation of the rotation layered shells will be considered in the case of impact of acting on it of surface forces and temperature field. In order to study the tasks of this class, let's introduce on the coordinate surface the system of curvilinear coordinates S, θ, where S is the arc length of the coordinate surface of the meridian from the parallel circle and θ represents the central angle in the parallel circle of the coordinate surface from the selected plane.

From basic equations and relations given in this work we obtain for the solution of the above mentioned class of problems the system of nonlinear differential equations in the curvilinear coordinate system S, θ (see [5]).

For the study of the corrugated layered cylindrical shell let us restrict our self with the consideration of the deformation of one corrugation. As a particular example is considered the deformation of the shell with fixed ends by acting normal surface force and temperature field. During the solution of the problem it is assumed that the coordinate surface crosses the middle of the layer.

The parametric equation of the meridian of coordinate surface has the form

$$r = R - \alpha \cos \frac{2\pi}{L} z, \quad z = z, \quad (0 \leq z \leq L),$$

Where r is the distance from rotation axis to the coordinate surface, while L is the length-period of corrugation. Let's designate accordingly by h_1, h_2, h_3 the outer, middle and internal layers of shell; by E_1^i, E_2^i the elasticity modulus of i-th $(i = 1, 2, 3)$ layer of shell; v_{12}^i, v_{21}^i are the Poisson coefficients; G_{13}^i–are the modulus of in-plane shear; α_{1T}^i, α_{2T}^i are the coefficient of thermal expansion.

The problem is solved for the following values of listed variables:

$$R = 20; \quad \alpha = 1; \quad L = 24; \quad h_1 = 0.5; \quad h_2 = 2; \quad h_3 = 0.5; \quad E_1^1 = 1.5 \cdot 10^5;$$
$$E_2^1 = 3 \cdot 10^5; \quad E_1^2 = 2 \cdot 10^4; \quad E_2^2 = 3 \cdot 10^4; \quad E_1^3 = 1.5 \cdot 10^5; \quad E_2^3 = 3 \cdot 10^5; \quad \nu_{12}^1 = 0.2,$$
$$\nu_{21}^1 = 0.34; \quad \nu_{12}^2 = 0.1; \quad \nu_{21}^2 = 0.15; \quad \nu_{12}^3 = 0.2; \quad \nu_{21}^3 = 0.34; \quad G_{13}^1 = 0.15 \cdot 10^5;$$
$$G_1 3^2 = 0.15 \cdot 10^4; \quad G_1 3^3 = 0.35 \cdot 10^5; \quad \alpha_{1T}^1 = 0.8 \cdot 10^{-5}; \quad \alpha_{2T}^1 = 1.2 \cdot 10^{-5};$$
$$\alpha_{1T}^2 = 0.5 \cdot 10^{-3}; \quad \alpha_{2T}^2 = 1.2 \cdot 10^{-3}; \quad \alpha_{1T}^3 = 0.7 \cdot 10^{-5}; \quad \alpha_{2T}^3 = 0.16 \cdot 10^{-5}.$$

The following table gives the obtained values of solution of the problem for function w, for $q_3 = 50$ and for the given values T.

z	$q_3 = 50$							
	W							
	T=0		T=50		T=100		T=150	
	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	1.1894	1.1372	2.0601	1.9679	2.9308	2.7737	3.8015	3.5443
8	1.2609	1.1967	2.2304	2.1032	3.1999	2.9787	4.1694	3.8194
12	0.3147	0.2983	0.8240	0.7584	1.3332	1.1993	1.8425	1.6274
16	1.2562	1.1926	2.2236	2.0971	3.1911	2.9708	4.1585	3.8099
20	1.1942	1.1415	2.0673	1.9744	2.9405	2.7823	3.8137	3.5546
24	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

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