Reports of Enlarged Sessions of the
Seminar of I. Vekua Institute
of Applied Mathematics
Volume 34, 2020

STUDY OF THE BASIC BOUNDARY VALUE PROBLEMS OF STATIONARY OSCILLATIONS OF THE THEORY OF THERMOELASTICITY WITH MICROTEMPERATURES AND MICROROTATION BY THE POTENTIAL METHOD

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Abstract. In this paper, the basic boundary value problems of stationary oscillations for thermoelastic isotropic microstretch materials with microtemperatures and microrotation, are studied. To this end, there are constructed the matrix of fundamental solutions of the corresponding system of partial differential equations, and the mapping properties of the corresponding volume and layer potential operators are investigated. With the help of the potential method, the basic boundary value problems are reduced to the corresponding system of singular integral equations, and the existence theorems of solutions are proved.

Keywords and phrases: Microstretch materials, potential method, microtemperatures.

AMS subject classification (2010): 74A15, 74B10, 74F20.

1 Introduction. The main goal of our investigation is analysis of the basic boundary value problems for the pseudo-oscillation equations of the theory of thermoelasticity for isotropic materials with microstructure, whose microelements possess microtemperatures.

A theory of thermoelasticity with microtemperatures, in which the microelements can stretch and contract independently of their translations has been studied by Ieşan [1]. This is the simplest thermomechanical theory of elastic bodies that takes into account the microtemperatures and the inner structure of the materials. This model has been investigate by various authors (see e.g.,[2], [3], [4], [5], [6]).

2 Basic equations and boundary value problems. The system of homogeneous differential equations of the stationary oscilation of the thermoelasticity theory of microstretch materials with microtemperatures and microdilatation for isotropic bodies [6] in the two-dimensional case written in the form

$$((\mu+\varkappa)\Delta+\rho\sigma^2)u+(\lambda+\mu)\,grad\,div\,u-\varkappa R^\top(\partial)\omega+\mu_0\,grad\,v-\beta_0\,grad\,\theta=0, \quad (1)$$

$$(\varkappa_6\Delta+\varkappa_0)w+(\varkappa_4+\varkappa_5)\,grad\,div\,w+i\sigma\mu_1 R^\top(\partial)\omega+$$
$$+i\sigma\mu_2\,grad\,v-\varkappa_3\,grad\,\theta=0, \quad (2)$$

$$(\gamma\Delta+\delta)\omega+\varkappa R(\partial)u-\mu_1 R(\partial)w=0, \quad (3)$$

$$(a_0\Delta+\eta_0)v-\mu_0\,div\,u-\mu_2\,div\,w+\beta_1\theta=0, \quad (4)$$

$$(\varkappa_7\Delta+i\sigma c)\theta+i\beta_0 T_0\sigma\,div\,u+\varkappa_1\,div\,w+i\beta_1 T_0\sigma v=0, \quad (5)$$

where Δ is the two-dimensional Laplace operator, $u=(u_1,u_2)^\top$ is a displacement vector, $w=(w_1,w_2)^\top$ is a microtemperature vector, ω is the microrotation function, v is the microdilatation function, θ is the temperature, measured from a fixed absolute temperature T_0 $(T_0>0)$, σ is a friquency parameter,

$$\sigma=\sigma_1+i\sigma_2,\ \sigma_2>0,\ \sigma_1\in R,\ \delta=I_1\sigma^2-2\varkappa,\ \varkappa_0=i\sigma b-\varkappa_2,\ \eta_0=I\sigma^2-\eta,\ c=a\,T_0,$$

$\lambda,\ \mu,\ \varkappa,\ \eta,\ \beta_0,\ \beta_1,\ \mu_0,\ \mu_1,\ \mu_2,a_0,b,I,I_1,\ \varkappa_j,\ j=1,2,\cdots,7$ are the real constants characterizing the mechanical and thermal properties of the body, ρ is the mass density, the superscript $(\cdot)^\top$ denotes transposition operation,

$$R(\partial)u=\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2},\quad R(\partial)w=\frac{\partial w_2}{\partial x_1}-\frac{\partial w_1}{\partial x_2}.$$

Let $\Omega^+\subset R^2$ be a bounded domain with boundary $\partial\Omega$. We denote $\Omega^-=R^2\setminus\overline{\Omega}^+$.

Problem. Find in the domain Ω^+ (Ω^-) such a regular vector $U=(u,w,\omega,v,\theta)^\top\subset C^2(\Omega^\pm)\cap C^1(\overline{\Omega}^\pm)$ that satisfies in the domain the system of differential equations (1)-(5), and on the boundary $\partial\Omega$, satisfies one of the following boundary conditions

$(I)^\pm$ **(The Dirichlet problem)**

$$\{U(z)\}^\pm=f(z),\quad z\in\partial\Omega,$$

$(II)^\pm$ **(The Neumann problem)**

$$\{P(\partial,n)U(z)\}^\pm=f(z),\quad z\in\partial\Omega,$$

where $f=(f^{(1)},f^{(2)},f_3,f_4,f_5)^\top$, $f^{(j)}=(f_1^{(j)},f_2^{j})^\top$ j=1,2, are given vector-functions and $f_l,\ l=3,4,5$ are given functions on the boundary $\partial\Omega$, $n(z)$ is the outward normal unit vector passing at a point $z\in\partial\Omega$ with respect to the domain Ω^+, and $P(\partial,n)U$ is the generalized thermo-stress vector.

In the case of the exterior problems for the domain Ω^- the vector $U=(u,w,\omega,v,\vartheta)^\top$ should satisfy the following decay conditions at infiniti

$$U(x)=O(|x|^{-1}),\quad \frac{\partial}{\partial x_k}U(x)=o(|x|^{-1}),k=1,2.$$

3 Existence results for boundary value problems. Let us denote by $L(\partial,\sigma)$ the matrix differential operator of order 7×7, generated by the left hand side expressions in system (1)-(5). Assume that

$$L^*(\partial,\sigma)U:=L^\top(-\partial,\sigma)U.$$

Let us introduce the generalized single and double layer potentials

$$V(g)(x) = \int_{\partial\Omega} \Gamma(x-y,\sigma)\, g(y)\, d_yS, \quad x \in \mathbb{R}^2 \setminus \partial\Omega, \tag{6}$$

$$W(h)(x) = \int_{\partial\Omega} \big[\mathcal{P}^*(\partial_y, n(y))\Gamma^\top(x-y,\sigma)\big]^\top h(y)\, d_yS, \quad x \in \mathbb{R}^2 \setminus \partial\Omega, \tag{7}$$

where $\Gamma(x-y,\sigma)$ is the fundamental matrix of the system (1) - (5), $g = (g_1, g_2, \cdots, g_7)^\top$ and $h = (h_1, h_2, \cdots, h_7)^\top$ are the density vector-functions defined on $\partial\Omega$, while a density vector-function $\psi = (\psi_1, \psi_2, \cdots, \psi_7)^\top$ is defined on $\Omega^\pm$, $P^*(\partial_y, n(y))$ is the boundary differential operator of the $L^*(\partial,\sigma)$.

The following theorems are valid.

Theorem 1. *For any $g \in C^{0,\delta'}(\partial\Omega)$, $h \in C^{1,\delta'}(\partial\Omega)$, and any $x \in \partial\Omega$*

$$\begin{aligned}
&\{\, V(g)(x)\,\}^\pm = V(g)(x) = \mathcal{H}\, g(x),\\
&\{\, P(\partial_x, n(x))\, V(g)(x)\,\}^\pm = \big[\,\mp 2^{-1} I_7 + \mathcal{K}\,\big]\, g(x),\\
&\{\, W(h)(x)\,\}^\pm = \big[\,\pm 2^{-1} I + \mathcal{N}\,\big]\, h(x),\\
&\{P(\partial_x, n(x))\, W(h)(x)\,\}^+ = \{\, P(\partial_x, n(x))\, W(h)(x)\,\}^- = \mathcal{L}\, h(x),
\end{aligned}$$

where

$$\begin{aligned}
&\mathcal{H}\, g(x) := \int_{\partial\Omega} \Gamma(x-y,\sigma)\, g(y)\, d_yS\,,\\
&\mathcal{K}\, g(x) := \int_{\partial\Omega} \big[\, P(\partial_x, n(x))\, \Gamma(x-y,\sigma)\,\big]\; g(y)\, d_yS\,,\\
&\mathcal{N}\, h(x) := \int_{\partial\Omega} \big[\, P^*(\partial_y, n(y))\, \Gamma^\top(x-y,\sigma)\,\big]^\top h(y)\, d_yS\,,\\
&\mathcal{L}\, h(x) := \lim_{\Omega^\pm \ni z \to x \in \partial\Omega} P(\partial_z, n(x)) \int_{\partial\Omega} \big[\, P^*(\partial_y, n(y))\, \Gamma^\top(z-y,\sigma)\,\big]^\top h(y)\, d_yS\,.
\end{aligned}$$

Theorem 2. *Let $\partial\Omega \in C^{2,\nu}$ and $f \in C^{1,\tau}(\partial\Omega)$ with $0 < \tau < \nu \le 1$. Then the boundary value problem $(I)^\pm$ is uniquely solvable in the space $C^{1,\tau}(\overline{\Omega^\pm})$ and the solution is represented by the double layer potential $W(h)$ defined by (7), where density $h \in C^{1,\tau}(\partial\Omega)$ is a unique solution of the integral equation $\big[\pm 2^{-1} I_7 + \mathcal{N}\big] h = f$ on $\partial\Omega$.*

Theorem 3. *Let $\partial\Omega \in C^{1,\nu}$ and $f \in C^{0,\tau}(\partial\Omega)$ with $0 < \tau < \nu \le 1$. Then the boundary value problem $(II)^\pm$ is uniquely solvable in the space $C^{1,\tau}(\overline{\Omega^\pm})$ and the solution is represented by the single layer potential $V(g)$ defined by (6), where $g \in C^{0,\tau}(\partial\Omega)$ is a unique solution of the integral equation $\big[\mp 2^{-1} I_7 + \mathcal{K}\big] g = f$ on $\partial\Omega$.*

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Received 28.05.2020; revised 28.07.2020; accepted 30.09.2020.

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