Reports of Enlarged Sessions of the
Seminar of I. Vekua Institute
of Applied Mathematics
Volume 34, 2020

MATHEMATICAL AND COMPUTER MODELING OF POLITICAL CONFLICTS IN THE CASE OF VARIABLE COEFFICIENTS

Temur Chilachava George Pochkhua Leila Sulava Maia Chakaberia

Abstract. Earlier we proposed mathematical models for resolving political conflicts through economic cooperation between two politically mutually opposing sides (possibly a country or country and its legal subject). In case of constant parameters of models, with some dependencies between coefficients, exact analytical solutions are found and conflict resolution conditions are established. This paper considers mathematical models in the general case, when the variation of demographic factors of the sides is taken into account, and over time the coefficients of aggressiveness and cooperation of parts of the population of the sides, respectively, both preventing and facilitating cooperation, are changed. Numerous computer simulations were performed in the Matlab software environment in the case of exponential as well as trigonometric functions of model coefficients. The cases where the sign is changed during the consideration of models, derivatives of aggressiveness coefficients and cooperation between the sides were considered. On the basis of specific examples that model some existing political conflicts, numerical solutions have been obtained, appropriate schedules have been constructed and conflict resolution conditions have been found.

Keywords and phrases: Mathematical models with variable coefficients, computer modeling.

AMS subject classification (2010): 97M10, 97M70, 37N99.

Introduction. Creation of mathematical models is more original in social sphere, because, they are more difficult to substantiate. In [1] a new nonlinear continuous mathematical model of linguistic globalization is considered. In [2] a new nonlinear mathematical model of process of three level assimilation which is described by four-dimensional dynamic systems is offered. We proposed to create new nonlinear mathematical models of economic cooperation between two politically (not military opposition) mutually warring sides (two countries or a country and its legal region) which consider economic or other type of cooperation between different parts of population aimed at the peaceful resolution of conflicts [3–8].

1 General mathematical model in the case of variable coefficients.

The nonlinear mathematical model (the dynamic system) of economic cooperation between two political warring sides, offered by us, has the form:

$$\begin{cases} \frac{dN_1(t)}{dt} = -\alpha_1(t)\left[a(t)-N_1(t)\right]\left[b(t)-N_2(t)\right]+\beta_1(t)N_1(t)N_2(t)+F_1(t,N_1(t)) \\ \frac{dN_2(t)}{dt} = -\alpha_2(t)\left[a(t)-N_1(t)\right]\left[b(t)-N_2(t)\right]+\beta_2(t)N_1(t)N_2(t)+F_2(t,N_2(t)) \end{cases},$$

$$N_1(0)=N_{10},\ N_2(0)=N_{20}.$$

Model 1:

$$F_1(t,N_1(t))\equiv 0,\ F_2(t,N_2(t))\equiv 0.$$

Model 2:

$$F_1(t,N_1(t))=-\delta_1(t)N_1(t),\ F_2(t,N_2(t))=-\delta_2(t)N_2(t).$$

Model 3:

$$F_1(t,N_1(t))=\gamma_1(t)\left[a(t)-N_1(t)\right],\ F_2(t,N_2(t))=\gamma_2(t)\left[b(t)-N_2(t)\right].$$

Model 4:

$$F_1(t,N_1(t))=-\delta_1(t)N_1(t),\ F_2(t,N_2(t))=\gamma_2(t)\left[b(t)-N_2(t)\right].$$

We use the following exponential functions:

$$a(t)=a_0e^{n_1\frac{t}{T}},\ b(t)=b_0e^{n_2\frac{t}{T}},\ \alpha_1(t)=\alpha_{10}e^{n_3\frac{t}{T}},\ \alpha_2(t)=\alpha_{20}e^{n_4\frac{t}{T}},\ \beta_1(t)=\beta_{10}e^{n_5\frac{t}{T}},$$

$$\beta_2(t)=\beta_{20}e^{n_6\frac{t}{T}},\ \delta_1(t)=\delta_{10}e^{n_7\frac{t}{T}},\ \delta_2(t)=\delta_{20}e^{n_8\frac{t}{T}},\ \gamma_1(t)=\gamma_{10}e^{n_9\frac{t}{T}},\ \gamma_2(t)=\gamma_{20}e^{n_{10}\frac{t}{T}}$$

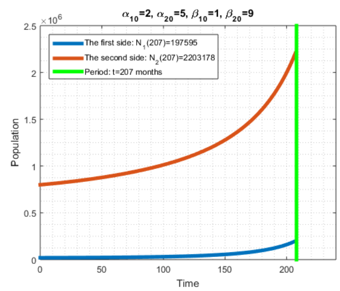
and also trigonometric functions:

$$\alpha_1(t)=\alpha_{10}\sqrt{2}\sin\left(\frac{2t}{T}+\frac{\pi}{4}\right),\ \alpha_2(t)=\alpha_{20}\sqrt{2}\sin\left(\frac{2.3t}{T}+\frac{\pi}{4}\right),$$

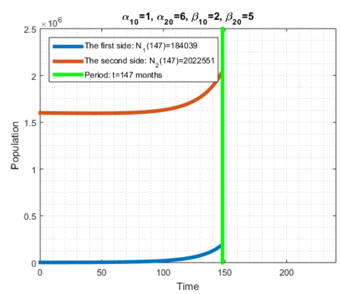
$$\beta_1(t)=\beta_{10}\sqrt{2}\sin\left(\frac{t}{T}+\frac{\pi}{4}\right),\ \beta_2(t)=\beta_{20}\sqrt{2}\sin\left(\frac{2t}{T}+\frac{\pi}{4}\right),$$

where
$a(t),b(t)$ - the population according to the first and second sides in time-point t;
$N_1(t),N_2(t)$ - number of the citizens of the first and second sides in time-point t, wishing or already being in economic cooperation and inclined to the subsequent peaceful resolution of the conflict;
$\alpha_1(t),\alpha_2(t)$ - the coefficients of aggression (alienation) of the sides;
$\beta_1(t),\beta_2(t)$ - the coefficients of cooperation of the sides;
$\delta_1(t),\delta_2(t)$ - the coefficients of coercion to aggression (alienation) of the sides;
$\gamma_1(t),\gamma_2(t)$ - the coefficients of coercion to cooperation of the sides;
n_1,n_2 - the coefficients of demographic factors of the sides;
n_3,n_4 - the coefficients of aggression (alienation) factors of the sides;
n_5,n_6 - the coefficients of cooperation factors of the sides;
n_7,n_8 - the coercion coefficients to aggression (alienation) factors of the sides;
n_9,n_{10} - the coercion coefficients to cooperation factors of the sides;
a_0,b_0 - the population of the first and second sides;
T - time interval for model (conflict) consideration.

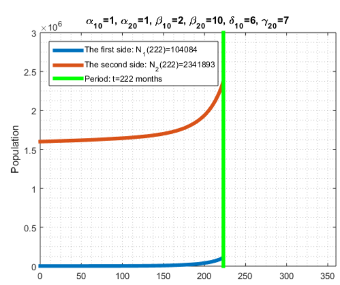
2 General computer modeling in the case of variable coefficients. Numerical calculations were performed and some results are shown in the below cases, where $a_0 = 2 \cdot 10^5$, $b_0 = 4 \cdot 10^6$, power of α_1 and α_2 coefficients is -10, power of β_1 and β_2 coefficients is -8, power of δ_1 coefficient is -3, power of γ_2 coefficients is -6,:
Case 1: Model 1, $N_{10} = 2 \cdot 10^4, N_{20} = 8 \cdot 10^5, n_1 = 0.05, n_2 = 0.1, T = 240.$



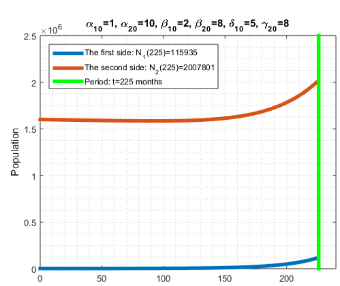
Case 2: Model 1, $N_{10} = 2 \cdot 10^3, N_{20} = 16 \cdot 10^5, n_1 = -0.05, n_2 = 0, T = 240.$



Case 3: Model 4, $N_{10} = 2 \cdot 10^3, N_{20} = 16 \cdot 10^5, n_1 = 0.05, n_2 = 0.1, n_3 = 1.5, n_4 = 2.5, T = 360.$



Case 4: Model 4, $N_{10} = 2 \cdot 10^3, N_{20} = 16 \cdot 10^5, n_1 = -0.05, n_2 = 0, n_3 = 1.5, n_4 = 2.5, T = 240.$



Conclusion. In all of the above cases, we assume that the conflict has been resolved if the following conditions are met:

$$\begin{cases} \frac{a(t)}{2} < N_1(t) \le a(t) \\ \frac{b(t)}{2} < N_2(t) \le b(t) \end{cases}, \qquad t \ge t_1.$$

R E F E R E N C E S

1. Chilachava, T. Research of the dynamic system describing globalization process. 3rd International Conference on Applications of Mathematics and Informatics in Natural Sciences and Engineering, AMINSE 2017; Tbilisi; Georgia; 7 December 2017 through 9 December 2017. *Springer Proceedings in Mathematics and Statistics,* **276** (2019), 67–78.

2. Chilachava, T., Pinelas, S., Pochkhua, G. Research of three-level assimilation processes. Differential and Difference Equations and Applications 2019, ICDDEA, Lisbon. *Springer Proceedings*, 2020 (to appear).

3. Chilachava, T., Pochkhua, G. Research of the dynamic system describing mathematical model of settlement of the conflicts by means of economic cooperation, *GESJ: Computer Science and Telecommunications*, **3**, 55 (2018) 18–26.

4. Chilachava, T., Pochkhua, G. About a possibility of resolution of conflict by means of economic cooperation. Problems of management of safety of difficult systems. *The XXVI International Conference, Moscow*, (2018), 69–74.

5. Chilachava, T., Pochkhua, G. Research of the nonlinear dynamic system describing mathematical model of settlement of the conflicts by means of economic cooperation. 8th International Conference on Applied Analysis and Mathematical Modeling, *ICAAMM 2019, Proceedings Book*, (2019), 183–187.

6. Chilachava, T., Pochkhua, G., Kekelia N., Gegechkori, Z. Research of the dynamic systems describing mathematical models of resolution of conflict by means of economic cooperation at bilateral or unilateral counteraction. *Tskhum-Abkhazian Academy of Sciences, Proceedings Book*, **XVII-XVIII** (2019), 12—23.

7. Chilachava, T., Pochkhua, G. Research of the dynamic systems describing mathematical models of resolution of the conflicts by means of economic cooperation. *Tskhum-Abkhazian Academy of Sciences, Proceedings Book*, **XVII-XVIII** (2019), 24-–37.

8. Chilachava, T., Pochkhua, G., Kekelia N., Gegechkori, Z. Research of conflict resolution dynamic systems describing by mathematical models. *Rep. Enlarged Sess. Seminar I. Vekua Inst. Appl. Math.*, **33** (2019), 1–4.

Received 30.05.2020; revised 21.07.2020; accepted 30.09.2020.

Author(s) address(es):

Temur Chilachava, George Pochkhua, Leila Sulava, Maia Chakaberia
Sokhumi State University
Politkovskaya str. 61, 0186 Tbilisi, Georgia
E-mail: temo_chilachava@yahoo.com, gia.pochkhua@gmail.com,
leilasamadash@gmail.com, chakaberiam@gmail.com