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RESEARCH OF THE DYNAMIC SYSTEMS DESCRIBING MATHEMATICAL MODELS OF RESOLUTION OF CONFLICT

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Abstract. In this work new mathematical models are offered where it is meant: in the first case – the governments of both sides, influencing various levers of pressure upon the citizens inclined to mutual economic cooperation, interfere with the process of economic cooperation, and in the second case – the government of one side interferes, and the government of the second and external the sides promote cooperation. In both cases the dynamic systems describing dynamics of parts of the population of the sides focused on cooperation are obtained. In case of constancy of coefficients of mathematical models special points of nonlinear systems of the differential equations are found. In both models, at some dependence between constant coefficients of model, the first integrals and analytical solutions are found. The obtained exact solutions allow within these mathematical models and dependences between its coefficients, to define conditions at which economic cooperation will be able to peacefully resolve a political conflict. In both cases through parameters of management it is possible to define conditions under which perhaps or resolution of conflicts, i.e. definition of the minimum economic investment or power influences with external (minimization of external financial expenses) or inside for permission or continuation of the conflict is impossible.

Keywords and phrases: Mathematical models of resolution of conflict, dynamic systems, exact solutions.

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Introduction. As it is known, there is a very large history of mathematical modeling of physical processes, in biology, ecology, chemistry, medicine and especially in epidemiology. Creation of mathematical models is more original in **social sphere,** because, they are more difficult to substantiate [1]. In 2005, Mathematicians Robert Aumann and Thomas Schelling won the Nobel Prize in Economics for the scientific work cycle **"Understanding of the problems of the conflict and cooperation through the game theory". Lee Kuan Yew,** author of the Singaporean **"Economic Miracle"**, noted: **"If you want economic growth, do not break out the war with neighbors, establish trade relations with them, instead". Prof. T. Chilachava** proposed to create new nonlinear mathematical models of economic cooperation between two politically (not military opposition) inter conflicting sides (possible states or country and its legal entities), which envisages economic or other type of cooperation between the part of population of the sides, direction towards the rapprochement of the sides and the peaceful resolution of the conflict [2]. He conducted this research with his PhD student **G. Pochkhua** [3 - 5].

1 Mathematical model of solving cooperation conflict. Let's discuss the first case, when the governments of both sides put the pressure on those

citizens, who collaborate economically, in order to disrupt the economic cooperation process. In this case, the mathematical model (dynamic system) is following:

$$\begin{cases} \frac{dN_1(t)}{dt} = -\alpha_1(t)(a - N_1(t))(b - N_2(t)) + \beta_1(t)N_1(t)N_2(t) - \gamma_1(t)N_1(t), \\ \frac{dN_2(t)}{dt} = -\alpha_2(t)(a - N_1(t))(b - N_2(t)) + \beta_2(t)N_1(t)N_2(t) - \gamma_2(t)N_2(t), \end{cases} \tag{1}$$

$$N_1(0) = N_{10}, \qquad N_2(0) = N_{20}, \tag{2}$$

where $N_1(t),\ N_2(t)$ represent the amount of the citizens willing to cooperate, accordingly $a,\ b$ are the amount on the citizens the first and the second sides, $\alpha_1(t),\ \alpha_2(t)$ are the aggressive (alienation) coefficients, $\beta_1(t),\ \beta_2(t)$ are the cooperation coefficients, $\gamma_1(t),\ \gamma_2(t)$ are the coefficients of the pressure by the governments of the sides on the citizens, who are willing to cooperate.

The weak condition of the solving conflict is fulfillment of the following system

$$\begin{cases} N_1(t) > \frac{a}{2}, \\ N_2(t) > \frac{b}{2}, \end{cases} \qquad t = t_*. \tag{3}$$

The strong condition of the solving conflict is fulfillment of the following system

$$\begin{cases} N_1(t) \geq \frac{2a}{3}, \\ N_2(t) \geq \frac{2b}{3}, \end{cases} \qquad t = t_{**}. \tag{4}$$

Let's consider the special case for system (1)

$$\begin{cases} k\alpha_1 = \beta_1, \\ k\alpha_2 = \beta_2, \\ \gamma_1 = \gamma_2 = \gamma, \end{cases} \qquad k \neq 1. \tag{5}$$

For discussed (5) special case (1), (2) the first integral of Cauchy's problem is represented as

$$\frac{N_1(t)}{\alpha_1} - \frac{N_2(t)}{\alpha_2} = \left(\frac{N_{10}}{\alpha_1} - \frac{N_{20}}{\alpha_2}\right) e^{-\gamma t}. \tag{6}$$

From system (1), according to ratio (6), for function $N_1(t)$, we will get Riccati's equation

$$\begin{aligned} \frac{dN_1(t)}{dt} &= (k-1)\alpha_2 N_1^2(t) \\ &+N_1(t)\left[(-k+1)\alpha_1\alpha_2\left(\frac{N_{10}}{\alpha_1} - \frac{N_{20}}{\alpha_2}\right)e^{-\gamma t} + \alpha_1 b + \alpha_2 a - \gamma\right] \\ &-\alpha_1 a\left[b + \alpha_2\left(\frac{N_{10}}{\alpha_1} - \frac{N_{20}}{\alpha_2}\right)e^{-\gamma t}\right]. \end{aligned} \tag{7}$$

The exact solution of the Cauchy's problem (7), (2) has the form

$$N_1(t) = N_{1par} + \overline{N}_1(t),\ z(t) \equiv \frac{1}{\overline{N_1}(t)},\ N_{1par}(t) = pe^{-\gamma t} + q, \tag{8}$$

$$p = \alpha_1 \left(\frac{N_{10}}{\alpha_1} - \frac{N_{20}}{\alpha_2} \right),\ q = -\frac{\alpha_1 ab}{\gamma - a\alpha_2},\ \gamma \neq a\alpha_2,\ k = 1 + \frac{\gamma - a\alpha_2}{\alpha_2 a} = \frac{\gamma}{a\alpha_2},$$

$$\begin{aligned} z(t) = \exp\left[\tfrac{(k-1)\alpha_1\alpha_2}{\gamma}\left(\tfrac{N_{10}}{\alpha_1} - \tfrac{N_{20}}{\alpha_2}\right)\left(e^{-\gamma t} - 1\right) + \left(\alpha_1 b + \alpha_2 a\,(k-1)\right)t\right] \times \\ \times \left[\tfrac{1}{N_{10} - p - q} - (k-1)\,\alpha_2 A\right], \end{aligned} \tag{9}$$

$$A \equiv \int_0^t \exp\left[\frac{(k-1)\alpha_1\alpha_2}{\gamma}\left(\frac{N_{10}}{\alpha_1} - \frac{N_{20}}{\alpha_2}\right)\left(-e^{-\gamma t} + 1\right) - \left(\alpha_1 b + \alpha_2 a\,(k-1)\right)t\right] dt,$$

$$\frac{N_{20}}{\alpha_2} \neq \frac{ab}{a\alpha_2 - \gamma},\ N_1(t) = \frac{1}{z(t)} + N_{1par}. \tag{10}$$

From (6) it is easy to receive

$$N_2(t) = \frac{\alpha_2}{\alpha_1} N_1(t) - \alpha_2 \left(\frac{N_{10}}{\alpha_1} - \frac{N_{20}}{\alpha_2} \right) e^{-\gamma t}. \tag{11}$$

2 The mathematical model of solving problem through the economic cooperation in case of one sided resistance of the government structure. Let's discuss the second case, when the government of the first side prevents, but the second and external sides support economic cooperation. In this case, the mathematical model (dynamic system) is following:

$$\begin{cases} \frac{dN_1(t)}{dt} = -\alpha_1(t)(a - N_1(t))(b - N_2(t)) + \beta_1(t)N_1(t)N_2(t) - \gamma_1(t)N_1(t), \\ \frac{dN_2(t)}{dt} = -\alpha_2(t)(a - N_1(t))(b - N_2(t)) + \beta_2(t)N_1(t)N_2(t) + \gamma_2(t)(b - N_2(t)), \end{cases} \tag{12}$$

$$N_1(0) = N_{10}, \qquad N_2(0) = N_{20}, \tag{13}$$

$\gamma_1(t)$ is the coefficient of the pressure by the government of the first side on citizens, supporting cooperation, $\gamma_2(t)$ is the coefficient of pressure to cooperate, by the government of the second side toward aggressive citizens.

For tasks (12), (13), let's discuss case (5).

We get the differential equation of the first order

$$\frac{1}{\alpha_1}\frac{dN_1(t)}{dt} - \frac{1}{\alpha_2}\frac{dN_2(t)}{dt} = -\gamma\left(\frac{N_1(t)}{\alpha_1} - \frac{N_2(t)}{\alpha_2}\right) - \frac{\gamma b}{\alpha_2}.$$

From this one, according to (13), we get the first integral of equation (12), (13)

$$\frac{N_1(t)}{\alpha_1} - \frac{N_2(t)}{\alpha_2} = \left(\frac{b}{\alpha_2} + \frac{N_{10}}{\alpha_1} - \frac{N_{20}}{\alpha_2}\right) e^{-\gamma t} - \frac{b}{\alpha_2}. \tag{14}$$

Accordingly, from system (12), according to (14) we get Riccati's equation for the function $N_1(t)$

$$\begin{aligned} \frac{dN_1(t)}{dt} &= (k-1)\alpha_2 N_1^2(t) \\ &+ N_1(t)\left[(-k+1)\alpha_1\alpha_2\left(\tfrac{b}{\alpha_2} + \tfrac{N_{10}}{\alpha_1} - \tfrac{N_{20}}{\alpha_2}\right)e^{-\gamma t} + k\alpha_1 b + \alpha_2 a - \gamma\right] \\ &- \alpha_1\alpha_2 a\left[\tfrac{b}{\alpha_2} + \tfrac{N_{10}}{\alpha_1} - \tfrac{N_{20}}{\alpha_2}\right]e^{-\gamma t}. \end{aligned} \tag{15}$$

The exact solution of Cauchy's problem (15), (13) has the form

$$N_1(t) = N_{1par} + \overline{N}_1(t),\ z(t) \equiv \frac{1}{\overline{N_1}(t)},\ N_{1par}(t) = pe^{-\gamma t} + q, \tag{16}$$

$$p = \alpha_1 \left(\frac{b}{\alpha_2} + \frac{N_{10}}{\alpha_1} - \frac{N_{20}}{\alpha_2} \right),\ q = -\frac{k\alpha_1 b}{\alpha_2(k-1)},\ \alpha_2 a = \gamma, \tag{17}$$

$$\begin{aligned} z(t) &= \exp\left[\frac{(k-1)\alpha_1}{a} \left(\frac{b}{\alpha_2} + \frac{N_{10}}{\alpha_1} - \frac{N_{20}}{\alpha_2} \right) \left(e^{-a\alpha_2 t} - 1 \right) + k\alpha_1 bt \right] \\ &\times \left[\tfrac{1}{N_{10}-p-q} - (k-1)\,\alpha_2 A \right], \end{aligned} \tag{18}$$

$$A \equiv \int_0^t \exp\left[\frac{(k-1)\alpha_1}{a} \left(\frac{b}{\alpha_2} + \frac{N_{10}}{\alpha_1} - \frac{N_{20}}{\alpha_2} \right) \left(-e^{-a\alpha_2 t} + 1 \right) - k\alpha_1 bt \right] dt.$$

From (14) it is easy to get

$$N_2(t) = \frac{\alpha_2}{\alpha_1} N_1(t) - \alpha_2 \left(\frac{b}{\alpha_2} + \frac{N_{10}}{\alpha_1} - \frac{N_{20}}{\alpha_2} \right) e^{-\gamma t} + b. \tag{19}$$

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