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SEMI-DISCRETE AND FINITE-DIFFERENCE SCHEMES FOR ONE-DIMENSIONAL ANALOG OF THE MITCHISON NONLINEAR SYSTEM WITH SOURCE TERM

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Abstract. One-dimensional analog of the Mitchison nonlinear partial differential system with source term is considered. The semi-discrete scheme with respect to spatial variable and finite-difference scheme for the numerical solution of the posed initial-boundary value problem are constructed. Convergence of these schemes are given.

Keywords and phrases: Nonlinear partial differential equations, vein formation, semi-discrete scheme, finite-difference scheme, convergence theorems.

AMS subject classification (2010): 35Q80, 35Q92, 65N06, 65Y99.

Two-dimensional model describing the vein formation of young leaves and some qualitative and structural properties of solutions of this model are given in [1]. In [2] investigations for one-dimensional analog are carried out. In biological modeling there are many works where this and many models of similar processes are also presented and discussed (see, for example, [3]-[6] and references therein). Many scientific works are devoted to the investigation and numerical resolution of different kinds of initial-boundary value problems for the model described in [1] and its one- and multi-dimensional analogs (see, for example, [7]-[17] and references therein). Let us consider the following initial-boundary value problem for one-dimensional analog of the vein formation model [1] with source term $|S|^{q-2}S$, $q \geq 2$:

$$
\begin{gathered}
\frac{\partial S}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial S}{\partial x}\right) - |S|^{q-2}S, \quad (x,t) \in (0,1)\times(0,T], \\
\frac{\partial D}{\partial t} = -D + g\left(D\frac{\partial S}{\partial x}\right), \quad (x,t) \in (0,1)\times(0,T], \\
S(0,t) = 0, \quad D\frac{\partial S}{\partial x}\bigg|_{x=1} = \psi, \quad t \in [0,T], \\
S(x,0) = S_0(x), \quad D(x,0) = D_0(x) \geq \delta_0 = const > 0, \quad x \in [0,1],
\end{gathered} \tag{1}
$$

where $0 < g_0 \leq g(\xi) \leq G_0$, g_0, G_0, T and ψ are positive constants, g, S_0 and D_0 are given sufficiently smooth functions and D, S are unknown functions.

The purpose of our note is to construct approximate solution of problem (1) by using semi-discrete scheme with respect to spatial variable and fully-discrete finite-difference scheme.

Using known notations, the following semi-discrete scheme is constructed and its convergence in the norm of the usual space C_h is studied:

$$
\begin{gathered}
\frac{ds}{dt} - (ds_{\bar{x}})_x + |s|^{q-2}s = 0, \\
\frac{d\,d}{dt} = -d + g\left(ds_{\bar{x}}\right), \\
s(x,0) = S_0(x), \quad d(x,0) = D_0(x), \\
s(0,t) = 0, \quad d_{M-1/2}s_{\bar{x},M} + \frac{h}{2}\left(\frac{dd}{dt}\right)\bigg|_{x=1} = \psi.
\end{gathered}
\tag{2}
$$

Here the function s is defined on the $\bar{\omega}_h \times [0,T]$, where

$$\bar{\omega}_h = \{x_i = ih, \quad i = 0,1,...,M; \quad h = 1/M\}$$

and the function d is defined on the $\omega_h^* \times [0,T]$, where

$$\omega_h^* = \{x_i = (i-1/2)h, \quad i = 1,2,...M\}$$

respectively.

Assume, problem (1) has a sufficiently smooth solution. The following statement takes place.

Theorem 1. *The semi-discrete scheme (2) converges to the solution of problem (1) in the norm of the space C_h with the rate $O(h^2)$.*

On the grids:

$$\bar{\omega}_h \times \omega_\tau, \quad \omega_h^* \times \omega_\tau,$$

where

$$\omega_\tau = \{t_j = j\tau, \quad j = 0,1,...,N, \quad \tau = T/N\},$$

based on (2) the fully-discrete finite difference scheme for problem (1) is also constructed and investigated. This scheme has the following form:

$$
\begin{gathered}
s_{t,i}^j = (d_i^{j+1}s_{\bar{x},i}^{j+1})_x - |s_i^{j+1}|^{q-2}s_i^{j+1}, \\
d_{t,i-1/2}^j = -d_{i-1/2}^{j+1} + g\left(d_{i-1/2}^{j+1}s_{\bar{x},i}^{j+1}\right), \\
s_i^0 = S_{0,i}, \quad d_{i-1/2}^0 = D_{0,i-1/2}, \\
s_0^j = 0, \quad d_{M-1/2}^j s_{\bar{x},M} + \frac{h}{2}s_{t,M}^j = \psi.
\end{gathered}
\tag{3}
$$

The following statement takes place.

Theorem 2. *The finite difference scheme (3) converges to the solution of problem (1) in the norm of the space C_h with the rate $O(\tau + h^2)$.*

Using discrete analog (3), algorithm described in [17] and based on well known iterative procedures [18] many numerical experiments were carried out. Numerical results for tests examples fully agree with theoretical investigation.

R E F E R E N C E S

1. Mitchison G,J. A model for vein formation in higher plants. *Proc. T. Soc. Lond. B.,* **207** (1980), 79-109.

2. Bell, J., Cosner, C., Bertiger, W. Solutions for a flux-dependent diffusion model. *SIAM J. Math. Anal.,* **13** (1982), 758-769.

3. Candela, H., Martinez-Laborda, A., Luis Micol, J. Venation pattern formation in Arabidopsis thaliana vegetative leaves. *Develop. Biol.,* **205** (1999), 205-216.

4. Freeman, C., Graham, H., Emlen, M. Developmental stability in plants. *Symmetries, stress and epigenesis. Genetica,* **89** (1993), 97-119.

5. Hardwick, R. Physiological consequences of modular growth in plants. *Philos. Trans. Roy. Soc. London B, Biol. Sci.,* **313** (1986), 161-173.

6. Roussel, M.R., Slingerland, M.J. A biochemically semi-detailed model of auxin mediated vein formation in plant leaves. *Biosystems,* **109** (2012), 475-487.

7. Dzhangveladze, T. Averaged model of sum approximation for a system of nonlinear partial differential equations (Russian). *Proc. I. Vekua Inst. Appl. Math.,* **19** (1987), 60-73.

8. Dzhangveladze, T., Tagvarelia, T. Convergence of a difference scheme for a system of nonlinear partial differential equations (Russian). *Proc. I. Vekua Inst. Appl. Math.,* **40** (1990), 77-83.

9. Jangveladze, T,. Investigation and numerical solution of some systems of nonlinear partial differential equations. *Rep. Enlarged Sess. Semin. I. Vekua Appl., Math.,* **6** (1991), 25-28.

10. Jangveladze, T., Kiguradze, Z., Nikolishvili, M. On approximate solution of one nonlinear two-dimensional diffusion system. *Rep. Enlarged Sess. Semin. I. Vekua Appl., Math.,* **23** (2009), 42-45.

11. Jangveladze, T., Nikolishvili, M., Tabatadze, B. On one nonlinear two-dimensional diffusion system. *Proceedings of the 15th WSEAS Int. Conf. Applied Math.,* **10** (2010), 105-108.

12. Nikolishvili, M. Numerical resolutions of one nonlinear partial differential system. *Rep. Enlarged Sess. Semin. I. Vekua Appl., Math.,* **24** (2010), 93-96.

13. Kiguradze, Z., Nikolishvili, M., Tabatadze, B. Numerical resolutions of one system of nonlinear partial differential equations. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.,* **25** (2011),76-79.

14. Jangveladze, T. On some properties and approximate solution of one system of nonlinear partial differential equations. *International Workshop on the Qualitative Theory of Differential Equations, QUALITDE - 2013, Dedicated to the 100th birthday anniversary of Professor L. Magnaradze,* (2013), 58-60.

15. Jangveladze, T., Kiguradze, Z., Gagoshidze, M., Nikolishvili, M. Stability and convergence of the variable directions difference scheme for one nonlinear two-dimensional model. *International Journal of Biomathematics,* **8** (2015), 1550057 (21 pages).

16. Jangveladze, T., Kiguradze, Z., Gagoshidze, M. Economical difference scheme for one multi-dimensional nonlinear system. *Acta Mathematica Scientia,* (2019), 39B(4): 1-18 (accepted) https://doi.org/10.1007/s10473-019

17. Katsadze, T. On the numerical solution of one-dimensional analog of Mitchison nonlinear partial differential system. *Rep. Enlarged Sess. Semin. I. Vekua Appl. Math.,* **31** (2017), 71-74.

18. Rheinboldt W.C. Methods for Solving Systems of Nonlinear Equations. *SIAM, Philadelphia,* 1970.

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