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NUMERICAL SOLUTION OF NONLINEAR DEFORMATION TASK IN THE CASE OF AXISYMMETRIC LOADING OF A LAYERED CYLINDRICAL SHELL BY LOCAL SURFACE FORCE

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Abstract. Based on one of the variants of the improved theory, in the case of axisymmetric loading of a layered cylindrical shell by local surface force, for solution of the nonlinear deformation task the system of decision differential equations is obtained for this class. A particular case of deformation of cylindrical shell is considered, an appropriate analysis based on the results obtained from numerical realization of the example is stated.

Keywords and phrases: Layered shell, non-linear deformation, nonuniformity of in-plane shear deformation.

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The paper deals with the sandwich shells that are consisting from the layers with different mechanical properties. For the study of the mode of deformation of this class of shells it is desirable to use the constructed on theory of breaks hypothesis. The essence of the breaks hypothesis is the following: the arranged on the normal of the coordinate surface element of the shell after deformation becomes as break that gives the possibility along the thickness of the sandwich shell to consider the non-uniformity of the shear deformation.

Let us state the constructed based on the breaks hypothesis the main equations and ratios of the nonlinear deformation theory of sandwich shells in the curvilinear coordinates α, β system.

The expressions of tangential displacements will be [4]:

$$\begin{aligned} u_\alpha^{(i)} &= u + a_1^{(i)}\gamma_\alpha^{(0)} + \gamma\left(\psi_\alpha - a_2^{(i)}\gamma_\alpha^{(0)}\right), \\ u_\beta^{(i)} &= v + b_1^{(i)}\gamma_\beta^{(0)} + \gamma\left(\psi_\beta - b_2^{(i)}\gamma_\beta^{(0)}\right), \end{aligned} \tag{1}$$

where u, v are the tangential displacements of the coordinate surface, ψ_α, ψ_β are the full rotation angles of the normal of coordinate surface, $\gamma_\alpha^{(0)}$, $\gamma_\beta^{(0)}$ are the shear deformation of layer, through which the coordinate surface passes. The coefficients including the tangential displacement expressions (1) are given in [3].

In case of presentation of tangential displacements as (1) the deformation components will be presented as follows:

$$\begin{aligned} &\varepsilon_{\alpha\alpha}^{(\gamma)} = \varepsilon_{\alpha\alpha}^{(i)} + \gamma\varkappa_{\alpha\alpha}^{(\gamma)}, \quad \varepsilon_{\alpha\beta}^{(\gamma)} = \varepsilon_{\alpha\beta}^{(i)} + \gamma 2\varkappa_{\alpha\beta}^{(i)}, \\ &\varepsilon_{\beta\beta}^{(\gamma)} = \varepsilon_{\beta\beta}^{(i)} + \gamma\varkappa_{\beta\beta}^{(\gamma)}, \quad \varepsilon_{\alpha\gamma}^{(\gamma)} = \gamma_\alpha^{(i)}, \quad \varepsilon_{\gamma\gamma}^{(\gamma)} = 0, \quad \varepsilon_{\beta\gamma}^{(\gamma)} = \gamma_\beta^{(i)}. \end{aligned} \tag{2}$$

The expressions of the values of included in the expressions of the deformation (2), $\varepsilon_{\alpha\alpha}^{(i)}$, $\varepsilon_{\beta\beta}^{(i)}$, ... , $\varkappa_{\alpha\beta}^{(i)}$ are stated in [4-6].

In the theory of shells, the relations between the forces and deformation components are expressed by the elasticity relations, those expressions are stated in [4-6]. The elasticity relations in the theory of shells perform the same role as the Hooke's law in the theory of elasticity.

Equilibrium equations of elements of layered shells will have the form:

$$\begin{aligned}
&\tfrac{\partial BN_\alpha}{\partial\alpha} + \tfrac{\partial BN_{\beta\alpha}}{\partial\beta} + \tfrac{\partial A}{\partial\beta}N_{\alpha\beta} - \tfrac{\partial B}{\partial\alpha}N_\beta + ABk_1Q_\alpha^* + ABq_1 = 0,\\
&\tfrac{\partial BN_\beta}{\partial\alpha} + \tfrac{\partial BN_{\alpha\beta}}{\partial\alpha} + \tfrac{\partial B}{\partial\alpha}N_{\beta\alpha} - \tfrac{\partial A}{\partial\beta}N_\alpha + ABk_2Q_\beta^* + ABq_2 = 0,\\
&\tfrac{\partial BQ_\alpha^*}{\partial\alpha} + \tfrac{\partial BQ_\beta^*}{\partial\beta} + ABk_1N_\alpha - ABk_2N_\beta + ABq_3 = 0,\\
&\tfrac{\partial BM_\alpha}{\partial\alpha} + \tfrac{\partial BM_{\beta\alpha}}{\partial\beta} + \tfrac{\partial A}{\partial\beta}M_{\alpha\beta} - \tfrac{\partial B}{\partial\alpha}M_\beta + ABQ_\alpha = 0,\\
&\tfrac{\partial BM_\beta}{\partial\beta} + \tfrac{\partial BM_{\alpha\beta}}{\partial\alpha} + \tfrac{\partial A}{\partial\beta}M_{\beta\alpha} - \tfrac{\partial A}{\partial\beta}M_\alpha + ABQ_\beta = 0,
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
Q_\alpha^* &= Q_\alpha - (N_\alpha + k_1M_\alpha)\theta_\alpha - (N_{\alpha\beta} + k_1M_{\alpha\beta})\theta_\beta,\\
Q_\beta^* &= Q_\beta - (N_{\beta\alpha} + k_2M_{\beta\alpha})\theta_\alpha - (N_\beta + k_2M_\beta)\theta_\beta.
\end{aligned} \tag{4}$$

Further let's consider the tasks of axisymmetric nonlinear deformation of the sandwich shells of rotation. In this case we mean the curvilinear coordinate $\alpha = S$ - that represents the length of the meridian arc, and $\beta = \theta$ - is the central angle of the parallel circle. The following system of solving nonlinear differential equations to solve these tasks based on the basic equations and ratios is stated in our paper

$$\begin{aligned}
&\frac{dN_s}{ds} = a_{11}^*N_s + a_{12}^*Q_s^* + a_{13}^*M_s + a_{14}^*u + a_{15}^*w + a_{16}^*\psi_s + d_{12}\Phi + f_1,\\
&\frac{dQ_s^*}{ds} = a_{21}^*N_s + a_{22}^*Q_s^* + a_{23}^*M_s + a_{24}^*u + a_{25}^*w + a_6^*\psi_s + d_{22}\Phi + f_2,\\
&\frac{dM_s}{ds} = a_{31}^*N_s + a_{32}^*Q_s^* + a_{33}^*M_s + a_{34}^*u + a_{35}^*w + a_{36}^*\psi_s + d_{31}(N_s + k_1M_s)\Phi_s + f_3,\\
&\frac{du}{ds} = a_{41}^*N_s + a_{42}^*Q_s^* + a_{43}^*M_s + a_{44}^*u + a_{45}^*w + a_{46}^*\psi_s + d_{42}^*\Phi_s^2 + d_{43}Q_s^*\Psi_s + d_{45}Q_s^{*2}\\
&+d_{44}(N_s + k_1M_s)\Psi_s^2 + d_{46}(N_s + k_1M_s)Q_s^*\Psi_s + d_{47}(N_s + k_1M_s)^2\Psi_s^2 + d_{48}\Phi + f_4,\\
&\frac{dw}{ds} = a_{52}^*Q_s^* + a_{54}^*u + a_{56}^*\Psi_s + d_{51}(N_s + k_1M_s)\Phi_s,\\
&\frac{d\Psi_s}{ds} = a_{61}^*N_s + a_{62}^*Q_s^* + a_{63}^*M_s + a_{64}^*u + a_{65}^*w + a_{66}^*\psi_s + d_{61}(N_s + k_1M_s)\Psi_s\\
&+d_{62}\Psi_s^2 + d_{63}Q_s^*\Psi_s + d_{64}(N_s + k_1M_s)\Psi_s^2 + d_{65}Q_s^{*2} + d_{66}(N_s + k_1M_s)Q_s^*\Psi_s\\
&+d_{67}(N_s + k_1M_s)^2\Psi_s^2 + d_{68}\Phi + f_6,
\end{aligned} \tag{5}$$

where

$$\Phi = \frac{1}{c_0 - c_1 N_s - c_2 M_s - c_3 \Psi_s} \left\{ \frac{1}{c_0}(c_1 N_s - c_2 M_s - c_3 \Psi_s)[a_1 N_s + a_2 Q_s^* + a_3 M_s \right.$$
$$+ a_4 u + a_5 w + (a_1 - q)\Psi_s] - \frac{1}{c_0^2} q_3 (c_1 N_s + c_2 M_s + c_3 \Psi_s)^2$$
$$+ (d_1 N_s + d_2 Q_s^* + d_3 M_s + d_4 u + d_5 w + d_6 \Psi_s)\Psi_s + (N_s + k_1 M_s) \qquad (6)$$
$$\times (b_1 N_s + b_2 Q_s^* + b_3 M_s + b_4 u + b_5 w + b_6 \Psi_s + b_7 Q_s^{*2} + b_8 Q_s^* \Psi_s + b_9 \Psi_s^2)$$
$$\left. + (N_s + k_1 M_s)^2 (b_2 \Psi_s + 2 b_7 Q_s^* \Psi_s + b_8 \Psi_s^2) + b_7 (N_s + k_1 M_s)^3 + M_s \Psi_s \frac{dk_1}{ds} \right\}.$$

The coefficients of the equations (5) system and (6) expressions are determined by geometric and mechanical characteristics of the shell [6].

If we add the boundary conditions to the equations (5) system, we get the nonlinear boundary task.

In the case of a particular case it is considered the mode of deformation of the simply supported by edges orthotropic cylindrical shell, in the case of acting on it contour compression axial N_S forces and local normal surface forces.

Let's designate as h_1, h_2, h_3 the thickness of the outer, middle and inner layers of cylindrical shell accordingly; as E_1^i, E_2^i the modules of elasticity in accordance with coordinate axe direction; v_{12}^i, v_{21}^i are the Poison coefficients; G_{13}^i is a shear modulus where $i = 1,\ 2,\ 3$ are numbers of layers of shell; R is the radius of coordinate surface of the cylindrical shell and the l is the length of cylinder. The task has been solved for the following listed values: $R = 50$; $l = 60$; $h_1 = 0.3$; $h_2 = 2$; $h_3 = 0.3$; $E_1^1 = 1.5 \cdot 10^6$; $E_2^1 = 3 \cdot 10^6$; $E_1^2 = 2 \cdot 10^2$; $E_2^2 = 3 \cdot 10^2$; $E_1^3 = 1.2 \cdot 10^4$; $E_2^3 = 2.5 \cdot 10^4$; $v_{12}^1 = 0.2$; $v_{21}^1 = 0.34$; $v_{12}^2 = 0.1$; $v_{21}^2 = 0.14$; $v_{12}^3 = 0.14$; $v_{21}^3 = 0.17$; $G_{13}^1 = 0.15 \cdot 10^6$; $G_{13}^2 = 0.15 \cdot 10^2$; $G_{13}^3 = 0.35 \cdot 10^4$.

q	ω			
	1		2	
	Linear	Non-linear	Linear	Non-linear
	ε=10			
0	−0.1896	−0.1828	−0.1776	−0.1715
−10	−0.9813	−0.8847	−1.3821	−1.4112
−20	−1.7731	−1.5781	−2.5866	−2.6508
	ε=5			
0	−0.1896	−0.1828	−0.1776	−0.1715
−10	−0.6311	−0.5692	−1.1536	−1.3485
−20	−1.0725	−0.9554	−2.1294	−2.5255
	ε=2.5			
0	−0.1896	−0.1828	−0.1776	−0.1715
−10	−0.4179	−0.3823	−0.8463	−1. 1899
−20	−0.6461	−0.5818	−1.5157	−2.2083

Table

The numerical data in the $S = l/2$ section of the results of calculation of ω function derived from the solution of the proposed task is given in the table. The numerical data stated in the table are constructed based on Kirchhoff-Love (1) as well as broken lines (2) hypotheses, as well as solutions made using linear and non-linear theories. In particular, the acting on cylindrical body compression contour force $N_S = -200$, and on the cylindrical surface strip $S \in \left(\frac{l}{2} - \varepsilon,\ \frac{l}{2} + \varepsilon\right)$ acts as $q_3 = q \sin \pi \frac{S - \frac{l}{2} + \varepsilon}{2\varepsilon}$ the normal force.

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