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ON THE STATIONARY SOLUTION AND NUMERICAL APPROXIMATION FOR ONE-DIMENSIONAL MAXWELL SYSTEM

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Abstract. Initial-boundary value problem with Dirichlet boundary conditions for one-dimensional Maxwell system is considered. Corresponding system of nonlinear partial differential equations, describing the process of the penetration of an electromagnetic field into a substance is obtained when the vector of the magnetic field depends on two components. Necessary and sufficient condition for linear stability of stationary solution is given. Using corresponding finite difference scheme various numerical experiments are fulfilled. Possibility of Hopf bifurcation is shown which is verified by numerical experiments. Some graphical illustrations are given.

Keywords and phrases: Nonlinear system of partial differential equations, initial-boundary value problem, stationary solution, linear stability.

AMS subject classification (2010): 45K05, 65N06.

The nonlinear systems of partial differential equations describe various applied processes. It is very important to study properties of the solutions of these models. The construction and investigation of discrete analogues of initial-boundary problems is very important as well. There are many authors who investigate of such kind models [1] - [7].

One nonlinear partial differential systems arise in the mathematical modeling of the process of a magnetic field penetrating into a substance. In some physical assumption, the corresponding one-dimensional system of Maxwell's equations has the following form [8] - [10]:

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(W^{\alpha}\frac{\partial U}{\partial x}\right),\quad \frac{\partial V}{\partial t}=\frac{\partial}{\partial x}\left(W^{\alpha}\frac{\partial V}{\partial x}\right),$$
$$\frac{\partial W}{\partial t}=-aW^{\beta}+bW^{\gamma}\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}^2\right)\right], \tag{1}$$

where a, b are positive constants and α, β, γ are real numbers which will be specified later.

In the cylinder $[0,1]\times[0,T]$ for the system (1) let us consider the following initial-boundary value problem:

$$U(0,t)=0,\quad V(0,t)=0,$$
$$U(1,t)=\psi_1,\quad V(1,t)=\psi_2, \tag{2}$$
$$U(x,0)=U_0(x),\quad V(x,0)=V_0(x),\quad W(x,0)=W_0(x)>0,$$

where ψ_1, ψ_2, $\psi_1^2+\psi_2^2 \neq 0$, are constants and $U_0(x)$, $V_0(x)$, $W_0(x)$ are known functions of their arguments.

It is easy to check that the unique stationary solution U_s, V_s, W_s of problem (1), (2) is given by:

$$U_s = \psi_1 x, \quad V_s = \psi_2 x,$$
$$W_s = \left[\frac{b}{a}\left(\psi_1^2+\psi_2^2\right)\right]^{\frac{1}{\beta-\gamma}}. \tag{3}$$

The following statement takes place [7], [9], [10].

Theorem 1. *If $2\alpha+\beta-\gamma>0, \beta \neq \gamma$ then the stationary solution*

$$\left(\psi_1 x, \psi_2 x, \left(\frac{\beta}{\alpha}\left(\psi_1^2+\psi_2^2\right)\right)^{\frac{1}{\beta-\gamma}}\right)$$

of problem (1), (2) is linearly stable if and only if the following inequality is fulfilled

$$\alpha(\gamma-\beta)\left[\frac{\beta}{\alpha}\left(\psi_1^2+\psi_2^2\right)\right]^{\frac{\beta-\alpha-1}{\beta-\gamma}} < \pi^2.$$

From inequality above, it is clear that, if $\gamma<\beta$, then solution of problem (1), (2) is always linearly stable. Assume, $\gamma>\beta$, $\beta-\alpha-1\neq 0$ and consider the value

$$\psi_s = \left[\frac{\pi^2}{\gamma-\beta} a^{\frac{\gamma-\alpha-1}{\beta-\gamma}} b^{\frac{\alpha-\beta+1}{\beta-\gamma}}\right]^{\frac{\beta-\gamma}{\beta-\alpha-1}}.$$

If $\beta-\alpha-1<0$, then for $\psi \in (0,\psi_s), \psi=\psi_1^2+\psi_2^2$ solution of problem (1), (2) is linearly stable, and when $\psi>\psi_s$ it is unstable. So, there appears possibility of Hoph bifurcation. The small perturbations may cause transformation of solution in periodic oscillations. This phenomena is confirmed by numerical experiments below.

Applying finite-difference scheme constructed and investigated in the work [8] various numerical experiments are carried out which fully agree with the theoretical findings.

Using these results the graphical illustrations are given for stability of solution (see, Figures 1 and 2) as well as fixed the bifurcation of solution (see, Figure 3).

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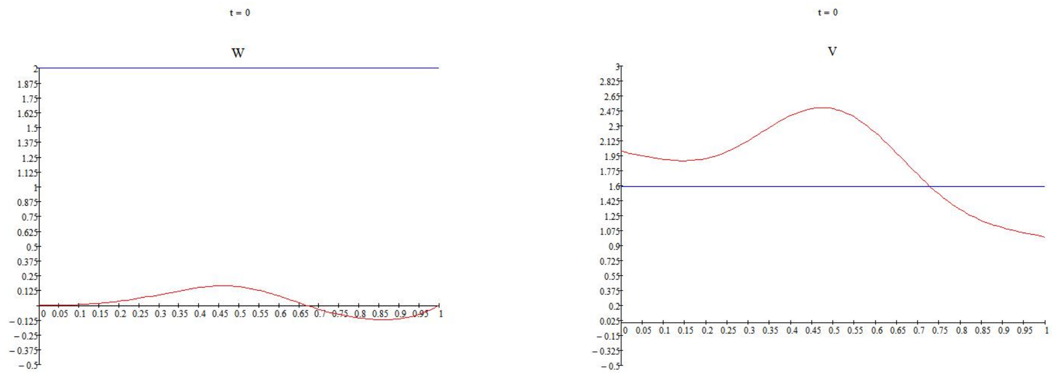

Figure 1: Initial Solution

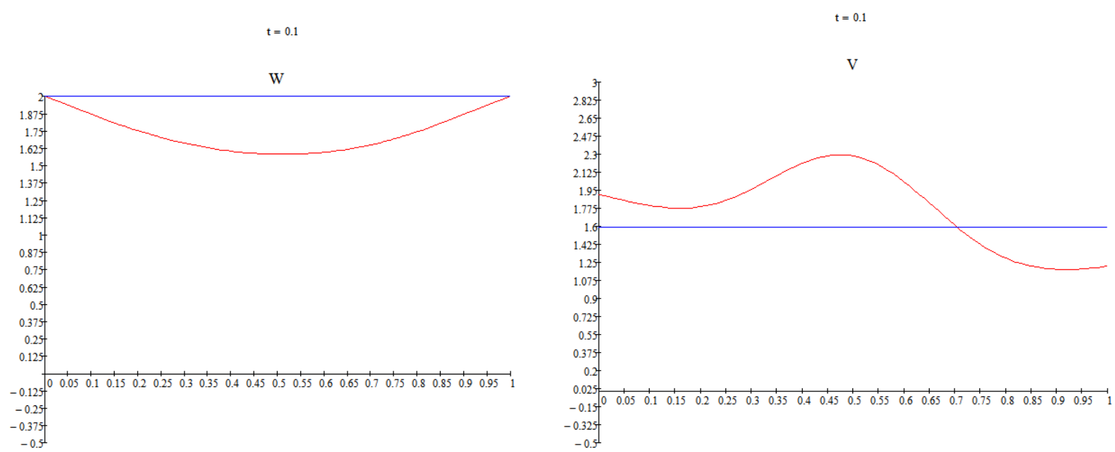

Figure 2: Stabilization of Solution

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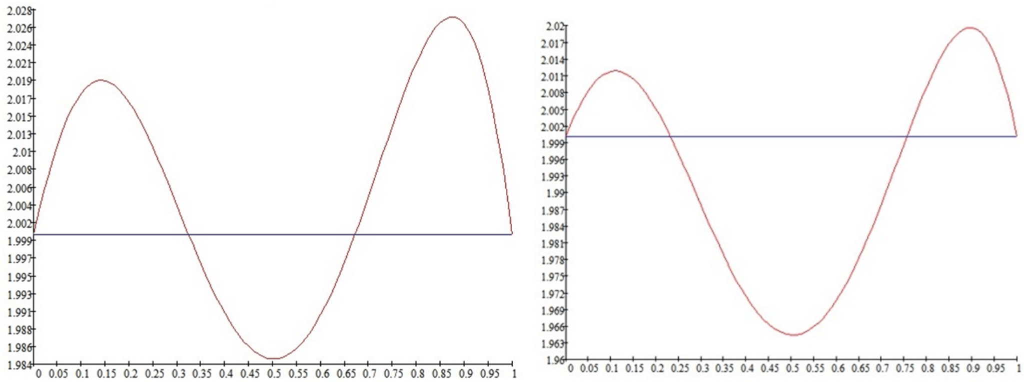

Figure 3: Bifurcation

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