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ONE METHOD OF ANALYTIC REPRESENTATION AND CLASSIFICATION OF A WIDE SET OF GEOMETRIC FIGURES WITH "COMPLEX" CONFIGURATION AND THEIR MOVEMENTS *

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Abstract. In this paper we give an analytic representation of one special class of geometric figures with a double toroidal basis line.

Keywords and phrases: Analytic representation, toroidal lines, torus, generalized Möbius-Listing's bodies

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In 2006, in [1], an analytical representation of a wide class of geometric figures that had plane baselines was given. This representation made it possible not only to describe the so-called GTR (Generalized Twisting and Rotated) and GML (Generalized Mbius-Listing's) figures but also to explore many of the geometric properties of these figures.

$$\begin{aligned} X(\tau,\psi,\theta,t) &= T_1(t)+\cos(\theta+M(t))[R(\theta,t)+r(\tau,\psi,\theta,t)\cos(n(\theta)+\mathrm{g}(t)+\psi)] \\ Y(\tau,\psi,\theta,t) &= T_2(t)+\sin(\theta+M(t))[R(\theta,t)+r(\tau,\psi,\theta,t)\cos(n(\theta)+\mathrm{g}(t)+\psi)] \\ Z(\tau,\psi,\theta,t) &= T_3(t)+K(\theta,t)+r(\tau,\psi,\theta,t)\sin(n(\theta)+\mathrm{g}(t)+\psi)] \end{aligned} \tag{1}$$

All possible admissible frames, as well as the geometric and physical meaning of the functions $R(\theta,t), r(\tau,\psi,\theta,t), M(t), n(\theta), g(t)$ and vector $T(t) \equiv (T_1(t)_1(t), T_2(t), T_3(t))$ entering into the analytic representation (1) are given in [1,2]. New results were obtained when the baselines (i.e. function $R(\theta,t)$) and radial cross sections (i.e. function $r(\tau,\psi,\theta,t)$) of these figures were described using the Johan Gielis superformula.

$$\rho(\theta) \equiv \left[\left| \frac{\cos\left(\frac{\lambda_1\theta}{4}\right)}{a} \right|^{K_2} + \left\| \frac{\sin\left(\frac{\lambda_2\theta}{4}\right)}{b} \right\|^{K_3} \right]^{-\frac{1}{k_1}} \tag{2}$$

Various methods for specifying geometric lines and surfaces are very convenient for a further study of the properties of these geometric shapes and lines (for example, see [3]).

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At this stage an analytical representation figures baseline which is toroidal (i.e. space) line is given. This type of figure appears when cutting GML bodies along parallel surface of the basis line. Thus, an analytical description of these figures plays an important role in the further study of quite interesting and important problems.

Words, a simple example of this class of figures, can be described as follows:

1. A certain torus is given and a cylinder is wound on its surface in a certain way.
2. There is one cylinder wound around this geometric surface.

Now, without loss of generality, we will give the simplest form of an analytic representation and describe in general terms the geometric and physical values of functions that are new in comparison with representation (1).

$$
\begin{aligned}
&X(\tau,\psi,\theta,t) = T_1(t) + \cos(\theta + M(t)) \left\{ r_1(\theta,t) + r_2(\theta,t)\cos\frac{n_2\theta}{m_2} + r_3(\tau,\psi,t)\cos\left(\frac{n_3\theta}{m_3}\right) \right\} \\
&Y(\tau,\psi,\theta,t) = T_2(t) + \sin(\theta + M(t)) \left\{ r_1(\theta,t) + r_2(\theta,t)\cos\frac{n_2\theta}{m_2} + r_3(\tau,\psi,t)\cos\left(\frac{n_3\theta}{m_3}\right) \right\} \\
&Z(\tau,\psi,\theta,t) = T_3(t) + r_2(\theta,t)\sin\frac{n_2\theta}{m_2} + r_3(\tau,\psi,t)\sin\left(\psi + \frac{n_3\theta}{m_3}\right)
\end{aligned} \tag{3}
$$

Here τ,θ,ψ - are space coordinates (local coordinates or parameters in parallelogram:

$\tau \in [\tau_*, \tau^*]$ and $\tau_* \le \tau^*$ usually are non-negative constants; and $\psi \in [0.2\pi]$; $\theta \in [0, 2\pi h]$ where $h \in R$); t is a time coordinate.

• Function $r_1(\theta,t)$ describes the geometric shape and behavior in time of a "large" radius or plane baseline in terms of papers [1, 2] of the primary torus (toroidal surface).

Shape is a circle, if this function is a constant or circle with a time-varying radius if this function depends only on time argument. This plane curve may be of quite complex shape, and can be described as a super formula J. Gielis (2).

Behavior - The initial toroidal surface can change in time, and this change and its speed are given by this function, depending on the time argument.

• In the analytic representation (3), the term describes the behavior and shape of the line wound on the initial toroidal surface (or the behavior of the small radius of the torus). In general these two functions determine the toroidal surface and its movements. And the parameters and determine the rule of winding by a given line of the toroidal surface

Shape is a circle, if the function is a constant or circle with a time-varying radius if this function depends only on time argument; i.e. we have a classic torus if both functions and are constants or torus with changing radii, if these functions depend only on the time argument. This plane curves may be of quite complex shapes, and can be described as a super formula J. Gielis (2), with different parameters; i.e. we apply formula (2) two times but with different values of the parameters.

Behavior - The shape can change in time, and this change and its speed are given by these functions, depending on the time argument.

Rule of winding around torus is determined by two parameters. Parameter n_2 determines the number of rings around the torus, while the other parameter determines the amount of circumvention of the torus after which the above n_2 rings appear (i.e.

$\nu = n_2/m_2$). In representation (3) we give a regular winding rule, in reality it can be of a rather complex kind that will be described by a function g(θ) (similar representation (1)).

The direction of the winding is determined by the sign before the fraction; The winding is counter-clockwise or clockwise.(some exaples are presented in Fig. 1)

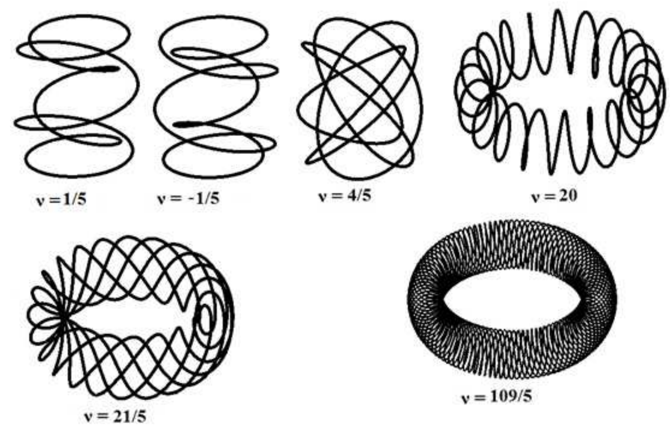

Fig. 1.

• Function $r_3(\tau, \psi, t)$ determines the radial cross section of the body itself. This body has the basic line described above the toroidal line and for each value of the argument θ defines the shape of the radial section. But this shape may be changed with time.

A very interesting case where all three functions $r_1(\theta, t), r_2(\theta, t)$ and $r_3(\theta, t)$ depend on a time argument and are periodic - these are the patterns of three different oscillatory movements that can not be described without such a superposition.

• The vector function $T(t)$ defines, as before in (1), a parallel transfer of the whole body without changing the geometric shape and structure, and the function $M(t)$ determines the rotation of this body around the axis OZ.

REFERENCES

1. Tavkhelidze, I., Ricci, P.E. Classification of a wide set of geometric figures, surfaces and lines - (Trajectories), 124^0. *Rendiconti Accademia Nazionale dell Scienze detta dei XL Memorie di Matematica a Applicazioni*, **XXX**, 1 (2006), 191-212.

2. Tavkhelidze, I., Caratelli, D., Gielis, J., Ricci, P.E., Rogava, M., Transirico M. On a geometric model of bodies with "complex" configuration and some movements - modeling in mathematics. *Chapter 10, Atlantis Transactions in Geometry 2*, DOI 10.2991/978-94-6239-261-8_10- Springer (2017), 129-158.

3. Goemans W., van de Woestyne I. Clelia curves, twisted surfaces and Plücker's conoidin Euclidean and Minkowski 3-space. *Contemporary Mathematics*, **674** (2016), 59-73.

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