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ON THE NUMERICAL SOLUTION OF TWO-DIMENSIONAL MITCHISON NONLINEAR MODEL

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Abstract. In the note for the construction of numerical solution of two-dimensional Mitchison nonlinear partial differential system the variable directions difference scheme and the difference scheme corresponding to the average method are used. Practical realization of those algorithms is fulfilled and comparative analysis of the obtained results are carried out. Numerical experiments are in accordance with theoretical findings. On basis of experiments the corresponding tables of data is given.

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By the nonlinear partial differential equations a lot of natural processes are described. Among them, is one of the important mathematical model, that describes vein formation in the leaves of plants. This model was proposed by J. Michison [1].

Investigations for one-dimensional analog of Michison system are carried out in [2].

Beginning from the basic works the methods of constructing of effective algorithms for the numerical solution of the multi-dimensional problems of the mathematical physics and the class of problems solvable with the help of these algorithms were essentially extended [3] - [7]. Those algorithms mainly belong to the methods of splitting-up or sum approximation. Some schemes of the variable directions are constructed and studied in the following work too [8].

Some questions of construction and investigation of the schemes of variable directions and the average model of sum approximation as well as difference schemes for one-dimensional case for the type Michison systems are discussed in the papers [9] - [15].

Let us take special form of function f and consider the following two-dimensional system:

$$\begin{gathered}\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(V_1\frac{\partial U}{\partial x}\right)+\frac{\partial}{\partial y}\left(V_2\frac{\partial U}{\partial y}\right),\\ \frac{\partial V_1}{\partial t}=-V_1+g_1\left(V_1\frac{\partial U}{\partial x}\right),\frac{\partial V_2}{\partial t}=-V_2+g_2\left(V_2\frac{\partial U}{\partial y}\right)\end{gathered}\tag{1}$$

with the initial

$$U(x,y,0)=U_0(x,y),\quad V_1(x,y,0)=V_{10}(x,y),\quad V_2(x,y,0)=V_{20}(x,y)\tag{2}$$

and boundary conditions

$$U(x,0,t)=U(x,1,t)=U(0,y,t)=U(1,y,t)=0,\quad t\in[0,T],\quad x,y\in[0,1]. \tag{3}$$

Here g_α, U_0, $V_{\alpha 0}$ $\alpha=1,2$ are given sufficiently smooth functions, such that:

$$\begin{gathered} V_{\alpha 0}\geq \delta_0,\quad \delta_0=const>0,\quad (x,y)\in\bar{\Omega}, \\ g_0\leq g_\alpha(\xi_\alpha)\leq G_0, |g_\alpha^{'}(\xi_\alpha)|\leq G_1,\quad \xi_\alpha\in R, \\ \alpha=1,2, \end{gathered} \tag{4}$$

where δ_0, g_0, G_0, G_1 and T are some positive constants.

Later we shall follow known notations for the construction of the grid on the domain $\bar{Q}$:

$$\begin{gathered} \bar{\omega}_h=\{(x_i,y_j)=(ih,jh)\}, \\ \bar{\omega}_{1h}=\{(x_i,y_j)=((i-1/2)h,jh)\}, \\ \bar{\omega}_{2h}=\{(x_i,y_j)=(ih,(j-1/2)h)\}, \\ i,j=0,...,M,Mh=1, \\ \omega_h=\Omega\cap\bar{\omega}_h,\gamma_h=\bar{\omega}_h/\omega_h,\bar{\omega}_h=\omega_h\cup\gamma_h, \\ \omega_\tau=\{t_k=k\tau,k=0,...,N,N\tau=T\}. \end{gathered} \tag{5}$$

Following the known notations [7], let us correspond to problem (1) the following difference scheme of variable directions [10]:

$$\begin{gathered} u_{1t}=(\hat{v}_1\hat{u}_{1\bar{x}})_x+(v_2u_{2\bar{y}})_y, u_{2t}=(\hat{v}_1\hat{u}_{1\bar{x}})_x+(\hat{v}_2\hat{u}_{2\bar{y}})_y, \\ v_{1t}=-\hat{v}_1+g_1(v_1u_{1\bar{x}}), v_{2t}=-\hat{v}_2+g_2(v_2u_{2\bar{y}}), \\ u_1(x,y,0)=U_0(x,y), u_2(x,y,0)=U_0(x,y),\quad (x,y)\in\bar{\omega}_h, \\ v_1(x,y,0)=V_{10}, v_2(x,y,0)=V_{20},\quad (x,y)\in\bar{\omega}_{2h}, \\ u_1(x,y,t)=u_2(x,y,t)=0, \\ (x,y,t)\in\gamma_h\times\omega_\tau. \end{gathered} \tag{6}$$

For problem (1) let us also consider the following difference scheme corresponding to the average method [9]:

$$\begin{gathered} u_{1t}=(\hat{v}_1\hat{u}_{1\bar{x}})_x,\quad u_{2t}=(\hat{v}_2\hat{u}_{2\bar{y}})_y, \\ v_{1t}=-\hat{v}_1+g_1(v_1u_{1\bar{x}}), v_{2t}=-\hat{v}_2+g_2(v_2u_{2\bar{y}}), \\ u_1(x,y,0)=U_0(x,y), u_2(x,y,0)=U_0(x,y),\quad (x,y)\in\bar{\omega}_h, \end{gathered} \tag{7}$$

$$v_1(x,y,0) = V_{10}, v_2(x,y,0) = V_{20}, \quad (x,y) \in \bar{\omega}_{2h},$$
$$u_1(x,y,t) = u_2(x,y,t) = 0,$$
$$(x,y,t) \in \gamma_h \times \omega_\tau,$$
$$u = \eta_1 u_1 + \eta_2 u_2, \quad \eta_1\eta_2 > 0, \eta_1 + \eta_2 = 1,$$
$$u_1 = u, \quad u_2 = u.$$

Using algorithms, proposed in (6) and (7) let us carry out comparative analysis of the numerical results for schemes above.

Let us take the right side of the corresponding nonhomogeneous system (1) so that the solution of problem (1), (2) is:

$$\begin{aligned} U(x,y,t) &= xy(1-x)(1-y)(1+t), \\ V_1(x,y,t) &= 1 + xy(1-x)(1-y)(1+t+t^2), \\ V_2(x,y,t) &= 1 + xy(1-x)(1-y)(1+t+t^3). \end{aligned} \tag{8}$$

CPU time and errors for variable directions difference scheme (6) are given in the table 1 and the CPU time and errors for the scheme (7) are given in the table 2.

Table 1: CPU time and error for solution U, V_1 , V_2 applying variable directions difference scheme (6).

t	CPU time	Error U	Error V_1	Error V_2
0.2	0.074	0,00013912790131447	0,00000712766408961	0,00002916084998672
0.4	0.148	0,00022425859907783	0,00001730244454379	0,00009005618525060
0.6	0.224	0,00031286373416026	0,00004804529821700	0,00017715471240609
0.8	0.301	0,00040788793632886	0,00009668298990784	0,00028758192640277
1.0	0.378	0,00051151056363487	0,00016425091499817	0,00041715052893787

Table 2: CPU time and error for solution U, V_1 , V_2 applying variable directions difference scheme (7).

t	CPU time	Error U	Error V_1	Error V_2
0.2	0.079	0,00006973950435170	0,00001634140038553	0,00001662571352523
0.4	0.150	0,00007422011594080	0,00003786305693865	0,00003781271060488
0.6	0.221	0,00007890208614024	0,00006202878270467	0,00005790906416947
0.8	0.295	0,00008480943243865	0,00008875495749039	0,00007978157763566
1.0	0.369	0,00009205402490850	0,00011818090303972	0,00010625023389577

The approximation error for variable direction difference scheme (6) is smaller compared with scheme (7). However, CPU time is better for scheme (7) than scheme (6). We have experimented a number of other experiments and observed the same situations. In all cases results of numerical experiments are in accordance with theoretical findings.

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