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# ON THE NUMERICAL SOLUTION OF ONE-DIMENSIONAL ANALOG OF MITCHISON NONLINEAR PARTIAL DIFFERENTIAL SYSTEM

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**Abstract**. Finite-difference scheme for the numerical solution of one-dimensional analog of Mitchison nonlinear partial differential system is considered. Many numerical experiments are conducted and on the basis of that graphical illustrations are constructed.

**Keywords and phrases**: Nonlinear partial differential equations, vein formation, finite difference scheme.

**AMS subject classification (2010)**: 35Q80, 35Q92, 65N06, 65Y99.

Two-dimensional model describing the vein formation of young leaves is given and some qualitative and structural properties of solutions of this model are established in [1]. In [2] investigations for one-dimensional analog are carried out. In biological modeling there are many works where this and many models of similar processes are also presented and discussed (see, for example, [3]-[6] and references therein). Many scientific works are devoted to investigation and numerical resolution of different kinds of initial-boundary value problems for the model described in [1] and its one-dimensional and multi-dimensional analogs (see, for example, [7]-[17] and references therein). Let us consider the following initial-boundary value problem for one-dimensional analog of the vein formation model [1]. One-dimensional analog of the Mitchison model was investigated in [2] by J.Bell, C.Cosner and W.Bertiger. They considered the following problem:

$$
\begin{aligned}
&\frac{\partial S}{\partial t} = \frac{\partial}{\partial x}\left(d\frac{\partial S}{\partial x}\right), \quad (x,t)\in(0,1)\times(0,T],\\
&\frac{\partial d}{\partial t} = -d + g\left(d\frac{\partial S}{\partial x}\right), \quad (x,t)\in(0,1)\times(0,T],
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
&S(0,t)=0, \quad d\frac{\partial S}{\partial x}\bigg|_{x=1} = \psi, \quad t\in[0,T],\\
&S(x,0)=S_0(x), \quad d(x,0)=d_0(x)\geq\delta_0=const>0, \quad x\in[0,1],
\end{aligned}
\tag{2}
$$

where $0<g_0\leq g(\xi)\leq G_0$, $g_0, G_0, T, \psi$ are positive constants, $g, S_0, d_0$ are given sufficiently smooth functions and $d$, $S$ are unknown functions.

Let us consider the system which is analogical to system (1), where $\varphi(x,t)$ and $f(x,t)$ are given functions:

$$
\begin{cases}
\dfrac{\partial d}{\partial t} = -d + g\left(d\dfrac{\partial s}{\partial x}\right) + \varphi(x,t), \\[2ex]
d\,\dfrac{\partial S}{\partial x}\bigg|_{x=1} = \psi, \\[2ex]
\dfrac{\partial S}{\partial t} = \dfrac{\partial}{\partial x}\left(d\dfrac{\partial S}{\partial x}\right) + f(x,t)
\end{cases}
$$

and correspond to it the following finite-difference scheme:

$$
\begin{cases}
\dfrac{d_i^{j+1} - d_i^j}{\tau} = -d_i^{j+1} + g\left(d_i^j \dfrac{s_i^j - s_{i-1}^j}{h}\right) \\[2ex]
d_M^{j+1} \dfrac{s_M^j - s_{M-1}^j}{h} = \psi, \\[2ex]
\dfrac{s_i^{j+1} - s_i^j}{\tau} = \dfrac{d_{i+1}^{j+1} \frac{s_{i+1}^{j+1} - s_i^{j+1}}{h} - d_i^{j+1} \frac{s_i^{j+1} - s_{i-1}^{j+1}}{h}}{h} + f_i^{j+1}
\end{cases} \tag{3}
$$

After some transformations (3) gets the following form:

$$
\begin{cases}
d_i^{j+1} - d_i^j = \dfrac{1}{\tau}\left(d_i^j + \tau g_i^j + \tau\varphi_i^j\right), \\[2ex]
s_M^{j+1} = \dfrac{\psi h}{d_M^{j+1}} + s_{M-1}^{j+1}, \\[2ex]
-\left(h^2 + \tau d_1^{j+1} + \tau d_2^{j+1}\right) S_1^{j+1} + \tau d_2^{j+1} S_2^{j+1} = -\tau d_1^{j+1} S_0^{j+1} - h^2 S_1^j - \tau h^2 f_1^{j+1}, \\[2ex]
\tau d_i^{j+1} S_{i-1}^{j+1} - \left(h^2 + \tau d_i^{j+1} + \tau d_{i+1}^{j+1}\right) S_i^{j+1} + \tau d_{i+1}^{j+1} S_{i+1}^{j+1} = -h^2 S_i^j - \tau h^2 f_i^{j+1}, \\[2ex]
\tau d_{M-1}^{j+1} S_{M-2}^{j+1} - \left(h^2 + \tau d_{M-1}^{j+1}\right) S_{M-1}^{j+1} = \tau h\psi - h^2 S_{M-1}^j - \tau h^2 f_{M-1}^{j+1},
\end{cases} \tag{4}
$$

where $i = 0, 1, ..., M$, $j = 0, 1, ...N$, $Mh = 1$, $N\tau = T$, $S_i^j = S(hi, \tau j)$, $d_i^j = d(hi, \tau j)$, $g_i^j = g(hi, \tau j)$, $\varphi_i^j = \varphi(hi, \tau j)$, $f_i^j = f(hi, \tau j)$.

For the test experiment, we take the right-hand sides such that the exact solution is given by

$$
\begin{gathered}
S(x,t) = 10(x - x^2)(1+t), \quad d(x,t) = 10(x - x^2)(1 + t + t^2), \\[2ex]
g(\xi) = \frac{1}{1+\xi^2} + 1, \quad 0 < g_0 = 1 \le g(\xi) \le 2 = G_0,
\end{gathered} \tag{5}
$$

Some graphical illustrations of those numerical results for experiment (5) are given in Fig.1. and Fig.2. The intermediates of absolute error are indicated below the graphs.

The graphs in Fig.1. illustrate numerical results for the case $t = 0.5$ end $\psi = 0.9$

The graphs in Fig.2. illustrate numerical results for the case $t = 0.5$ end $\psi \approx d^0_M \frac{S^0_M - S^0_{M-1}}{h} \approx 0.891804977$

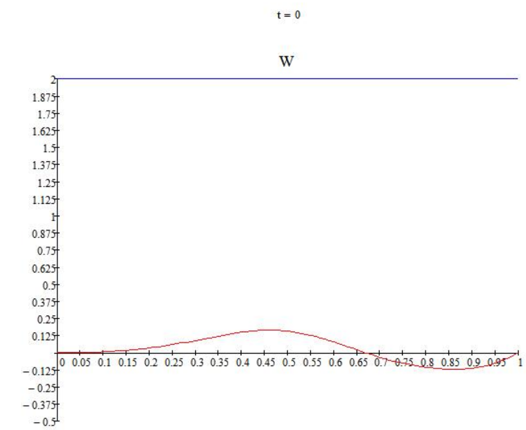


Figure 1: $\psi = 0.9$ Absolute error $\in [0.005373114; 0.019314282]$.

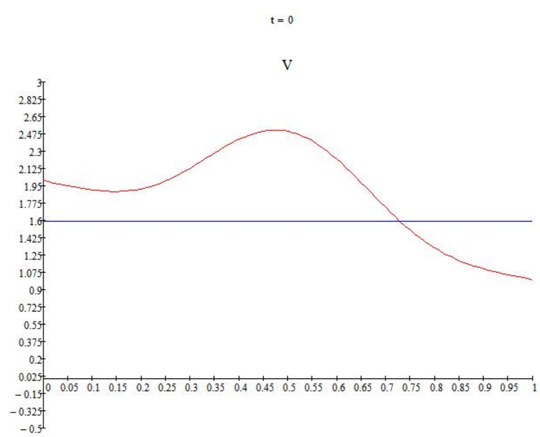


Figure 2: $\psi \approx 0.891804977$ Absolute error $\in [0.004595881; 0.011474271]$.

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