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MODELLING OF GAS CONVEYANCE IN PIPE NETWORK

Teimuraz Davitashvili

Abstract. The present work is devoted to the one dimensional mathematical model studding gas flow in the inclined and branched pipeline. A simplified mathematical model governing the dynamics of gas non-stationary flow in the inclined, branched pipeline is constructed. Formula describing gas pressure distribution along the branched pipeline is presented.

Keywords and phrases: Modeling, gas flow, inclined, branched pipeline.

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Gas distribution network is a complex system containing a number of essential elements in production, transportation, storage and distribution. At present in the natural gas transportation service pipelines have acquired popularity in comparison with the railway, marine auto service systems and become an object of studding [1]-[9]. Owing to the friction forces and heat transfer moving gas endure losses of energy in pipeline[3, 7, 8, 9]. For supporting the normal operating conditions it is necessarily to feed gas periodically with enough energy by compression stations. For solving this problem determination of the gas pressure and flow rate temporally and spatial distribution along the pipeline is a needful step. Studding of the gas flow even in the simple horizontal pipeline is not easy task and it becomes more complicated for inclined and branched pipelines. The purpose of this study is to determine distribution of gas pressure along the inclined and branched pipeline. For studding this problem we based on the following nonlinear system of PDE's governing gas one-dimensional flow in the branched and inclined pipeline [1, 3, 4, 7, 9].

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} + q^* \delta(x - x^*) = 0, \tag{1}$$

$$\frac{\partial(\rho v S)}{\partial t} + \frac{\partial(PS + \rho v^2 S)}{\partial x} + |\tau| \pi D + \rho S g \sin\theta = 0, \tag{2}$$

where $\rho(x,t)$ is density, $P(x,t)$ is pressure, $v(x,t)$ is speed, S is the cross-sectional area of pipeline, D is diameter, τ is the tangential stress between gas and the inner wall of gas pipe, δ is Dirac function and x^* is placement of an offshoot in the pipeline, q^* is volumetric gas consumption in the branch-line θ, is inclination angle. For convenience and practical purposes (as consumer demand is usually expressed in mass flow terms), let us replace the speed v in equations (1)-(2) by the term of flow rate q, where q is defining by the following equation (3).

$$q = \rho v S, \tag{3}$$

If we represent the tangential stress by the Fanning friction factor, $f = \dfrac{2|\tau|}{\rho v^2}$, also taking into account the relation $P = c^2\rho$, where c is velocity of sound, valid for an isothermal processes and Dirac's function in the following approximation:

$$\delta(x - x^*) = \frac{\alpha}{\pi[1 + \alpha^2(x - x^*)^2]},$$

where $\alpha \to \infty$, then we have

$$\frac{\partial P}{\partial t} + \frac{c^2}{S}\frac{\partial q}{\partial x} + \frac{c^2 q^* \alpha}{\pi[1 + \alpha^2(x - x^*)^2]} = 0, \tag{4}$$

$$\frac{\partial q}{\partial t} + S\frac{\partial P}{\partial x} + \frac{c^2}{S}\frac{\partial}{\partial x}\left(\frac{q^2}{P}\right) + \frac{2fc^2 q|q|}{DSP} + \frac{Sg\sin\theta}{c^2}P = 0, \tag{5}$$

The system of equations (4), (5) with the additional constraints $0 \le x < L$, $t \ge 0$ is solving by the following initial and boundary conditions

$$P(x,0) = P_0(x), \quad q(x,0) = q_0(x), \tag{6}$$

$$P(0,t) = P_1(t), \quad q(L,t) = q_1(t). \tag{7}$$

With the purpose of finding approximate analytical solution of the system of equations (4), (5) it is necessary to make some more simplifications which leads us to estimate the magnitude of each term in the system of equations (4), (5) for typical values of the variables involved in the process. Let us consider high pressure gas transmission pipeline, where the dynamic variations are taking hours to complete a significant change [1, 7-9]. These conditions together with the estimated magnitude for the terms in the gas dynamic equation gives possibility to ignore the third and first terms in the momentum equation (the hypothesis that the boundary conditions do not change quickly and that the capacity of gas duct is relatively large). Then, the set of PDE's can be expressed by the following system of Equations (8), (9),

$$\frac{\partial P}{\partial t} + \frac{c^2}{S}\frac{\partial q}{\partial x} + c^2 q^* \delta(x - x^*) = 0, \tag{8}$$

$$\frac{\partial P}{\partial x} = -\frac{2fc^2q^2}{DS^2P} - \frac{g\sin\theta}{c^2}P. \tag{9}$$

Following Charny [4] using linearization trick in the equation (9) and introducing the following denotations $\overline{\lambda} = \dfrac{|q|}{P}$; $\phi(x) = c^2 q^* \delta(x - x^*)$ we have:

$$\frac{\partial P}{\partial t} + \frac{c^2}{S}\frac{\partial q}{\partial x} + \phi(x) = 0, \tag{10}$$

$$\frac{\partial P}{\partial x} = -\frac{2f\overline{\lambda}c^2 q}{DS^2} - \frac{g\sin\theta}{c^2}P, \tag{11}$$

Differentiation of equation (11) gives

$$\frac{\partial^2 P}{\partial x^2} = -\frac{2f\overline{\lambda}c^2}{DS^2}\frac{\partial q}{\partial x} - \frac{g\sin\theta}{c^2}\frac{\partial P}{\partial x},$$

further substituting $\frac{\partial q}{\partial x} = -\frac{S}{c^2}\left(\phi(x) + \frac{\partial P}{\partial t}\right)$ from the equation (10) into the last equation, gives

$$\frac{\partial^2 P}{\partial x^2} = \frac{2f\overline{\lambda}}{DS}\frac{\partial P}{\partial t} - \frac{g\sin\theta}{c^2}\frac{\partial P}{\partial x} + \frac{2f\overline{\lambda}}{DS}\phi(x). \tag{12}$$

Equation (12) can be rewriting in the following form

$$\frac{\partial P}{\partial t} = a\frac{\partial^2 P}{\partial x^2} + b\frac{\partial P}{\partial x} + \phi(x), \tag{13}$$

where $a = \frac{DS}{2f\overline{\lambda}}, \quad b = -\frac{DSg\sin\theta}{2f\overline{\lambda}c^2}, \quad \phi(x) = \frac{DSc^2q^*}{2f\overline{\lambda}}\frac{\alpha}{\pi[1+\alpha^2(x-x^*)^2]}$.

As it is known the substitution

$$P(x,t) = U(x,t)e^{\beta t+\mu x}$$

where $\beta = -\frac{b^2}{4a}, \quad \mu = -\frac{b}{2a}$, leads to the following non-homogeneous equation

$$\frac{\partial U}{\partial t} = a\frac{\partial^2 U}{\partial x^2} + e^{-\beta t-\mu x}\phi(x), \tag{14}$$

The equation (14) with the additional constraints $0 \le x < L, \quad t \ge 0$ is solving by the following initial and boundary conditions

$$U(x,0) = U_0(x), \tag{15}$$

$$U(0,t) = U_0(0,t), \quad \frac{\partial U(L,t)}{\partial x} = 0. \tag{16}$$

And exact solution of the second order non-homogeneous heat (diffusion) linear parabolic partial differential equation with (15) and (16) has the following form:

$$U(x,t) = \int\limits_{-\infty}^{+\infty} U_0(\varsigma)G(x,\xi,t)d\xi + \int\limits_0^t\int\limits_{-\infty}^{+\infty}\phi(\varsigma,\tau)G(x,\xi,\tau)d\xi d\tau,$$

where $G(x,\xi,t) = \frac{1}{2\sqrt{\pi a t}}\sum\limits_{n=-\infty}^{n=+\infty}\left\{exp\left[-\frac{(x-\varsigma+2nL)^2}{4at}\right] + exp\left[-\frac{(x+\varsigma+2nL)^2}{4at}\right]\right\}$.

And finally we have:

$$P(x,t) = e^{\beta t+\mu x}\left[\int\limits_{-\infty}^{+\infty} U_0(\varsigma)G(x,\xi,t)d\xi + \int\limits_0^t\int\limits_{-\infty}^{+\infty}\phi(\varsigma,\tau)G(x,\xi,\tau)d\xi d\tau\right].$$

Thus for simplified mathematical model derived from the nonlinear system of one-dimensional partial differential equations governing the dynamics of gas non-stationary flow in the inclined, branched pipeline the analytical expressions of gas pressure and flow rate distribution along the branched pipeline is obtained.

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Author(s) address(es):

Teimuraz Davitashvili
I. Javakhishvili Tbilisi State University
Faculty of Exact and Natural Sciences
I. Vekua Institute of Applied Mathematics
University str. 2, 0186 Tbilisi, Georgia
E-mail: tedavitashvili@gmail.com