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MODELING OF CURRENCY EXCHANGE RETURN VARIANCE

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Abstract. This paper considers currency exchange rate volatility problem modeling and using these models in exchange return forecasting. On the biases of the ARCH and GARCH class models there are developed models that describe Georgian LARI exchange rates volatility according USD and EUR currencies.

Keywords and phrases: forecasting, currency exchange rates; ARCH and GARCH models.

AMS subject classification (2010): 62P20.

Trading currency position can be considered as a financial instrument. In financial trading, one of the key tasks is to try to capture the movement of the underlying asset, which is usually known as volatility. The volatility is the conditional standard deviation of underlying assets return (r_t) and is denoted by (σ_t). This volatility depends on the trading each day and some previous days [1]. As other financial time series, one main characteristics of the volatility of currency exchange return is that it appears in clusters (see Fig. 1). In recent years, especially with regard to financial applications, ARCH [2] and Generalize ARCH (GARCH) models have received ample attention for dealing heteroskedasticity [3]. The aim of this paper is to assess empirically the adequacy of this class of models in currency exchange return volatility forecasting. To accomplish this, we consider three currency (USD, EUR and Georgian LARI) exchange rate sequences and evaluate how well the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model replicates the empirical nature of these sequences. To assess the forecast accuracy of GARCH model we need time series to be stationary. One way to make financial time series stationary is to use continuously compound rate of return. If we denote the exchange rate at time t by P_t, we can transform the sequence of exchange rates as follows: $r_t = \ln(P_t) - \ln(P_{t-1})$, where (r_t) is the continuously compound rate of return at time t. The compounded daily return, (r_t) can be computed simply by taking first difference of the natural logarithms of daily prices. The GARCH (n, m) model can be expressed as:

$$\begin{aligned} &\eta_t = \sigma_t \varepsilon_t, \\ &\sigma_t^2 = \omega + \sum_{i=1}^{n} \alpha_i \eta_t^2 + \sum_{j=1}^{m} \beta_j \sigma_{t-j}^2, \end{aligned} \tag{1}$$

where $\varepsilon_t \approx N(0,1)iid, \eta_t = r_t - \mu_t$, the parameter α_i is the ARCH parameter and β_j is the GARCH parameter and $\omega \geq 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ and $\sum\limits_{i=1}^{max(n,m)} (\alpha_i + \beta_i) < 1$.

In this paper we use a large sample size (more than 2 300 observations) in order to get best results in estimating standard errors even with heteroskedasticity. We will investigate if our large set of financial data can be fitted to a time series model, and which model will provide the best fit. Fig. 1 shows the continuously compounded daily returns. This figure shows behavior of currency trading return and clearly demonstrate some kind of dependence between conditional variances in consecutive moments. In other words, there is ARCH affect and we will examine GARCH model for these time series.

The GARCH model also takes into account volatility clustering and tail behavior- the important characteristics of financial time series. It provides an accurate assessment of variances and covariances through its ability to model time-varying conditional variances. GARCH allows for modeling the serial dependence of the volatility. Due to the conditional property of GARCH, the mechanism depends on the observations of the immediate past, thus including past variances into explanation of future variances. Financial return volatility data is highly influenced by time dependence, which can cause volatility clustering. Time series such as this can be parameterized using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, which can then be used to forecast volatility.
Other two time series, USD/EUR, and LARI/EUR, figures like figure 1 and indicate that

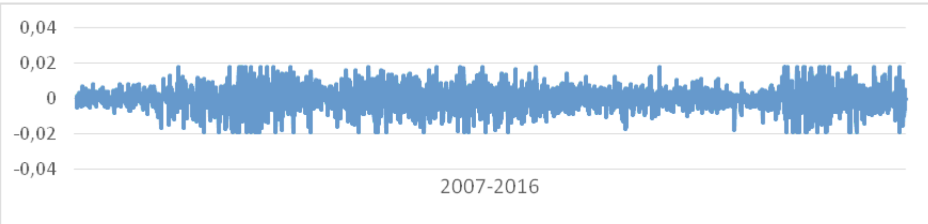

Figure 1: Time series of logarithmic relative exchange rates of LARI/USD

there are an ARCH effect and there are some stationary parts and much more non stationary parts. The financial return volatility data is highly influenced by time dependence, which, in these cases, is evidenced in volatility clustering. USD/EUR Exchange Rate. We used GARCH (1, 1), GARCH (1, 2) and GARCH (2, 1) models and have obtained following results: 1. GARCH (1, 1): $\sigma_t^2 = 0.0011093 + 0.0354899\eta_{t-1}^2 + 0.9612536\sigma_{t-1}^2$, 2. GARCH (2, 1): $\sigma_t^2 = 0.0010388 + 0.086820\eta_{t-1}^2 + 0.0560561\eta_{t-2}^2 + 0.9660324\sigma_{t-1}^2$, 3. GARCH (1, 2): $\sigma_t^2 = 0.0020431 + 0.0628202\eta_{t-1}^2 + 0.1186592\sigma_{t-1}^2 + 0.8124835\sigma_{t-2}^2$. All these model have the same ACF functions as is shown in figure 2. Except two residuals,

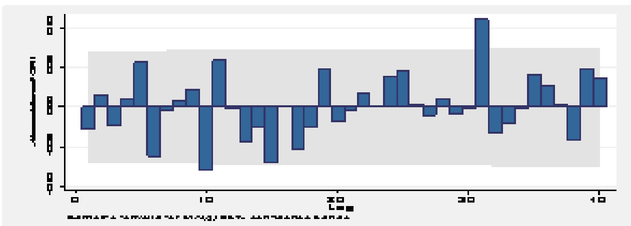

Figure 2: Diagrame of autocorrelation function for model residuals

all others are within 2 standard deviations of the sample autocorrelation. GARCH (1, 2) do not fit conditions given above in (1). For the left, now we have to check normality of these models residuals distribution. Both of them are slightly skewed to the right side, we will see that both are slightly skewed to the right side (skewness - 0.03), but both have about the same kurtosis of about 2.77 which do not gives enough arguments to reject formality of their residuals. LARI/EUR Exchange Rate Let consider the same GARCH models as previous. 1. GARCH (1, 1): $\sigma_t^2 = 0.0019377 + 0.0473765\eta_{t-1}^2 + 0.9486566\sigma_{t-1}^2$ 2. GARCH (2, 1): $\sigma_t^2 = 0.0019363 + 0.0484219\eta_{t-1}^2 + 0.0011433\eta_{t-2}^2 + 0.9487533\sigma_{t-1}^2$ 3. GARCH (1, 2): $\sigma_t^2 = 0.0019851 + 0.0484657\eta_{t-1}^2 + 0.9234984\sigma_{t-1}^2 + 0.0239673\sigma_{t-2}^2$ Figure 3 exhibits ACF plots of these models residuals. For all these GARCH models

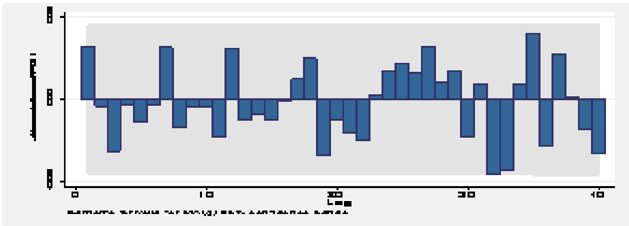

Figure 3: Diagram of autocorrelation function for model residuals

residuals skewness are almost the same with negative sign (-0.098) and kurtosis - 2.93. We can not reject the hypothesis that residuals of these models follows a normal distribution based on these evidences. In this case, only GARCH (1, 1) and GARCH (1, 2) models are most appropriate. LARI/USD Exchange Rate. The ACF, as the name implies, shows an autocorrelation or relationship among observations. Fig. 4 gives evidence that shows existence of autocorrelation in this time series. In other words, there is serial dependence in the variance of the data. A geometrically decaying ACF plot would indicate that we should use MA type model. Notice that the first lag of the ACF plot is close to zero, indicating that our data set does not appear to have much correlation between observations.The PACF (see Fig. 4) is used to determine the appropriate order of a fitted ARIMA data set. The PACF is used to determine the appropriate order of a fitted MA data set. The autocorrelation function of MA (2) model's residuals is shown

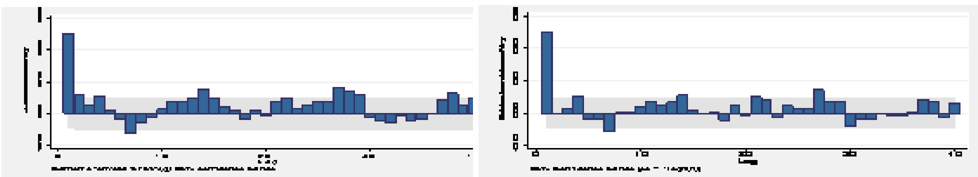

Figure 4: Diagrams of autocorrelation function and partial autocorrelation function

in Fig. 5. According to the ACF the data looks almost random (see Fig. 5) and certainly shows no easily discernible pattern. This would support the appearance of the time series plot since the plot looks a lot like white noise except for the change in spread (variation) of observations. Such heteroskedasticity would most likely not be evident in a truly random data set. Now we will combine GARCH model with MA (2). The results

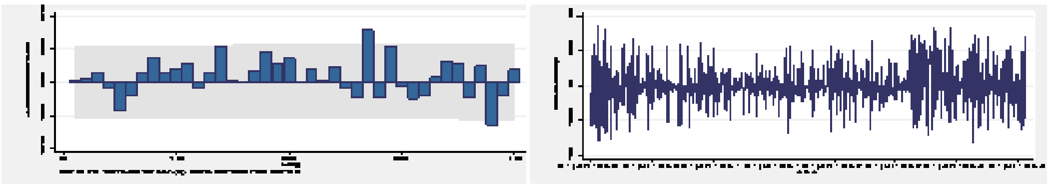

Figure 5: Diagram of autocorrelation function and MA(2) model residual plot.

of this are: MA (2) and GARCH (1, 1): $r_t = 0,01094 + 0.3245992\varepsilon_{t-1} + 0.1330251\varepsilon_{t-2}$, $\sigma_t^2 = 0.000146 + 0.125862\eta_{t-1}^2 + 0.8883365\sigma_{t-1}^2$ We don't represent other models because of their parameters some of which is insignificant and their less fitted characteristics to given time series patterns. Our choices of the best models in above sections are based on assessing the residuals of the considered models. For this goal, we looked up of ACF plots of residuals, probability plots of the residuals and assessed each model with respect to the Ljung - Box statistic. Then, to check the normality assumption of the errors, we used the normal probability plots and histograms of the fitted GARCH models which showed that their errors are very close to normal distribution. The skewness and kurtosis values did not show exactly symmetric matters of errors but tails are not too much heavier than normal distribution. In addition, we simulated data from all GARCH models and evaluated the simulation data with respect to the given empirical time series. The comparison of these characteristics of considered models is shown. In general, GARCH (1, 1) model was the best in all cases.

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