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RENORMDYNAMICS OF COUPLING CONSTANTS AND MASSES

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Abstract. In the Standard Model of Particle Physics (SM), the values of the coupling constants and masses of particles depends on the scale according to the Renormdynamic motion equations. For the electron and nucleon masses, electrodynamic and pion-nucleon fine structure constants we have an empirical relation: $m_e/\alpha \simeq m_N/\alpha_{\pi N}$. We take the relation $m/\alpha = const$ as an integral of renormdynamic motion equations for m and α, find exact form of the β function in the minimal mass parametrization and find the exact solution of the corresponding renormdynamic motion equations. In a Fundamental Theory the values of the Fundamental Physical Constants will also be defined from the solutions of the corresponding renormdynamic motion equations. In SM, minimal supersymmetric extension of the SM, standard pion-nucleon field theory and other models is shown how to define the values of coupling constants and masses.

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We say that we find **New Physics** (NP) when either we find a phenomenon which is forbidden by SM in principal - this is the qualitative level of NP - or we find a significant deviation between precision calculations in SM of an observable quantity and a corresponding experimental value. In the Universe, matter has manly two geometric structures, homogeneous and hierarchical. The homogeneous structures are naturally described by real numbers with an infinite number of digits in the fractional part and usual archimedean metrics. The hierarchical structures are described with p-adic numbers with an infinite number of digits in the integer part and non-archimedean metrics, [1]. A discrete, finite, regularized, version of the homogenous structures are homogeneous lattices with constant steps and distance rising as arithmetic progression. The discrete version of the hierarchical structures is hierarchical lattice-tree with scale rising in geometric progression. There is an opinion that present day theoretical physics needs (almost) all mathematics, and the progress of modern mathematics is stimulated by fundamental problems of theoretical physics. In QFT the existence of a given theory means that we can control its behavior at some scales (short or large distances) by renormalization theory [2]. If the theory exists, than we want to solve it, which means to determine what happens on other (large or short) scales. This is the problem (and content) of Renormdynamics. The result of the Renormdynamics, the solution of its discrete or continual motion equations, is the effective QFT on a given scale (different from the initial one). We will call RDF functions $g_n = f_n(t)$ which are solutions of the RD motion equations $\dot{g}_n = \beta_n(g), 1 \leq n \leq N$. In the simplest case of one coupling constant the function $g = f(t)$ is constant, $g = g_c$ when $\beta(g_c) = 0$, or is invertible (monotone). Indeed,

$$\dot{g} = f'(t) = f'(f^{-1}(g)) = \beta(g). \tag{1}$$

Each monotone interval ends by UV and IR fixed points and describes corresponding phase of the system. The simplest case of the classical dynamics, the Hamiltonian system with one degree of freedom, is already two dimensional, so we have no analog of one charge renormdynamics. The regular Hamiltonian systems of the classical mechanics are defined on the even dimensional phase space, so there is no analog of the three dimensional renormdynamics for the coupling constants of the SM. The fixed points of renormdynamics belong to a set of solutions of the polynomial system of equations $\beta_n(g) = 0, 1 \leq n \leq N$, in the perturbative renormdynamics. Describing the solutions is the task of contemporary algebraic and computational geometry. The quantitative values and qualitative content of the given field theory depend on the scale (parameter, e.g. $\mu-$renormalization point, $g = g(\mu),\ A = A(\mu)$). In QCD e.g. the effective action has the following form:

$$S(\mu) = \frac{1}{g^2(\mu)} \int d^D x \mathcal{L}(A(\mu)), \tag{2}$$

variation with respect to the change of scale gives

$$\delta S = -2\frac{\beta(g)}{g\mu}\delta\mu S + \frac{1}{g^2}\int d^D x \frac{\delta\mathcal{L}}{\delta A}\delta A \tag{3}$$

and the following two statements are equivalent:

$$\delta S = 0,\ \beta(g) = 0 \Leftrightarrow \delta S = 0,\ \frac{\delta\mathcal{L}}{\delta A} = 0. \tag{4}$$

So, from renorminvariance of the effective action follows that at the conformal symmetric points, the motion equations for fields are satisfied. Generalization for the several coupling constants and other models is obvious. In string theory, the connection between conformal invariance of the effective theory on the parametric world sheet and the motion equations of the fields on the embedding space is well known [3]. A more recent topic in this direction is AdS/CFT Duality [4].

It was noted [5] that in valence quark parametrization $\alpha_s(m) = 2$, at a valence quark scale m. For the QCD running coupling considered in [6]

$$\alpha(q^2) = \frac{4\pi}{9\ln(\frac{q^2+m_g^2}{\Lambda^2})}, \tag{5}$$

where $m_g = 0.88GeV,\ \Lambda = 0.28GeV$, the $\beta-$function of renormdynamics is

$$\beta(\alpha) = -\frac{\alpha^2}{k}(1 - c\exp(-\frac{k}{\alpha})), k = \frac{4\pi}{9} = 1.40,\ c = \frac{m_g^2}{\Lambda^2} = (3.143)^2 = 9.88, \tag{6}$$

for a nontrivial (IR) fixed point we have $\alpha_{IR} = k/\ln c = 0.61$ For $\alpha(m) = 2$, at valence quark scale m we predict the gluon (or valence quark) mass as $m_g = \Lambda\exp(\frac{k}{2\alpha(m)}) = 1.42\Lambda = m_N/3,\ \Lambda = 220MeV$. It is nice to have a nonperturbative $\beta-$function like (6), but it is more important to see which kind of nonperturbative corrections we need to have a phenomenological coupling constant dynamics. In the Standard Model of Particle Physics (SM), the values of the coupling constants and masses of particles depend on the scale according to the Renormdynamic motion equations. One charge a, one mass m RD equations are $\dot{\alpha} = \beta(\alpha),\ \dot{m} = \gamma(\alpha)m$. For the electron and nucleon masses, electrodynamic and pion-nucleon fine structure constants we have an empirical relation: $m_e/\alpha \simeq m_N/\alpha_{\pi N}$ We take the relation $m/\alpha = const$ as an integral of renormdynamic motion equations for m and α and find exact form of the β function in the minimal mass parametrization: $\gamma(\alpha) = \gamma_1\alpha$,

$$(\ln\alpha)^{\cdot} = (\ln m)^{\cdot} \Rightarrow \beta(\alpha)/\alpha = \gamma(\alpha) = \gamma_1\alpha \Rightarrow \beta(\alpha) = \beta_2\alpha^2,\ \beta_2 = \gamma_1 \tag{7}$$

so, we have the following algebraic-diofant equations for the flavor and color content of the theory $\beta_2 = \gamma_1,\ \beta_n = 0,\ n \geq 3$ and prediction for the dimension of space-time: $D = 4$. Solutions of the motion equations are

$$\alpha(t) = \frac{\alpha_0}{1-\alpha_0\beta_2 t},\ m(t) = m_0|\alpha_0^{-1} - \beta_2 t|^{-\gamma_1/\beta_2} = \frac{m_0}{\alpha_0}\alpha(t). \tag{8}$$

In the 1870's G.J. Stoney [7], the physicist who coined the term "electron" and measured the value of elementary charge e, introduced as universal units of Nature for L, T, M :

$$l_S = \frac{e}{c^2}\sqrt{G},\ t_S = \frac{e}{c^3}\sqrt{G},\ m_S = \frac{e}{\sqrt{G}}. \tag{9}$$

M. Planck introduced [8] as universal units of Nature for L, T, M:

$$m_P = \sqrt{\frac{hc}{G}} = \frac{m_S}{\sqrt{\alpha}},\ l_P = \frac{h}{cm_P} = \frac{l_S}{\sqrt{\alpha}} = 11.7 l_S,\ t_P = \frac{l_P}{c} = \frac{t_S}{\sqrt{\alpha}}. \tag{10}$$

Stoney's fundamental constants are more fundamental just because they are less than Planck's constants :) Due to the value of $\alpha^{-1} = 137$, we can consider relativity theory and quantum mechanics as deformations of the classical mechanics when deformation parameter $c = 137$ (in units $e = 1, \hbar = 1$) and $\hbar = 137$ (in units $e = 1, c = 1$), correspondingly. These deformations have an analytic sense of p-adic convergent series.

There are different opinions about the number of fundamental constants [9]. According to Okun, there are three fundamental dimensionful constants in Nature: Planck's constant, $\hbar$; the velocity of light, c; and Newton's constant, G. According to Veneziano, there are only two: the string length L_s and c. According to Duff, there are not fundamental constants at all. Usually $L_s = l_p$, so, the fundamental area is $L_s^2 = 137 l_s^2$. The value $s_s = l_s^2-$ Stoney area, is more like on a fundamental area :) In mathematics we have two kinds of structures, discrete and continuous one. If a physical quantity has discrete values, it might have no dimension. If the values are continuous - the quantity might have a dimension, a unit of measure. These structures may depend on scale, e.g. on macroscopic scale condensed state of matter (and time) is well described as continuous medium, so we use dimensional units of length (and time). On the scale of atoms, the matter has a discrete structure, so we may count lattice sites and may not use a unit of length. If at small (e.g. at Plank) scale space (and/or time) is discrete, then we do not need a unit of length (time) for measuring, there is a fundamental length and we can just count.

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