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THE CONSISTENT CRITERIA OF HYPOTHESES FOR GAUSSIAN STATIONARY PROCESSES

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Abstract. In the present paper we prove the necessary and sufficient conditions for the existence of the consistent criteria of hypotheses for Gaussian stationary processes.

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Let (E, S) be a measurable space with a given family on probability measures $\{\mu_i, i \in I\}$. We use definitions from the [1].

Definition 1. The family $\{\mu_i, i \in I\}$ of probability measures $\{\mu_i, i \in I\}$ is called orthogonal (singular) if μ_i and μ_j are orthogonal for each $i \neq j$.

Definition 2. The family $\{\mu_i, i \in I\}$ of probability measures is called strongly separable if there exists a disjoint family of S–measurable sets $\{X_i, i \in I\}$ such that the relations are fulfilled: $\mu_i(X_i) = 1$, $\forall i \in I$.

Let $|\mathrm{H}|$ be the set of hypotheses $\mathrm{B}(|\mathrm{H}|)$ and be σ–algebra of subsets of $|\mathrm{H}|$ which contains all finite subsets of $|\mathrm{H}|$ [2].

Definition 3. The family of probability measures $\{\mu_H, H \in |\mathrm{H}|\}$ is said to admit consistent criteria of hypotheses if there exists at least one measurable map $\delta : (E, S) \rightarrow (|\mathrm{H}|, \mathrm{B}(|\mathrm{H}|))$, such that $\mu_H(x : \delta(x) = H) = 1$, $\forall H \in |\mathrm{H}|$.

Let $\xi(t)$, $t \in T = [0, T] \subset R$ be a Gaussian stationary process with the same correlation functions and let different mean values $H(t)$, $t \in [0, T]$, μ_H, $H \in |H|$ be corresponding probability measures on S and let $f_H(\lambda)$ be spectral densities of these processes. Let

$$0 < c_1 < f_{H_k}(\lambda)\, e^{a|\lambda|^\gamma} < c_2 < +\infty, \quad \forall k \in N, \tag{1}$$

where $a > 0$, $0 < \gamma < 1$.

Theorem 1. *The Gaussian stationary statistical structure* $\{E, S, \mu_{H_k}, k \in N\}$, $N = \{1, 2, ..., n, ...\}$ *admits a consistent criterion* δ *of Hypotheses if and only if the sequences*

$$m_n^+(t) = \prod_{k=1}^{n} \left(1 + c_k \frac{d}{dt}\right)(H_k(t) - H_i(t)), \quad \forall k, i \in N \tag{2}$$

or

$$m_n^-(t) = \prod_{k=1}^{n} \left(1 - c_k \frac{d}{dt}\right)(H_k(t) - H_i(t)), \quad \forall k, i \in N \tag{3}$$

diverge in $L^2[0, T]$, *where* $c_k = \frac{a \sin \pi\gamma}{\pi \cos \frac{\pi\gamma}{2}} \cdot \frac{1}{2^{k-1}}$.

Necessity. Since the family $\{\mu_{H_k}, k \in N\}$ admits a consistent criterion of Hypotheses, then there exists δ a measurable map of the space (E, S) to $(|\mathrm{H}|, \mathrm{B}(|\mathrm{H}|))$

such that $\mu_{H_i}(x : \delta(x) = H_i) = 1, \ \ \forall i \in N$. Let $X_i = \{x : \delta(x) = H_i\}$, then if is obvious, that $X_i \bigcap X_j = \emptyset$ for all $i \neq j$ and $\mu_i(X_i) = 1, \ \ \forall i \in N$. Therefore, the family of probability measures $\{\mu_{H_i}, i \in N\}$ is strongly separable. A strong reparability implies orthogonality. Let us assume, that sequence (2) and sequence (3) converge in $L^2[0,T]$, then measures μ_{H_i} and $\mu_{H_j}, \ \ \forall i \neq j$ are equivalent. The necessity is proved.

Sufficiency. As either the sequence (2) or the sequence (3) diverges in $L^2[0,T]$, all this implies, that Gaussian measures μ_{H_i} and $\mu_{H_j}, \ \ \forall i \neq j$ are orthogonal. The statistical structure $\{E, S, \mu_{H_i}, i \in N\}$ is orthogonal so $cardN = \chi_0$, then the statistical structure $\{E, S, \mu_{H_i}, i \in N\}$ is strongly separable. There exist such pair wise disjoint S – measurable sets $X_i, i \in N$ that $\mu_{H_i}(X_i) = 1, \ \ \forall i \in N$. Let us define δ as such a mapping $(E, S) \to (|\mathrm{H}|, \mathrm{B}(|\mathrm{H}|))$, that $\delta(H_i) = H_i, \ \ \forall i \in N$. We have $\{x : \delta(x) = H_i\} = X_i$ and $\mu_{H_i}(x : \delta(x) = H_i) = 1, \ \ \forall i \in N$. Theorem 1 is proved [3].

Let $f_{H_i}(\gamma), \ \ i \in N$ be spectral dentist Gaussian processes. Analogously we can prove the following.

Theorem 2. *The Gaussian statistical structure* $\{E, S, \mu_{H_i}, i \in N\}$, N=$\{1, 2, ..., n, ..\}$ *admits a consistent criterion* δ *of Hypotheses if and only if or the formula* $\int\limits_{-\infty}^{+\infty} \left[\frac{f_{H_i}(\lambda)}{f_{H_K}(\lambda)} - 1 - \ln \frac{f_{H_i}(\lambda)}{f_{H_k}(\lambda)}\right] d\lambda = \infty$ *or this formula* $\int\limits_{-\infty}^{+\infty} \left[\frac{f_{H_k}(\lambda)}{f_{H_i}(\lambda)} - 1 - \ln \frac{f_{H_k}(\lambda)}{f_{H_i}(\lambda)}\right] d\lambda = \infty$ *for all* $k \neq i$.

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