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APPLICATION OF THE EXTENSIVE AIR SHOWER ARRIVAL DIRECTION ESTIMATION METHOD TO GELATICA EXPERIMENT DATA

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Abstract. Arrival direction distribution of the Extensive Air Showers with a wide range of number of charged particles is studied by the EAS 4-detector arrays stationed in Tbilisi and Telavi. These stations are parts of the GELATICA net in Georgia (Georgian Large-area Angle and Time Coincidence Array). The method of shower's arrival direction estimation and accessible accuracy are discussed.

Keywords and phrases: Extensive air showers, arrival direction estimation.

AMS subject classification: 85A04, 85A05.

The process of Extensive Air Showers (EAS) development in the atmosphere with accompanied absorption manifests itself through the arrival direction distribution. That is why an interest to such investigations is long-standing [1-6]. Generally the set of any number of ionizing radiation detectors, arbitrarily distributed in the 3D-space, can be used as the volumetric EAS goniometer [7]. The triggering structure has to send a signal to the measuring part of installation for a start of timing based on the pulses from EAS goniometer detectors. The measuring part itself records the difference of signal arrival time with the trigger signal hit time. These delay periods, being multiplied by the EAS front velocity (which is approximately equal to the light velocity c), give the distances between the detectors and shower's front passed. This set of distances can be used to estimate the directional unit vector $\boldsymbol{n}$ of the tangent plane of the incoming shower front.

Any nonsingular plane is defined by four independent parameters, the three being components of directional unit vector $\boldsymbol{n}$ and the fourth one a distance between the plane and the coordinates' origin. So any volumetric EAS goniometer has to enclose at least four detectors for the plane parameters' estimation.

Usually it would be more than four detector units, and the corresponding set of linear equations becomes overfilled; therefore, it has to be solved by a linear least-squares method. The solution of the relevant normal set of equations is then the linear expression of the discussed measured distances. The azimuth φ and zenith θ angels of the shower's arrival direction are then estimated by a common definition of directional unit vector

$$\boldsymbol{n} = (n_x,\, n_y,\, n_z)^T = (sin(\theta)cos(\varphi),\, sin(\theta)sin(\varphi),\, cos(\theta))^T. \tag{1}$$

It is well known [8, 9] that the residual mean square in this method represents the in-dependent estimation of a basic dispersion σ^2 of three components of the directional unit vector $\boldsymbol{n}$. So, the dispersion matrix of this vector depends both on a random discordance in measured delay periods of detectors' pulses and on the mutual arrangement of detectors. Thus, it is possible to estimate the dispersions of the spheric angles, too.

There exists a special, but typical case of planar goniometers with the detectors located in common plane. In this case the normal set of equations becomes singular and only the restricted set (n_x, n_y) of unit vector components (in the detectors' plane) remains linearly attainable. The third ("zenith") component has to be estimated via the normalization requirement of $|\boldsymbol{n}| = 1$. Unfortunately, this nonlinear relation provides the illegal imaginary value of n_z component if it appears that the planar projection of $\boldsymbol{n}$ length random estimation exceeds unity in certain measurements. That is why the volumetric goniometers are especially preferred [7].

It is of some interest to compare the similar azimuthally symmetric goniometers with the same number of detectors that differ only in the distance between the tiers of detector planes.

The standard deviations of planar components (n_x, n_y) of directional unit vector turn out to be equal for azimuthally symmetric goniometers and the correlation coefficient between this components reduce to zero, while the standard deviation of vertical n_z component decrease with increasing distance between the tiers. The azimuth angle estimation depends on planar components only, thus the standard deviation estimations of this angle σ_φ are identical for all the three examples

$$\sigma_\varphi = (\sigma)/sin(\theta). \tag{2}$$

Here the trivial polar singularity shows itself (Fig.1).The estimation of standard deviation of zenith angle σ_θ depends on values of both planar and vertical components, so it is highly sensitive to the distance between the tiers. The value of σ_θ in the horizon vicinity increases with the increasing distance of tiers (Fig.1). In the ultimate case of flat goniometer it becomes singular

$$\sigma_\theta^{flat} = (\sigma)/cos(\theta). \tag{3}$$

The GELATICA experiment uses small planar 4-detector goniometers, arranged under the roofs of buildings of several universities in Georgia [10 - 12]. Detectors are usu-ally arranged in the corners of a square with $a \approx 10m$ sides.The signals, initiated by the passage of EAS charged particles through the detector, are read by the equipment, which measures the signals delays relative to the 4 fold pulse coincidence with $\tau = 1.25ns$ time slicing step. The data are stored in the form of integer values k_1, k_2, k_3 and k_4, corresponding to the numbers of delay slices for every four detector. For this special case of arrangement of the detectors in the corners of a square, the estimation of the horizontal $2D$ projection of the direction vector is

$$\vec{n} = \begin{pmatrix} n_x \\ n_y \end{pmatrix} \left(\frac{c\tau}{2a}\right) \begin{pmatrix} (k_1 + k_4) - (k_2 + k_3) \\ (k_1 + k_2) - (k_3 + k_4) \end{pmatrix} \tag{4}$$

with respect to the XOY reference frame. The dispersion of this vector components estimation is

$$\sigma^2 = \sigma_x^2 = \sigma_y^2 = (c\tau/2a)^2((k_1 + k_3 - k_2 - k_4)^2 \tag{5}$$

while the correlation vanishes. Only statistical uncertainty is taken into account. Two examples of EAS events direction estimated by TBS flat goniometer in Andronikashvili Institute of Physics in Tbilisi are shown in Fig.3 and Fig.4.

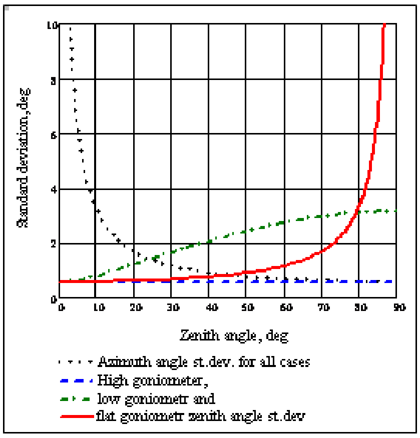


Fig.1. Dependences of standard deviations of estimations of azimuth and zenith angles by the three goniometers.

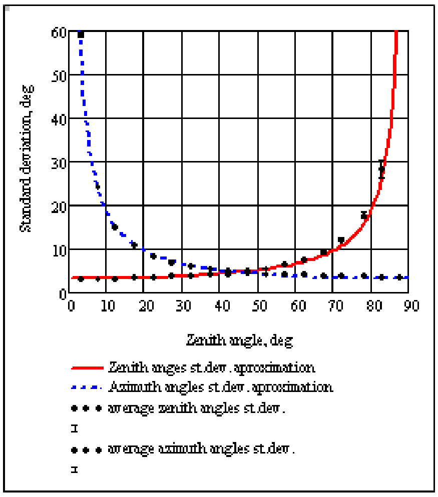
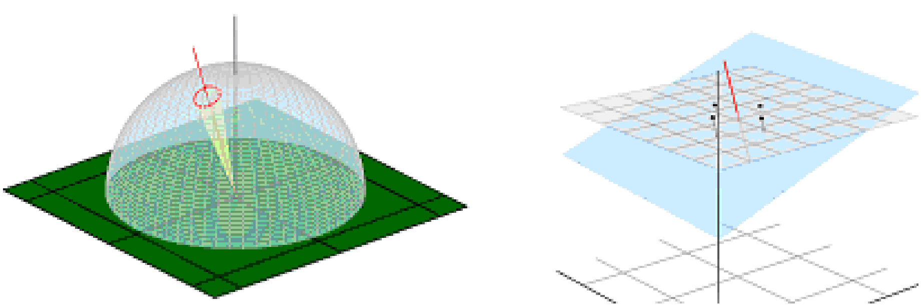
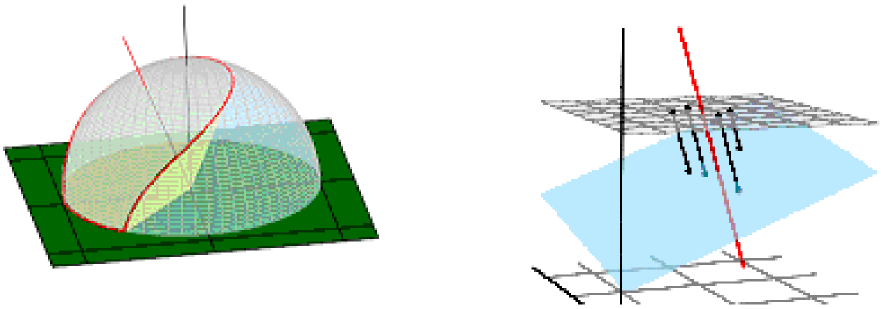


Fig.2. The standard deviations of measured spheric angles of the arrival directions.



A B

Fig.3. The example of good enough estimation of EAS arrival direction



A B

Fig.4. The example of unsatisfactory estimation of EAS arrival direction.
A. The imagined spatial picture of the EAS arrival direction estimation. The unit hemisphere represents the locus of possible directional unit vectors endpoints.
B. The estimated most likely position of the EAS front tangent plane at the 4 fold pulse coincidence instant time is shown.

The measured value of the residual root-mean-square standard deviation by (5) represents the visible deviations of the linear segments endpoints from front plane estimated. The average standard deviations of both the azimuth and zenith angles in the inter-vals of the zenith angle, which are measured by TBS goniometer, are shown in Fig.2. They follow dependences (2) and (3) (compare with Fig.1). So the goniometers of GELATICA net have capability of reliable estimation of EAS arrival direction in a wide ring around zenith for successive investigation of zenith angles distribution or for comparison of arrival directions of the spatially separated showers.

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