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ON ONE NUMERICAL METHOD OF APPROXIMATE SOLUTION OF BOUNDARY-CONTACT PROBLEM OF SOME DIFFICULT GEOMETRY MULTISTRUCTURES BODIES

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Abstract. In this paper stress-deformed state for some "ridge-form" multistructures having difficult geometry is studied. Particularly the boundary-contacted problem is considered. Two rectangle (particularly a square) form plates are connected by a beam. We consider classic linear boundary problems for plates (biharmonic equation), but for the beam nonlinear Kirchhoff type integro-differential equation is studied. In the equation of a beam we consider physical nonlinearity along with mathematical nonlinearity. The program in MATLAB is created and numerical experiments are made.

Keywords and phrases: Biharmonic equation, Kirchhoff type nonlinear integro-differential equation, finite-difference method.

AMS subject classification: 34B05, 34B15, 65M06, 65N06.

1. Introduction. The stress - deformed condition for some "bridge - form" multistructures with difficult geometry (two rectangular plates are connected by the beam, (see Fig.1)) is studied using numerical methods (finite-difference methods). Plate bending is represented by the biharmonic equation (see, for example [1], [4]). The equation of the beam by Kirchhoff type nonlinear integro-differential equation (see, for example [2] - [4]) is studied. The function of a plate bending in central points is found by direct numerical methods and the iterative method for definition of numerical values of function of a bend of a beam for the approached decision of nonlinear Kirchhoff type equation is used. In the equation of a beam we consider physical nonlinearity along with mathematical nonlinearity.

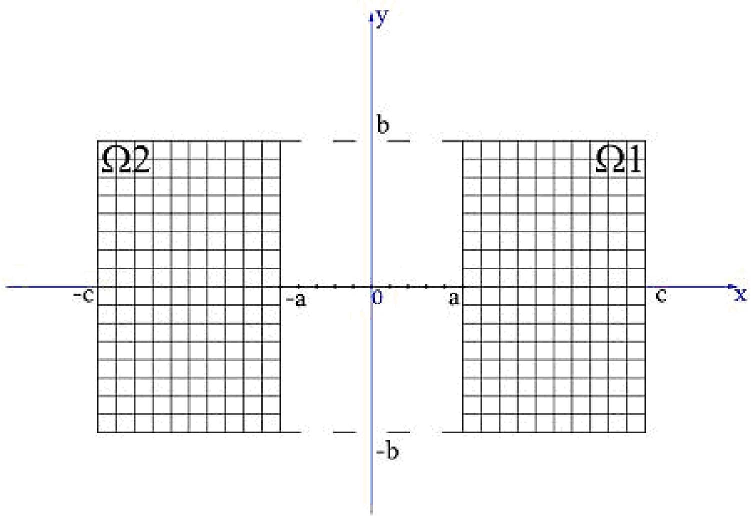

Fig.1

2. Statement of the problem. It is possible to dismantle the above boundary - contact problem in three separate tasks:

a) Boundary value problem for the right plate

$$D\Delta^2 w_1(x,y) = f_1(x,y),\ (x,y)\in\Omega_1=\{(x,y): a\le x\le c,\ \ -b\le y\le b\}, \tag{1}$$

$$w_1(x,\pm b)=0,\quad \frac{\partial^2}{\partial y^2}w_1(x,\pm b)=0,\quad a\le x\le c, \tag{2}$$

$$w_1(c,y)=0,\quad \frac{\partial}{\partial x}w_1(c,y)=0,\quad -b\le y\le b, \tag{3}$$

$$\frac{\partial^2 w_1}{\partial x^2}+\nu\frac{\partial^2 w_1}{\partial y^2}\Big|_{x=a}=0,\quad \frac{\partial^3 w_1}{\partial x^3}+(2-\nu)\frac{\partial^3 w_1}{\partial x\partial y^2}\Big|_{x=a}=0,\quad -b\le y\le b; \tag{4}$$

b) Boundary value problem for the left plate

$$D\Delta^2 w_2(x,y) = f_2(x,y),\ \ (x,y)\in\Omega_2=\{(x,y): -c\le x\le -a,\ -b\le y\le b\}, \tag{5}$$

$$w_2(x,\pm b)=0,\quad \frac{\partial^2}{\partial y^2}w_2(x,\pm b)=0,\quad -c\le x\le -a, \tag{6}$$

$$w_2(-c,y)=0,\quad \frac{\partial}{\partial x}w_2(-c,y)=0,\quad -b\le y\le b, \tag{7}$$

$$\frac{\partial^2 w_2}{\partial x^2}+\nu\frac{\partial^2 w_2}{\partial y^2}\Big|_{x=-a}=0,\quad \frac{\partial^3 w_2}{\partial x^3}+(2-\nu)\frac{\partial^3 w_2}{\partial x\partial y^2}\Big|_{x=-a}=0,\quad -b\le y\le b, \tag{8}$$

where $D=\dfrac{EH^3}{12(1-\nu^2)}$ is cylindrical rigidity, H is thickness of a plate, E is Young's modulus, ν is Poissons coefficient;

c) Boundary value problem for a beam

$$w_3^{''''}-\left[m_0+m_1\int_{-a}^{+a}\left(w_3'(t)\right)^2dt+m_2\left(\int_{-a}^{+a}\left(w_3'(t)\right)^2dt\right)^2\right]w_3''x)=f_3(x), \tag{9}$$

$$-a\le x\le a,$$

$$w_3(-a)=\alpha_2,\quad w_3(a)=\alpha_1, \tag{10}$$

$$\frac{d}{dx}w_3(-a)=0,\quad \frac{d}{dx}w_3(a)=0, \tag{11}$$

where $\alpha_1\approx w_1(a,0),\ \ \alpha_2\approx w_2(-a,0),\ \ m_0,m_1,m_2>0.$

3. The algorithm. In order to solve this boundary value problem we use the finite - difference method. Let's consider the case when the square is $c-a=2b$; On Ω_1 and Ω_2 squares let us make a regular square grid with steps $h_1=h_2=h$, $n_1=n_2=n$,

$$h_1=\frac{c-a}{n_1}=h_2=\frac{2b}{n_2}=h,\quad x_i=a+ih_1,\ i=0,1,2,\cdots,n_1,$$

or

$$x_i=-c+ih_1,\ i=0,1,2,\cdots,n_1,\quad y_j=-b+jh_2,\ j=0,1,2,\cdots,n_2.$$

The part of beam $[-a, a]$ section is divided $2n_3$ by step h_3,

$$h_3 = a/n_3, \quad x_i = -a + ih_3, \quad i = 0, 1, 2, \cdots, 2n_3.$$

Let's replace differential operators by the finite - difference analogues. Equations (1), (5) of the fourth order (biharmonic) differential operators are replaced by the difference equations using 3-point template with error $O(h^2)$. In (4), (8) third order differential operators for w_1, the right difference derivative is used for w_2 - the left difference derivative is used and they are taken on 4-point template with error $O(h)$. As for the first and second order differential operators, they are replaced on 2 and 3 point template with $O(h^2)$ error. In (9) beam equation differential operator, is replaced by $O(h_3^2)$ order difference scheme: for 4th order derivative on 5-point template and for 2nd order on 3-point template. In order to solve obtained nonlinear difference problem we use the iterative method.

Let's use the following marking for grid functions $w_{1,i,j} \approx w_1(x_i, y_j)$, $w_{2,i,j} \approx w_2(x_i, y_j)$, $w_{3,i} \approx w_3(x_i)$, $f_{1,i,j} = f_1(x_i, y_j)$, $f_{2,i,j} = f_2(x_i, y_j)$, $f_{3,i} = f_3(x_i)$.

In case of (1)-(4) and (5)-(8) problems we will get problem with five - block diagonal system of equations (see [4]).

In order to solve (9)-(11) nonlinear system of equations we use the difference method combined with iterative methods:

$$w_{3,i+2}^{(k+1)} - (4+\gamma)w_{3,i+1}^{(k+1)} + (6+2\gamma)w_{3,i}^{(k+1)} - (4+\gamma)w_{3,i-1}^{(k+1)} + w_{3,i-2}^{(k+1)} = F_{3,i},$$

$$i = 1, 2, \cdots, 2n-1; \quad k = 0, 1, 2, \cdots;$$

$$\gamma = h_3^2 \left(m_0 + m_1 tkf\left(w_3^{(k)}\right) + m_2 \left(tkf\left(w_3^{(k)}\right)\right)^2 \right);$$

$$tkf\left(w_3^{(k)}\right) = \left(\frac{w_{3,2}^{(k)} - w_{3,0}^{(k)}}{2h_3}\right)^2 + \cdots + \left(\frac{w_{3,2n}^{(k)} - w_{3,2n-2}^{(k)}}{2h_3}\right)^2;$$

$$F_{3,1} = h_3^4 f_{3,1} + (4+\gamma)\alpha_2; \quad F_{3,2} = h_3^4 f_{3,2} - \alpha_2;$$

$$F_{3,i} = h_3^4 f_{3,i} \quad, i = 3, 4, \cdots, 2n-3;$$

$$F_{3,2n-2} = h_3^4 f_{3,2n-2} - \alpha_1; \quad F_{3,2n-1} = h_3^4 f_{3,2n-1} + (4+\gamma)\alpha_1.$$

$w_{3,i}^{(0)}$, $i = 0, 1, \cdots, 2n$ is the initial approach.

Remark. we can take as initial approach

$$w_{3,0}^{(0)} = \alpha_2, \;\; w_{3,1}^{(0)} = 0, \;\; w_{3,2}^{(0)} = 0, \;\; \cdots, \;\; w_{3,2n-1}^{(0)} = 0, \;\; w_{3,2n}^{(0)} = \alpha_1;$$

For finding in central points of required functions of a deflection the following five - diagonal system of algebraic equations is obtained which was solved by direct numerical methods.

The system of programs in MATLAB on the basis of the above-stated algorithm is created which is intended for a wide range of consumers.

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