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PARABOLIC REGULARIZATION OF ONE-DIMENSIONAL ANALOG OF ONE NONLINEAR BIOLOGICAL MODEL

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Abstract. One-dimensional analog of the two-dimensional system of nonlinear partial differential equations arising in process of vein formation of young leaves is considered. Parabolic regularization of this system is studied. Finite difference scheme for initial-boundary value problem is constructed. Graphical illustrations of the tests experiments are given.

Keywords and phrases: Nonlinear system, partial differential equations, parabolic regularization, vein formation of young leaves, finite difference scheme.

AMS subject classification: 35Q80, 35Q92, 65N06, 65Y99.

Two-dimensional model describing the vein formation of young leaves is given and some qualitative and structural properties of solutions of this model are established in [1]. In [2] investigations for one-dimensional analog are carried out. In biological modeling there are many works where this and many models of similar processes are also presented and discussed (see, for example, [3]-[6] and references therein). Many scientific works are devoted to investigation and numerical resolution of different kinds of initial-boundary value problems for model described in [1] and its one-dimensional and multi-dimensional analogs (see, for example, [7]-[14] and references therein).

Let us consider the following initial-boundary value problem for one-dimensional analog of the vein formation model [1]:

$$
\begin{gathered}
\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(V\frac{\partial U}{\partial x}\right),\quad (x,t)\in(0,1)\times(0,T],\\
\frac{\partial V}{\partial t}=-V+g\left(V\frac{\partial U}{\partial x}\right),\quad (x,t)\in[0,1]\times(0,T],\\
U(0,t)=U(1,t)=0,\quad t\in[0,T],\\
U(x,0)=U_0(x),\quad V(x,0)=V_0(x)\geq\delta_0,\quad x\in[0,1],
\end{gathered}
\tag{1}
$$

where g, U_0, V_0 are known sufficiently smooth functions, $g_0\leq g(\xi)\leq G_0$; T, g_0, G_0, δ_0 are given positive constants.

The purpose of our note is to study the parabolic regularization of problem (1):

$$
\begin{gathered}
\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(V\frac{\partial U}{\partial x}\right),\quad (x,t)\in(0,1)\times(0,T],\\
\frac{\partial V}{\partial t}=-V+g\left(V\frac{\partial U}{\partial x}\right)+\varepsilon\frac{\partial^2 V}{\partial x^2},\quad (x,t)\in[0,1]\times(0,T],\\
U(0,t)=U(1,t)=0,\quad \frac{\partial V}{\partial x}(0,t)=\frac{\partial V}{\partial x}(1,t)=0,\quad t\in[0,T],\\
U(x,0)=U_0(x),\quad V(x,0)=V_0(x)\geq\delta_0,\quad x\in[0,1],
\end{gathered}
\tag{2}
$$

where ε is the given positive constant.

Investigation and numerical solution of nonlinear parabolic type models to which belongs the investigated problem (2) when $\varepsilon \neq 0$ are carried out in many works as well (see, for example, [12], [15] and references therein).

On $[0,1] \times [0,T]$ let us introduce a net with mesh points denoted by $(x_i, t_j) = (ih, j\tau)$, where $i = 0,1,...,M;\ j = 0,1,...,N$ with $h = 1/M,\ \tau = T/N$. The discrete approximation at (x_i, t_j) is designated by u_i^j, v_i^j and the exact solution of problem (2) by U_i^j, V_i^j.

Using the usual method of construction of discrete models let us consider the following finite difference scheme:

$$\frac{\tau}{h^2} v_i^{j+1} u_{i-1}^{j+1} - \left(1 + \frac{\tau}{h^2}\left(v_i^{j+1} + v_{i+1}^{j+1}\right)\right) u_i^{j+1} + \frac{\tau}{h^2} v_{i+1}^{j+1} u_{i+1}^{j+1} = -u_i^j, \tag{3}$$

$$\frac{\tau\varepsilon}{h^2} v_{i-1}^{j+1} - \left(1 + \tau + \frac{2\tau\varepsilon}{h^2}\right) v_i^{j+1} + \frac{\tau\varepsilon}{h^2} v_{i+1}^{j+1} = -v_i^j - \tau g\left(v_i^{j+1} \frac{u_{i+1}^{j+1} - u_{i-1}^{j+1}}{2h}\right), \tag{4}$$

$$u_0^j = u_M^j = v_{x,0} = v_{\bar{x},M} = 0, \quad j = 0,1,...,N, \tag{5}$$

$$u_i^0 = U_{0,i}, \quad v_i^0 = V_{0,i}, \quad i = 0,1,..,M. \tag{6}$$

The following statement takes place.

Theorem. *The finite difference scheme (3)-(6) converges to the solution of problem (2) in the norm of the space C_h with the rate $O(\tau + h)$.*

Note that, for solving the finite difference scheme (3)-(6) first we solve system (4) using iterative procedure and well known tridiagonal matrix algorithm and after we solve system (3) by the same algorithm, using in both cases suitable boundary and initial conditions from (5), (6). Numerous computer test experiments are made by using the above-mentioned algorithm and the numerical experiments are quite satisfactory and fully agree with the considered exact test solutions of problem (2) with suitable right parts.

Some graphical illustrations of those numerical results are given in Fig.1 and Fig.2.

The graphs in Fig.1 illustrate numerical results of problem (2) for the case $\varepsilon = 1$. Here, with the suitable right parts, the exact solutions, when $g(s) = \dfrac{1}{1+s^2}$ are:

$$U(x,t) = 10x(1-x)(1+t), \quad V(x,t) = 10x(1-x)\left(1+t+t^2\right).$$

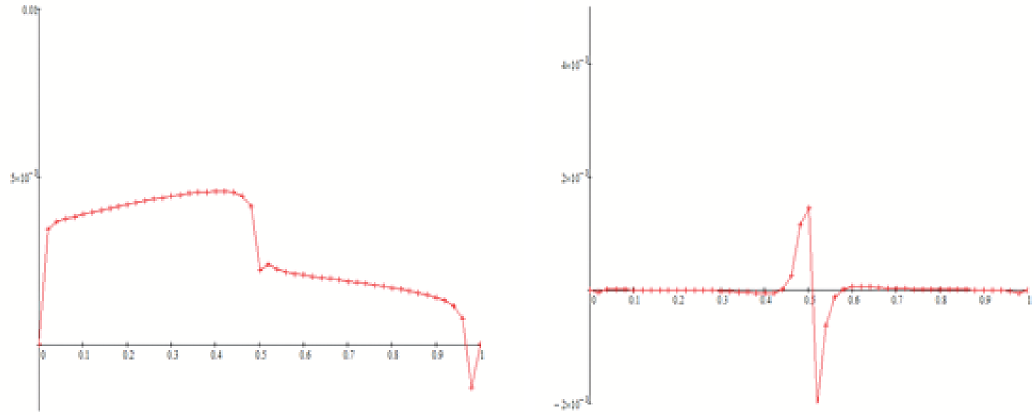

Fig.1. Exact (solid line) and numerical (marked with $\times$) solutions and differences between exact and numerical solutions (marked with $+$) when $\varepsilon = 1$.

The graphs below (see Fig.2) illustrate numerical results of problem (2) for the case $\varepsilon = 0,001$. For the same data as in the previous case, here we took the exact solutions as:

$$U(x,t) = 10x(1-x)(1+t), \quad V(x,t) = 10x(1-x)\left(1+t+t^2\right)$$

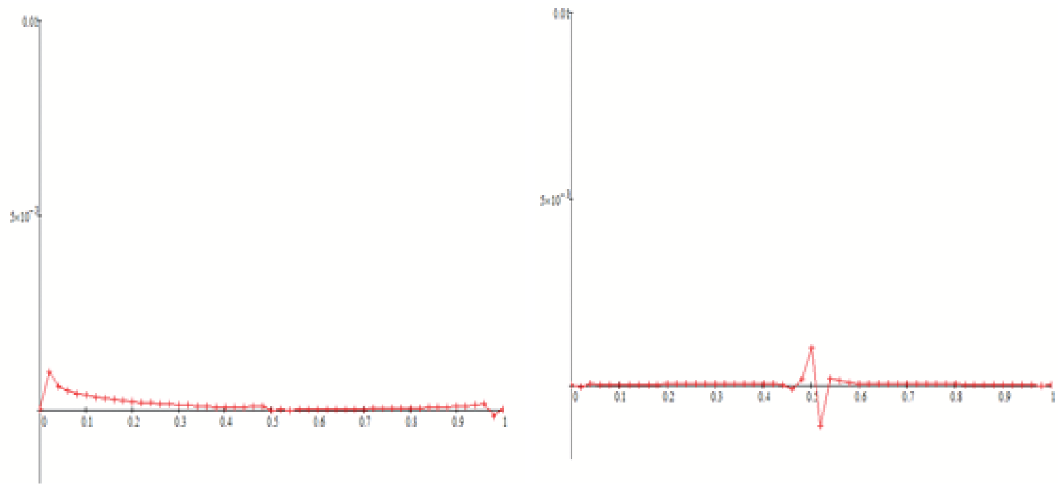

Fig.2. Exact (solid line) and numerical (marked with $\times$) solutions and differences between exact and numerical solutions (marked with $+$), when $\varepsilon = 0,001$.

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