Reports of Enlarged Session of the
Seminar of I. Vekua Institute
of Applied Mathematics
Volume 29, 2015

SOME NOTES ON THE CONSISTENT CRITERIA FOR CHECKING HYPOTHESES

Mumladze M., Zerakidze Z.

Abstract. In the paper we prove the necessary and sufficient conditions for the existence of the consistent criteria for statistical structures.

Keywords and phrases: Orthogonal, strongly structures, consistent criteria.

AMS subject classification: 62H05, 62H12.

Let there be given (E, S) measurable space and on this space let there be given $\{\mu_i, i \in I\}$ family of probability measures which depend on $i \in I$ parameter. Let us recall some definitions [1].

Definition 1. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called orthogonal (singular) if $(\forall i)\,(\forall j)\,(i \in I \& j \in I \& i \neq j \Rightarrow \mu_i \perp \mu_j)$.

Definition 2. A statistical structure $\{E, S, \mu_i, i \in I\}$ is said to be strongly separable, if there exists pairwaise disjoint S-measurable sets $\{X_i, i \in I\}$ such that the relation $(\forall i)\,(i \in I \Rightarrow \mu_i\,(X_i) = 1)$ is fulfilled [2].

Definition 3. Any assumption defining the form of distribution selection is called a Hypotheses.

Let $|\mathrm{H}|$ be the set of Hypotheses and let $\mathrm{B}\,(|\mathrm{H}|)$ be the σ-algebra of subsets in $|\mathrm{H}|$ which contains all finite subsets of $|\mathrm{H}|$.

Definition 4. A statistical structure $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ is said to admit consistent criteria for checking Hypotheses if there exists at least one measurable map $(E, S) \to (|\mathrm{H}|, \mathrm{B}\,(|\mathrm{H}|))$ such that $\mu_H\,(x : \delta\,(x) = H) = 1, \quad \forall H \in |\mathrm{H}|$.

Definition 5. A statistical structure $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ is said to admit a consistent criterion of any parametric function if for any real bounded measurable function $(|\mathrm{H}|, \mathrm{B}\,(|\mathrm{H}|)) \to (R, B\,(R))$ there exists at least one measurable function $f : (E, S) \to (R, B\,(R))$ such that $\mu_H\,(x : f\,(x) = g\,(H)) = 1, \quad \forall H \in |\mathrm{H}|$.

Definition 6. A statistical structure $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ a said to admit an unbiased criterion of any parametric function if for any real bounded measurable function $g : (|\mathrm{H}|, \mathrm{B}\,(|\mathrm{H}|)) \to (R, B\,(R))$ there exists at least one measurable function $l : (E, S) \to (R, B\,(R))$ such that $\int l\,(x)\,\mu_H\,(dx) = g\,(H), \quad \forall H \in |\mathrm{H}|$.

Let $S_1 \subset S$ be some σ-sub algebra of σ-algebra S and let μ be probability measure defined on S_1.

Denote by $S^{\sigma}_{\mu}\,(S, S_1)$ the set of all countable additive extensions of μ and by $\exp^{\sigma}_{\mu}\,(S, S_1)$ the set of all extreme points of. It's known, that exp $\exp^{\sigma}_{\mu}\,(S, S_1)$ may be empty [3].

The following theorem can be easily approved:

Theorem 1. *If a statistical structure* $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ *admitting a consistent criterion for checking Hypotheses, then this statistical structure* $\{E, S, \mu_H, H \in |\mathrm{H}|\}$

admits a consistent criterion for any parametric function and this statistical structure $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ *admits an unbiased criterion of any parametric function [4].*

Theorem 2. *The statistical structure* $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ *admits a consistent criteria of Hypotheses if and only if the statistical structure* $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ *admits an unbiased criterion of any parametric function and is strongly separable.*

Proof. Necessity. As the statistical structure $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ admits a consistent criterion of Hypotheses, so the statistical structure $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ admits an unbiased criterion of any parametric function and if is strongly separable (see theorem 1).

Sufficient. As the statistical structure $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ is strongly separable, there exists a disjoint family of S-measurable sets $\{B_H, H \in |\mathrm{H}|\}$ such that $\mu_H(B_H) = 1, \ \forall H \in |\mathrm{H}|$. As the statistical structure $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ admits an unbiased criterion for any parametric function, so there exists a subspace $G \subset B(E, S)$, containing l_E unit and $B(E, S)$ can be imagined as a topological sum of G and $H_0 = \mu^{-1}(0)$, where the functional $\mu(f) = \int_E f(x)\,\mu(dx)\,, \ f \in B(E, S)\,, \ \ \mu \in B'(E, S)$ and a statistical structure $\{E, S, \mu_H, H \in |\mathrm{H}|\}$ is strongly separable, subspace G is a grid towards canonical order on G.

We assume that is minimal σ- algebra of sub algebra S, all functions on G are measurable towards S_0. Then $G \subset B(E, S_0) \subset B(E, S)$. Since a subspace G contacts l_E and represents a grid, then $G \supset B(E, S)$ and that's why $G = B(E, S)$.

As family $\{\mu_H, H \in |\mathrm{H}|\}$ represents a dense subspace of $exS_{|H|}$ ($exS_{|H|}$ are extreme points of $S_{|H|}$), So, I_μ is an ideal in the set S_0, which contains zero measured sets for all $\mu \in \{\mu_H, H \in |\mathrm{H}|\}$ and consists only of an empty set.

Hence, there exist sets $\{A_H, H \in |\mathrm{H}|\}$ such that $\mu_H(A_H) = 1$ for $A_H \bigcap A_{H'} = \emptyset$ for $H \neq H'$and $E = \bigcup_{H \in |H|} A_H$ is a set S_0. It follows from the condition of this theorem that for every $T \in B(|H|)$ in G, there exists function f_T, which is a consistent criterion g_T parametric function. If $A = \{x : f_T(x) \neq 0\}$, then $\bigcup_{H \in |H|} A_H \subset A, \ \ A \bigcap A_H = \emptyset, \ \ \forall H \in T$ and hence $\bigcup_{H \in |H|} A_H = A$ implying that $\bigcup_{H \in |H|} A_H \subset S_0$.

Then, the mapping $\delta(x) = H$ if $x \in A_H$, $\forall H \in |H|$ is a consistent criterion of Hypotheses. Theorem 2 is proved.

Acknowledgement. Research partially supported by Shota Rustaveli National Science Foundation (Grant No FR/308/5-104/12).

R E F E R E N C E S

1. Zerakidze Z. On weakly divisible and divisible families of probability measures. *Bull. Acad Sci Georgian SSR*, **113** (1984), 273-275.

2. Aleksidze L., Mumladze M., Zerakidze Z. The consistent criteria of Hypotheses. *Modern stochastics: Theory and Applications*, **1** (2014), 3-11.

3. Peachky D. Extremely and monogenic additive set functions. *Proc. Amer Hath. Soc.*, **54** (1976), 193-196.

4. Mallev M., Serechenko A., Tarashchanskii M. Family of probability measures admitting consistent estimators, In Skorohod A.V. (ed) probability distributions in infinite dimensional spaces. *Inst.*

Math. Acad. Sci. Ukrainian SSR, Kiev, (1978), 125-134.

Received 29.05.2015; revised 11.10.2015; accepted 18.12.2015.

Authors' address:

M. Mumladze
Gori State Teaching University
53, Chavchavadze St., Gori 1400
Georgia
E-mail: mmumladze@mail.ru

Z. Zerakidze
Gori State Teaching University
53, Chavchavadze St., Gori 1400
Georgia
E-mail: zura.zerakidze@mail.ru