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# SUBCRITICAL TURBULENCE IN TWO-DIMENSIONAL MAGNETOHYDRODYNAMIC PLANE SHEAR FLOWS

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**Abstract**. We find and investigate via numerical simulations self-sustained two-dimensional turbulence in a plasma flow with a constant shear of velocity and threaded by a parallel uniform background magnetic field. This flow is spectrally stable, so the turbulence is subcritical by nature and hence it can be energetically supported just by a transient growth mechanism due to shear flow non-normality. This mechanism appears to be essentially anisotropic in the spectral (wavenumber) plane and operates mainly for spatial Fourier harmonics with streamwise wave numbers less than the ratio of flow shear to Alfvén speed. We focus on analysis of the character of nonlinear processes and the underlying self-sustaining scheme of the turbulence, i.e., on the interplay between linear transient growth and nonlinear processes, in the spectral plane. Our study, being concerned with a new type of energy-injecting process for turbulence the transient growthrepresents an alternative to the main trends of magnetohydrodynamic (MHD) turbulence research. We find similarity of the nonlinear dynamics to the related dynamics in hydrodynamic flows: to the bypass concept of subcritical turbulence. The essence of the analyzed nonlinear MHD processes appears to be a transverse redistribution of kinetic and magnetic spectral energies in the wave-number plane and differs fundamentally from the existing concepts of (anisotropic direct and inverse) cascade processes in MHD shear flows.

**Keywords and phrases**: MHD shear flows, non-modal approach, transient growth, instability, turbulence.

**AMS subject classification**: 65M06, 65N06, 65M60, 65M70.

**Introduction.** The direct nonlinear cascade - a central process in Kolmogorov's phenomenology - is a consequence of the existence of, so-called, inertial range in spectral/wavenumber space (**k**-space), which is free from the action of linear energy-exchange processes and occupied by nonlinear cascades only. In the Kolmogorov scheme, long wavelength perturbations grow exponentially, which are being transferred by the direct cascade, through the inertial range, to shorter wavelengths and ultimately to the dissipation region. In this course of events, the direct cascade together with unstable and dissipative linear phenomena constitute the well-known scheme of forced turbulence. The concept of the direct cascade took root in the turbulence theory and acquired dogmatic coloring. However, in spectrally stable shear flows, in which transient growth of perturbations is the only possibility, the balance of processes leading to the self-sustenance of turbulence should be completely different. The shear-induced transient growth mainly depends on the orientation (and, to a lesser degree, on the value) of wavevector of perturbations: the spatial Fourier harmonics (SFHs) of perturbations having a certain orientation of the wavevector with respect to shear flow, draw flow energy and get amplified, whereas harmonics having another orientation of the wavevector give energy back to the flow and decay. In other words, the linear energy-exchange processes in hydrodynamic (HD) shear flows are strongly anisotropic in **k**-space, cover an entire space without leaving a free room (i.e., inertial range) for the action of nonlinear processes only, unlike that in Kolmogorov's phenomenology. The strong anisotropy of the linear processes, in turn, leads to anisotropy of nonlinear processes in **k**-space. In this case, as shown in Horton et al (2010), even in the simplest HD shear flow with linear shear, the dominant nonlinear process turns out to be not a direct, but a *transverse cascade*, that is, transverse (angular) redistribution of perturbation harmonics in **k**-space.

**Statement of the problem.** We present the results of our direct numerical simulations (DNS), which demonstrate the dominance of the nonlinear transverse cascade of spectral energy in magnetohydrodynamic (MHD) plasma shear flows too (see details in Mamatsashvili et al 2014). Specifically, we consider the dynamics of two-dimensional perturbations in an incompressible MHD flow with a constant/linear shear of velocity, $\mathbf{U}_0 = (0, Sx)$, and uniform magnetic field, $\mathbf{B}_0 = (0, B_{0y})$, parallel to it. Such a simple configuration of an unbounded flow with a constant/linear shear of velocity profile corresponds, for example, to plasma flow in astrophysical accretion disks in the framework of the widely used local shearing box approximation with an imposed net toroidal magnetic field (e.g., Simon & Hawley 2009) as well as to flows of magnetized plasma in laboratory. It allows us to grasp key effects of shear on the perturbation dynamics and ultimately on a resulting MHD turbulent state in kinematically nonuniform plasma flows.

The dynamics of perturbations in this flow is governed by the following set of basic equations of incompressible non-ideal MHD

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U}\cdot\nabla)\,\mathbf{U} = -\frac{\nabla P}{\rho} + \frac{(\mathbf{B}\cdot\nabla)\,\mathbf{B}}{4\pi\rho} + \nu\nabla^2\mathbf{U}, \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{U}\times\mathbf{B}) + \eta\nabla^2\mathbf{B}, \tag{2}$$

$$\nabla\cdot\mathbf{U} = 0, \tag{3}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{4}$$

where $\rho$ is the fluid density, $\mathbf{U}$ is the velocity, $\mathbf{B}$ is the magnetic field and $P$ is the total pressure equal to the sum of thermal and magnetic pressures. $\nu$ and $\eta$ are the coefficients of kinematic viscosity and Ohmic resistivity.

Our goals are: *(i)* to investigate subcritical transition to turbulence by DNS in this magnetized flow, *(ii)* to describe the general behavior of nonlinear processes – transverse cascade – in wavenumber plane that appears to be a keystone of the turbulence's self-sustaining dynamics in this simple open system. The DNS of Eqs. (1)-(4) were performed using the SNOOPY code (http://ipag.obs.ujf-grenoble.fr/ lesurg/snoopy.html). White noise perturbations of velocity and magnetic field with rms amplitudes 0.84 (in Alfvén speed, $v_A$, units) are applied initially; the subsequent time-development of averaged kinetic and magnetic energies as well as Reynolds and Maxwell stresses are shown in Fig.1. In our simulations, we take the domain size $L_x = L_y = 400$ (in Alfvén lengths, $v_A S^{-1}$, units) and resolution $N_x \times N_y = 512 \times 512$. These parameters ensure the presence of harmonics in the simulation domain that can undergo efficient transient growth.

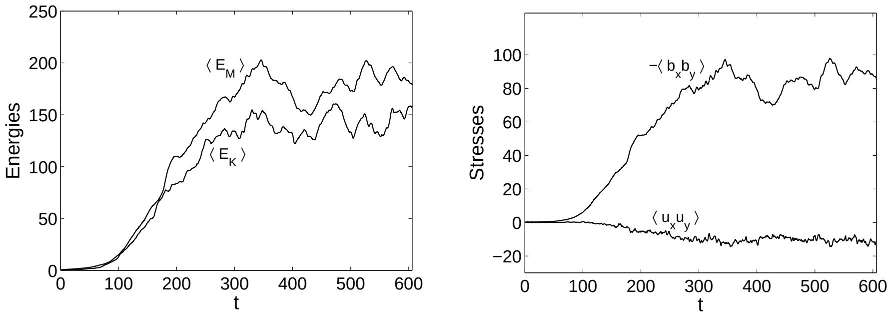


Fig.1. Evolution of the domain-averaged perturbed kinetic, $\langle E_K \rangle$, and magnetic, $\langle E_M \rangle$, energies and the Reynolds, $\langle v_x v_y \rangle$, and Maxwell, $\langle -b_x b_y \rangle$ stresses. In the beginning, they steadily grow as a result of shear-induced transient amplification of separate SFHs. Then, at about $t = 250S^{-1}$, the amplification saturates to a quasi-steady turbulent state that persists till the end of the run. The magnetic energy is a bit larger than the kinetic one.

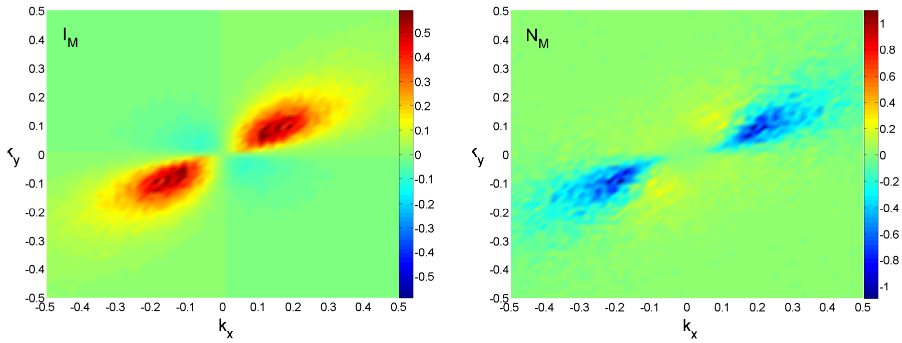


Fig.2. Maps of the time-averaged magnetic energy injection, $I_M$, and nonlinear transfer, $N_M$ terms in $\mathbf{k}-$plane in the state of quasi-steady turbulence. The magnetic energy injection occurs mostly at intermediate wavenumbers, $0.05 \lesssim k \lesssim 0.5$, on the $k_x/k_y > 0$ side where $I_M > 0$. Energy injection into turbulence appears to be mainly due to the Maxwell stresses, as seen also in Fig.1. The $N_M$ term transfers magnetic energy anisotropically/transversally in wavenumber plane, away from regions where they are negative $N_K < 0,\ N_M < 0$ to regions where they are positive $N_K > 0,\ N_M > 0$ and in this way provides positive feedback for $I_M$.

To understand the sustaining mechanism of the turbulence, we Fourier transformed basic MHD equations (1)-(4) and derived evolution equations for the perturbed kinetic and magnetic spectral energies in $\mathbf{k}$-plane (see Mamatsashvili et al 2014). In these spectral equations, using the simulation results, we calculated individual terms, which are divided into two types – terms of linear and nonlinear origin (Fig.2). The terms of linear origin – the Maxwell and Reynolds stresses – are responsible for energy exchange between the turbulence and the mean flow through transient amplification of perturbation harmonics due to shear. However, as we showed, only the positive Maxwell stress appears to be a dominant (magnetic) energy injector for the turbulence; it is much larger than the Reynolds stress (Fig.1), which has a negative sign and therefore cannot contribute to turbulent energy gain. Another linear term due to shear in these equations makes the spectral energies drift in the spectral plane parallel to the $k_x$-axis. The nonlinear terms, which do not directly draw the mean flow energy, act to redistribute this energy in spectral plane so as to continually repopulate perturbation harmonics that can undergo transient growth (Fig.2).

Thus, we demonstrated that in spectrally stable shear flows, the subcritical MHD turbulent state is sustained by the interplay of linear and nonlinear processes – the first supplies energy for turbulence via shear-induced transient growth mechanism of magnetic field perturbations (characterized by the Maxwell stresses) and the second plays an important role of providing a positive feedback that makes this transient growth process persist over long times and compensate for high-$k$ dissipation due to viscosity and resistivity. This balance/cooperation of energy injecting linear and redistributing nonlinear transfer terms relies on their anisotropy (i.e., dependence on wavevector angle) in $\mathbf{k}$-plane. The obtained energy spectra are, consequently, anisotropic in $\mathbf{k}$-plane. The studied self-sustenance scheme is consistent with the *bypass* concept of subcritical turbulence in spectrally stable shear flows and differs fundamentally from the existing concepts of (anisotropic direct and inverse) cascade processes in MHD shear flows.

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