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DYNAMICS, DISCRETE DYNAMICS AND QUANPUTERS

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Abstract. Hamiltonization method of the (discrete) dynamics with theory, construction and programming of Quanputers.

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1. Hamiltonization of dynamical systems. Let us consider the following system of ordinary differential equations [1]

$$\dot{x}_n = v_n(x) + j_n(t),\ 1 \leq n \leq N, \tag{1}$$

Lagrangian,

$$L = (\dot{x}_n - v_n(x) - j_n(t))\psi_n \tag{2}$$

and the corresponding motion equations

$$\dot{x}_n = v_n(x) + j_n(t), \dot{\psi}_n = -\frac{\partial v_m}{\partial x_n}\psi_m. \tag{3}$$

System (3) extends system (1) by linear equation for the ψ. The extended system can be put in the Hamiltonian form [2].

2. Quanputing. The idea of computations on quanputers is in finding the needed (value of the) state (wave function $\psi(t,x)$) from the initial, easy constructible, state ($\psi(0,x)$,) which is superposition of different states, including an interesting one, with the same weight. During the computation the weight of the interesting state is growing till the value when we can guess the solution of the problem and then test it, which is much more easier than to find it [3-5].

Let us consider the following nonlinear evolution equation

$$iV_t = \Delta V - \frac{1}{2}V^2 + J,$$

extended Lagrangian and Hamiltonian

$$L = \int dx^D(iV_t - \Delta V + \frac{1}{2}V^2 - J)\psi,\ H = \int dx^D(\Delta V - \frac{1}{2}V^2 + J)\psi$$

and corresponding Hamiltonian motion equations [6]

$$iV_t = \Delta V - \tfrac{1}{2}V^2 + J = \{V, H\},$$

$$i\psi_t = -\Delta\psi + V\psi = \{\psi, H\},$$

$$\{V(t,x), \psi(t,y)\} = \delta^D(x-y).$$

The solution of the problem is given in the form

$$|T) = U(T)|0),\ \psi(t,x) =< x|t),\ U(T) = Texp(-i\int_0^T H(t)).$$

Under the programming of the quanputer we understand construction of the potential V, or the corresponding Hamiltonian. For the given potential, we calculate corresponding source J.

The discrete version of the system can be put in the form [7]

$$S_m(n+1) = \Phi_n(S(n)) + J_m(n),$$

$$\Psi_m(n-1) = A_{mk}(S(n))\Psi_k(n),\ A_{mk}(S(n)) = \frac{\partial \Phi_k(S(n))}{\partial S_m(n)},$$

when the matrix A is regular, we obtain explicit form of the corresponding discrete dynamics

$$S_m(n+1) = \Phi_n(S(n)) + J_m(n),$$

$$\Psi_m(n) = A^{-1}_{mk}(S(n+1))\Psi_k(n).$$

Now the state vector $S(n)$ and wave vector $\Psi_m(n)$ may correspond not only to the discrete values of the potential $V(n,m) = S_m(n)$, and wave function $\psi(n,m) = \Psi_m(n)$ but also any representation of the computing process from theoretical to experimental realization on a quanputer, including algorithm of solution, higher level programm realization of the algorithm [8].

3. Complex polynomial equations and Nambu-poisson dynamics. We consider the following polynomial equation

$$P_N(z) - tz^{N+1} = 0,\ z \in C,\ t \in (0,\infty).$$

For small times t all zeros but one of this polynomial are near the zeros of the polynomial $P_N(z)$. The extra zero z_{N+1} is far from other zeros, for small t,

$$z_{N+1} = \frac{a_N}{t} + ...\ .$$

In regular case main zeros are linear functions of t, for small t.

For large times all $n+1$ zeros are near the zeros of the equation

$$a_0 - tz^{N+1} = 0,\ z_n = \sqrt[N+1]{a_0/t}\exp\left(2\pi i\frac{n}{N+1}\right),\ n = 0,1,...,N.$$

At a root x_c of multiplicity k we have

$$\frac{P_N^{(k)}(x_c)}{n!}(x-x_c)^k + ... = tx_c^{N+1},$$

$$x_n(t) = x_c + c_{n,k}t^{1/k},\ c_{n,k} = \left(\frac{x_c^{N+1}n!}{P_N^{(k)}(x_c)}\right)^{\frac{1}{k}}\exp(2\pi i\tfrac{n}{k}),\ 0 \le n \le k-1.$$

So we can define the multiplicity of the root k from the time dependence of the roots. It is interesting to know how extra zero approach with time to the other zeros and then all of them organized as sites of symmetric polygon on the circle with decreasing radius. Note that coefficients $a_n,\ 1 \leq n \leq N$ are known functions of zeros but do not depend on t - are invariants - integrals of motion. Having N integrals of motion $H_n,\ 1 \leq n \leq N$ we construct Nambu-Poisson dynamics for the roots [9-11]

$$\dot{x}_n = \{x_n, H_1, H_2, ..., H_N\},\ 1 \leq n \leq N.$$

As an example we consider quadratic deformation of the linear equation

$$a_0 + a_1 z - tz^2 = -t(z - z_1)(z - z_2) = 0,$$

$$a_0 = -tz_1 z_2,\ a_1 = t(z_1 + z_2).$$

As a 'time independent' Hamiltonian we take

$$H = -a_0/a_1 = \frac{z_1 z_2}{z_1 + z_2}$$

the motion equations we find from the time independence of a_0 and a_1

$$\dot{a}_0 = -z_1 z_2 - t(\dot{z}_1 z_2 + z_1 \dot{z}_2) = 0,$$

$$\dot{a}_1 = z_1 + z_2 + t(\dot{z}_1 + \dot{z}_2) = 0,$$

$$\dot{z}_1 = \frac{z_1^3 z_2}{a_0(z_1 - z_2)} = \{z_1, H\} = f_{12}\frac{\partial H}{\partial z_2},$$

$$\dot{z}_2 = \frac{z_2^3 z_1}{a_0(z_2 - z_1)} = \{z_2, H\} = f_{21}\frac{\partial H}{\partial z_1},$$

$$f_{12} = \frac{z_1 z_2 (z_1 + z_2)^2}{a_0(z_1 - z_2)} = \frac{a_1^2}{t^3(z_2 - z_1)}.$$

In the cubic deformation of the quadratic equation

$$a_0 + a_1 z + a_2 z^2 - tz^3 = -t(z - z_1)(z - z_2)(z - z_3) = 0$$

we have

$$a_0 = tz_1 z_2 z_3,\ a_1 = -t(z_1 z_2 + z_2 z_3 + z_3 z_1),\ a_2 = t(z_1 + z_2 + z_3),$$

$$\dot{z}_1 = \frac{z_1^4 z_2 z_3}{a_0(z_2 - z_1)(z_1 - z_3)} = \{z_1, H_1, H_2\} = f_{1nm}\frac{\partial H_1}{\partial z_n}\frac{\partial H_2}{\partial z_m},$$

$$f_{123} = \frac{z_1 z_2 z_3 (z_1 z_2 + z_2 z_3 + z_3 z_1)(z_1 + z_2 + z_3)}{a_0(z_2 - z_1)(z_3 - z_2)(z_1 - z_3)} = \frac{a_1 a_2}{t^3(z_1 - z_2)(z_1 - z_3)(z_3 - z_2)},$$

$$H_1 = \frac{z_1 z_2 z_3}{z_1 z_2 + z_2 z_3 + z_3 z_1},\ H_2 = \frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3}.$$

Introducing new time variable $\tau = a_1 a_2 t^{-2}/2$ we put the equation in the form

$$\frac{dz_1}{d\tau} = \{z_1, H_1, H_2\} = f_{1nm} \frac{\partial H_1}{\partial z_n} \frac{\partial H_2}{\partial z_m},$$

$$f_{123} = \frac{1}{(z_1 - z_2)(z_1 - z_3)(z_2 - z_3)}.$$

For the following generalization of the Weierstrass function $V_n(z)$

$$\int_{V_n(z)}^{\infty} \frac{dV}{\sqrt{P_n(V)}} = z,$$

$$P_n(V) = \frac{4}{(n-2)^2} V^n + C_{n-2} V^{n-2} + ... + C_0,$$

we have the following series (re)presentation [6]

$$V_n(z) = \wp_n(z, C_{n-2}, ..., C_0) = \frac{1}{z^{2/(n-2)}} - \frac{(n-2)^2}{4(n+2)} C_{n-2} z^{2/(n-2)} + ...$$

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