Reports of Enlarged Session of the
Seminar of I. Vekua Institute
of Applied Mathematics
Volume 29, 2015

ON THE INFLUENCE OF THE CANCER PROTEINS ON THE BLOOD FLOW

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Abstract. In the paper the connection of the blood flow with the amount of the cancer cells and proteins in the blood is investigated at the small arteriole level.Cancer proteins change viscosity and density of blood. The oxygen consumption process is described by the Stokes system and depends on viscosity and density of blood plasma. It is shown that when viscosity and density grow, oxygen consumption rate decreases. The velocity profile of oxygen consumption is constructed by using Maple for the different parameters.

Keywords and phrases: Blood flow, Stokes flow, cancer.

AMS subject classification: 92C30, 92C35, 92C37, 76D07.

At the human body cancer is originated by free radicals, radiation etc., when the immune system is weekend by different stresses. In this process normal cell circle disrupt and abnormal cells begin to grow. Blood supplies all cells by nutrients, but cancer cells grow faster, their consumption rate is much faster, than of normal cells [1,2,4,5]. Especially they consume glucose. When the volume of cancer reaches the critical size it becomes dangerous. At this stage cancer cells begin to circulate in the blood and their amount depends on the size of cancer and can be measured in the laboratory. These cells can promote metastases at the different parts of body. The viscosity and density of blood flow changes dramatically. Consequently, metabolism, especially oxygen consumption by normal cells changes [1,2,4,5,6,7]. The normal cells are in permanent oxidative stress [1,2,4,5].

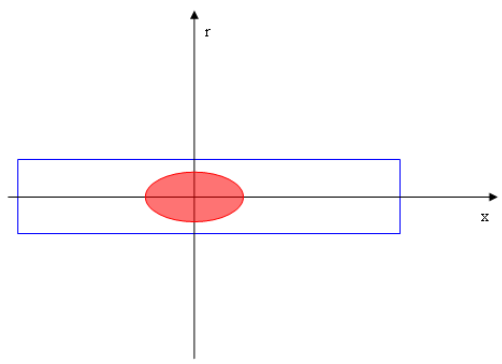


Fig. 1. Artificial image of the erythrocyte at the capillary

The human cardiovascular system is a transport system by means of which oxygen, carbon dioxide, and nutrients are carried by the blood vessels to and from various muscles and organs. Blood vessels are divided into several groups: large arteries, small arteries, arterioles, capillaries, venous, veins. Arterioles act as control velves through which blood is distributed into the capillaries. The blood pressure in each part of this system is different. The capillary is a cylinder with very thin walls (about $1\mu m$) containing blood plasma. Plasma is composed of 97% water and 3% other components. Viscosity and density of the blood plasma at the capillary is about 1. The radii of single capillary is about 4microm and the length-$1mm$.

Red blood cells Erythrocytes (RBC) play an important role in the oxygen transport process through the capillaries. They are most deformable blood cells with maximal thickness $8\mu m$. Adults human RBC are 25% of the total human body cell number. Each RBC contains 270 million of hemoglobin biomolecules. 97% of oxygen carried by blood from lungs is carried by hemoglobin, 3% is dissolved in blood plasma. RBC provides accelerate diffusion of oxygen and other substances at the maximal distance $50\mu m$ from the capillary. Velocity of RBC depends on heart rate and various from $0.1mm/sec$ to $1mm/sec$. RBC relies only 50% of its oxygen. Human being consumes about 260mL/min of oxygen at rest [1,2,3,12]. In every sec one RBC penetrate into the capillary from the arteriole end under the hydrostatic pressure and takes an ellipsoidal form. At the venous end hemoglobin at the RBC carries some of waste products and carbon dioxide (CO_2). The hydrostatic pressure at the arteriole end is about from $30mmHg$ to $40mmHg$ and at the venous end about $15mmHg$ to $20mmHg$. As a result of this pressure interstitial fluid with some plasma proteins filter through the walls of the capillaries into the tissue space. During the motion of RBC oxygen and carbon dioxide diffused to and from interstitial fluid of cells through the capillary walls. Interstitial fluid and blood plasma are very similar [1,2,3,12].

Oxygen consumption depends on the velocity of a single erythrocyte at the capillary level and the blood flow [2,3,6,7,8,12]. At the capillary level the erythrocyte takes the form of ellipsoid with the radiuses about $3\mu m$ and $4\mu m$. We consider the capillary as a cylinder with the ellipsoidal body (RBC) inside and choose the axisymmetric moving coordinate system (Fig.1, ox is axis of symmetry). As viscosity of blood plasma is low and flow at the capillary slow we can consider the axi symmetric motion of the ellipsoidal body at the cylinder (Fig 1). In this case we consider linearized Navier Stokes equation- Stokes axisymmetric system for the definition of the velocity of blood plasma through which O_2 is carried from the RBC to tissues [9,10,13]. In this case the motion is steady and the pressure fall is constant [3,12,13] i.e. the pressure P at the capillary is given by the formula $P = Cx + C_0, C$ and C_0 are the definite constants. The velocity of oxygen consumption depends on pressure falls and the velocity of blood plasma $\vec{V} = (V_x, V_r)$. In cylindrical coordinates this velocity satisfies the Stokes axisymmetric system [9,10,13]

$$\begin{aligned} \Delta V_x + \frac{1}{r}\frac{\partial V_x}{\partial r} &= \frac{C}{\rho\nu}, \\ \Delta V_r + \frac{1}{r}\frac{\partial V_r}{\partial r} - \frac{V_r}{r^2} &= 0, \end{aligned} \tag{1}$$

with the equation of continuity

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_r}{\partial r} + \frac{V_r}{r} = 0,$$

and the boundary conditions

$$\begin{aligned} &V_x|_{r=\pm h} = -V^0, \quad V_x|_S = 0, \\ &V_r|_{r=\pm h} = 0, \quad V_r|_S = 0, \end{aligned} \tag{2}$$

where V_x, V_r are the components of blood plasma velocity,$r = \sqrt{y^2+z^2}$, V^0 is the velocity of a single erythrocyte, S is a boundary of the erythrocyte, ρ is a density of plasma, ν-viscosity, h is a radii (width) of the capillary. The estimation of the flow velocity by system (1), (2) is more accurate, then by equation for the ideal fluid [11].

The solution of system (1), (2) is given by [9,10]:

$$V_x = 2q\left\{\frac{1}{(\sqrt{(x+a)^2+r^2})^3} - \frac{1}{(\sqrt{(x-a)^2+r^2})^3}\right\} \tag{3}$$

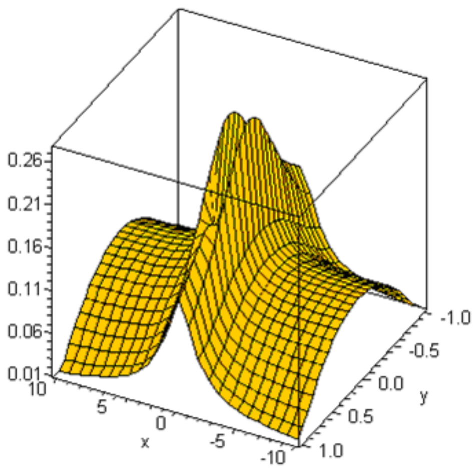
$$-3qr^2\left\{\frac{1}{(\sqrt{(x+a)^2+r^2})^5 3} - \frac{1}{(\sqrt{(x-a)^2+r^2})^5}\right\} + \frac{C}{\rho\nu}r^2 - C_1,$$

$$V_r = 3qr\left\{\frac{x+a}{(\sqrt{(x+a)^2+r^2})^3} - \frac{x-a}{(\sqrt{(x-a)^2+r^2})^3}\right\}, \tag{4}$$

where a, q and C_1 are the definite constants.

The velocity of blood plasma is equal to the oxygen consumption rate (here we will not consider the velocity of interstitial fluid under colloid-osmotic pressure, which is very low about $10^{-3} mm/sec$ [2,3]). From the formulas (3), (4) it is clear that if ρ and ν change, than the velocity also changes and consequently we can estimate changes of the oxygen consumption by the human body.

Below in Fig. 2, Fig. 3, the profile of velocity $|\vec{V}| = \sqrt{V_x^2+V_r^2}$ is constructed by using Maple and the following data $q = 0.1$, $a = 0.2$, $C = -10$, $C_1 = 1$. Fig.2 corresponds to the case of normal flow $\rho = \nu = 1$. In this case $|\vec{V}|_{max} = 2.6$. Fig.3 corresponds to the case of abnormal flow $\rho = \nu = 1.2$. In this case velocity decreases and



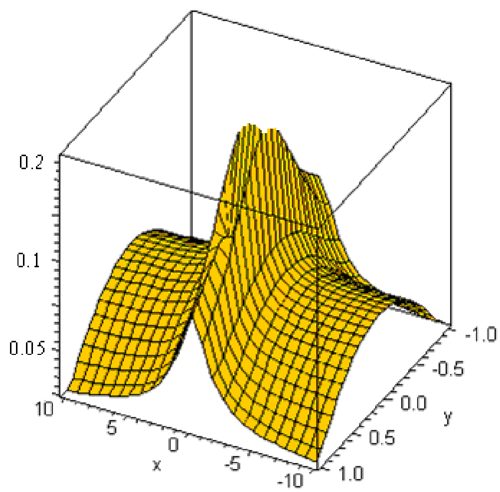


Fig. 2. $q = 0.1, \ a = 0.2, \ \rho = \nu = 1, \ |\vec{V}|_{max} = 2.6$.

Fig. 3. $q = 0.1, \ a = 0.2, \ \rho = \nu = 1.2, \ |\vec{V}|_{max} = 2$.

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Received 20.05.2015; revised 16.10.2015; accepted 20.12.2015.

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