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HYDRAULIC CALCULATION OF BRANCHED GAS PIPELINE BY QUASI-STATIONARY NONLINEAR MATHEMATICAL MODEL

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Abstract. In the present paper gas pressure and flow rate distribution along the main branched pipeline is investigated. The study is based on the solution of the simplified nonlinear, nonstationary partial differential equations describing gas quasi-stationary flow in the branched pipeline. The effective solutions of the quasi-stationary nonlinear partial differential equations are presented.

Keywords and phrases: Branched pipeline, gas quasistationary flow.

AMS subject classification: 76N15.

With the purpose of studying gas pressure and flow rate distribution in a pipeline we have been based on the following quasi-stationary, non-isothermal mathematical model describing gas flow in the branched main pipeline [1-4]

$$\frac{\partial P^2}{\partial x} = AQ^2, \tag{1}$$

$$\frac{\partial P}{\partial t} = B\frac{\partial Q}{\partial x} + q\delta(x - x^*), \tag{2}$$

where $Q(x,t)$ is volumetric gas flow rate, $P(x,t)$ is gas pressure, δ is Dirac function and $x*$ is placement of an offshoot in the pipeline, q is volumetric gas consumption in a branch-line $q = BV$, V is gas volumetric discharge consumption in branch-line in the offshoot, A and B are known values characterizing gas and pipe specifications [1-4]:

$$A = \frac{16\lambda P_0^2 T}{\pi^2 g R T_0 D^5}, \quad B = -\frac{4P_0T}{\pi T_0 D^2}, \quad q = B \cdot V,$$

where T is temperature (absolute) of gas, R is gas constant, Z is gas-compressibility factor, L, D, λ are length, diameter and hydraulic resistance factor of pipeline, is acceleration of gravitation, P_0 and T_0 are pressure and temperature of gas at standard conditions, V is gas volumetric consumption in the offshoot.

The system of equations (1), (2) with the additional constraints $0 \le x < L$, $t \ge 0$ is solved by the following initial and boundary conditions

$$P(x,0) = P_0(x), \tag{3}$$

$$P(0,t) = P_1(t), \quad Q(L,t) = Q_2(t). \tag{4}$$

Approximate analytical solution of the system of equations (1)-(2) for the simple pipelines (when q=0) is known. In the present paper we additionally consider a branch line and in our opinion this will be a new step for approximate analytical solution of the system of equations (1)-(2) with initial and boundary conditions (3)-(4). For this we use the following averaging [1, 3]

$$\frac{\partial P}{\partial t} \approx \frac{1}{L}\int\limits_0^L \frac{\partial P}{\partial t}dx = \varphi(t), \tag{5}$$

and well-known Dirac's function approximation [5]

$$\delta(x-x^*) = \frac{\alpha}{\pi\left[1+\alpha^2(x-x^*)^2\right]}, \tag{6}$$

where $\alpha \to \infty$.

Taking into consideration (6)

$$q\int\limits_x^L \delta(\xi - x^*)d\xi \approx \frac{q}{\pi}\left[\arctan\alpha(L-x^*) - \arctan\alpha(x-x^*)\right],$$

where $\alpha \to \infty$ and (5), then integration of equation (2) in the interval $[x, L]$ gives

$$\varphi(t)\cdot(L-x) = B\left(Q(L,t) - Q(x,t)\right) + \frac{q}{\pi}\left[\arctan\alpha(L-x^*) - \arctan\alpha(x-x^*)\right],$$

From the last equation we get

$$Q(x,t) = Q_2(t) - \frac{1}{B}\varphi(t)\cdot(L-x) + \frac{q}{\pi B}\left[\arctan\alpha(L-x^*) - \arctan\alpha(x-x^*)\right], \tag{7}$$

Solution (7) will be very useful for performing works of practical type in case of defining function $\varphi(t)$. Formula (7) after expansion in a Taylor series in the area of the point $x = L$ and keeping the first power terms gives:

$$Q(x,t)) = Q_2(t) - \frac{1}{B}\varphi(t)\cdot(L-x) - \frac{\alpha q(x-L)}{\pi B\left[1+\alpha^2(L-x^*)^2\right]}. \tag{8}$$

Integration of equation (1) in the interval $[0, x]$ gives

$$P^2(x,t) - P_1^2(t) = A\int\limits_0^x Q^2(x,t)dx.$$

If we take into account (8) in the last equality, after integration we get:

$$P^2(x,t) = P_1^2(t) + AQ_2^2(x_0,t)\cdot x + \frac{A}{3B^2}\varphi^2(t)\cdot\left[(x-L)^3 + L^3\right]$$

$$+\frac{A\alpha^2 q^2}{3\pi^2 B^2\left[1+\alpha^2(L-x^*)^2\right]^2}\left[(x-L)^3 + L^3\right]$$

$$+\frac{A}{B}\cdot Q_2(t)\cdot\varphi(t)\left[(x-L)^2 - L^2\right] - \frac{A\alpha\cdot q\cdot Q_2(t)}{\pi B\left[1+\alpha^2(L-x^*)^2\right]}\left[(x-L)^2 - L^2\right]$$

$$-\frac{2A\alpha\cdot q\cdot\varphi(t)\cdot\left[(x-L)^3+L^3\right]}{3\pi B^2\left[1+\alpha^2(L-x^*)^2\right]}. \tag{9}$$

Analysis of the terms orders in the right side of (9) shows that it is possible to neglect third and forth terms with respect to the rest components after expansion into a series the binomial and keeping first power terms. Taking into account that $\varphi^2(t)\approx 0$, then

$$P(x,t)=P_1(t)+\frac{A}{2P_1(t)}\left\{Q_2^2(t)x+\frac{1}{B}Q_2(t)\varphi(t)\cdot\left[(x-L)^2-L^2\right]\right.$$
$$+\frac{\alpha^2\cdot q^2\left[(x-L)^3+L^3\right]}{3\pi^2B^2\left[1+\alpha^2(L-x^*)^2\right]^2}-\frac{\alpha\cdot q\cdot Q_2(t)\left[(x-L)^2-L^2\right]}{\pi B\left[1+\alpha^2(L-x^*)^2\right]}$$
$$\left.-\frac{2\alpha\cdot q\cdot\varphi(t)\left[(x-L)^3-L^3\right]}{3\pi B^2\left[1+\alpha^2(L-x^*)^2\right]}\right\}. \tag{10}$$

Generally, at the standard working conditions gas feeding (supply) at the pipeline entrance point has stationary character that is why we assume that $P_1(t)=P_1=const$.

With the purpose of defining function $\varphi(t)$ first differentiating function $P(x,t)$ with respect to t then integrating of the obtained expression in the interval $[x,L]$ and in mind (5) gives:

$$\int_0^L\frac{\partial P(x,t)}{\partial t}dx=\frac{A}{2P_1}\int_0^L\left\{2Q_2(t)\cdot Q_2'(t)x+\frac{1}{B}(x^2-2Lx)\cdot\left[Q_2'(t)\varphi(t)+\varphi'(t)Q_2(t)\right]\right.$$
$$\left.-\frac{\alpha\cdot q\cdot Q_2'(t)\cdot\left[x^2-2Lx\right]}{\pi B\left[1+\alpha^2(L-x^*)^2\right]}-\frac{2\alpha\cdot q\cdot\varphi'(t)\left(x^3-3x^2L+3xL^2\right)}{3\pi B^2\left[1+\alpha^2(L-x^2)^2\right]}\right\}=\varphi(t)\cdot L. \tag{11}$$

$$\varphi(t)=\frac{A}{2P_1}Q_2(t)\cdot Q_2'(t)L-\frac{AL^2Q_2'(t)}{3P_1B}\varphi(t)-\frac{AL^2Q_2(t)}{3P_1B}\varphi'(t)$$
$$-\frac{\alpha AqQ_2'(t)L^2}{3P_1\pi B\left[1+\alpha^2(L-x^*)^2\right]}-\frac{A\alpha qL^3\varphi'(t)}{4P_1\pi B^2\left[1+\alpha^2(L-x^*)^2\right]}$$

In our notations:

$$a=\frac{AL^2Q_2(t)}{3P_1B}+\frac{A\alpha qL^3}{4P_1\pi B^2\left[1+\alpha^2(L-x^*)^2\right]},\qquad b=1+\frac{AL^2Q_2'(t)}{3P_1B},$$

$$d=\frac{AQ_2(t)Q_2'(t)L}{2P_1}-\frac{\alpha AqQ_2'(t)L^2}{3P_1\pi B\left[1+\alpha^2(1-x^*)^2\right]},\qquad E(t)=\frac{B}{a},\qquad N(t)=\frac{d}{a}$$

from expression (11) it follows that

$$\varphi'(t)+E(t)\cdot\varphi(t)=N(t). \tag{12}$$

Form equation (12) we then obtain:

$$\varphi(t)=e^{-\int E(t)dt}\left[\int N(t)\cdot e^{\int E(t)dt}dt+C\right]. \tag{13}$$

In (13) constant C can be obtained by the following equation $\varphi(0) = \varphi_0$, where $\varphi(0)$ can be defined from equation (7) by inserting $t = 0$ and $x = L$ by taking into consideration the following equality: $P(L, 0) = f(L)$.

It is notable to emphasize that formula (10) can be used for the definition of placement gas escape on the branched main pipeline. Namely for that we have to assume that x^* is unknown coordinate of the gas escape in equation (10) while we assume gas flow rate at the offshoot as the known function using one additional condition

$$P(L, \bar{t}) = \overline{P};$$

Some results of pressure distribution in the main pipeline with different location of offshoots are presented in Fig.1 and Fig.2.

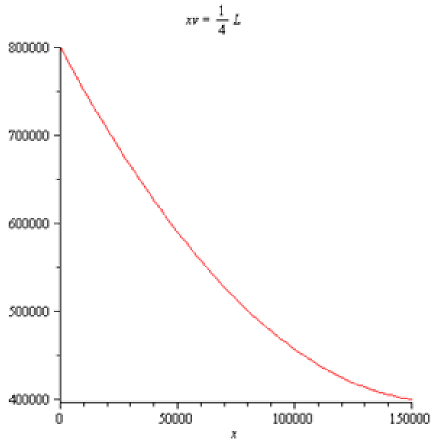

Fig.1 Pressure distribution in the branched pipe when the offshore located at the distance $L/4$ from the pipe's end point

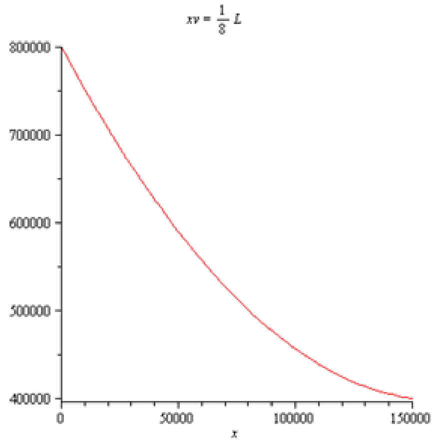

Fig.2 Pressure distribution in the branched pipe when the offshore located at the distance $L/8$ from the pipe's end point

R E F E R E N C E S

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