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APPROXIMATE SOLUTION OF ONE-DIMENSIONAL NONLINEAR MAXWELL MODEL WITH HEAT CONDUCTIVITY TERM

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Abstract. One-dimensional parabolic system of nonlinear partial differential equations is considered. The model is based on Maxwell's system which arises at describing penetration of a magnetic field into a substance. Semi-discrete scheme is constructed for the first type initial-boundary value problem. Graphs of numerical experiments are given.

Keywords and phrases: Nonlinear parabolic system, partial differential equations, semi-discrete scheme.

AMS subject classification: 35Q60, 35Q61, 35K55, 65M06.

One-dimensional model, which describes the process of diffusion of electromagnetic field with taking into account heat conductivity is studied. In particular, the following system is considered:

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(V^{\alpha}\frac{\partial U}{\partial x}\right), \tag{1}$$

$$\frac{\partial V}{\partial t}=V^{\alpha}\left(\frac{\partial U}{\partial x}\right)^{2}+\frac{\partial^{2}V}{\partial x^{2}}, \tag{2}$$

where α is a constant.

System of equations (1), (2) are interesting as for mathematics, also for physics and other scientific fields. As it is already mentioned, it represents one-dimensional analogue of system of equations, by which the electromagnetic field diffusion process with taking into account heat conductivity is described [1].

One must note that to investigation and approximate solution of (1), (2) and its multi-dimensional analogues many scientific works are dedicated (see, for example, [2]-[12] and references therein).

In the domain $Q=\Omega\times(0,T)$, where $\Omega=(0,1)$, let us consider the following initial-boundary value problem for system (1), (2):

$$U\left(x,t\right)=V\left(x,t\right)=0,\quad \left(x,t\right)\in\partial\Omega\times\left(0,T\right), \tag{3}$$

$$U\left(x,0\right)=U_{0}\left(x\right),\quad V\left(x,0\right)=V_{0}\left(x\right)\geq Const>0, \tag{4}$$

where U_0, V_0 are known functions defined on $\bar{\Omega}=[0,1]$, T is the fixed positive constant, $\partial\Omega$ is the boundary of Ω.

After introducing new notations $V^{1/2}=W$ and $2\alpha=\gamma$, problem (1) - (4) can be rewritten in the following equivalent form [2]:

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(W^{\gamma}\frac{\partial U}{\partial x}\right), \tag{5}$$

$$\frac{\partial W}{\partial t} = \frac{1}{2} W^{\gamma-1} \left(\frac{\partial U}{\partial x} \right)^2 + \frac{\partial^2 W}{\partial x^2} + \frac{1}{W} \left(\frac{\partial W}{\partial x} \right)^2, \tag{6}$$

$$U(x,t) = W(x,t) = 0, \quad (x,t) \in \partial\Omega \times (0,T), \tag{7}$$

$$U(x,0) = U_0(x), \quad W(x,0) = W_0(x) = V_0^{1/2}(x) \geq Const > 0. \tag{8}$$

Let us use well-known notations of time grid and forward derivative and consider the following semi-discrete scheme [2] constructed for problem (5) - (8):

$$\begin{gathered} u_t = \frac{d}{dx}\left(w^{\gamma} \frac{du}{dx} \right), \\ w_t = \frac{1}{2} w^{\gamma-1} \left(\frac{du}{dx} \right)^2 + \frac{d^2 w}{dx^2} + \frac{1}{w} \left(\frac{dw}{dx} \right)^2, \\ u_0 = U_0, \quad w_0 = W_0, \end{gathered} \tag{9}$$

with homogeneous boundary conditions (7).

The following statement takes place.

Theorem. *If $-1 \leq \gamma \leq 1$ and problem (5) - (8) has a sufficiently smooth solution U, W, then for the solution of the semi-discrete scheme (9) the following estimate holds*

$$\left\| U^j - u(t_j) \right\| + \left\| W^j - w(t_j) \right\| = O(\tau).$$

Here $\| \cdot \|$ is a discrete analog of the norm of the space $L_2(\Omega)$.

Let us note that semi-discrete and finite difference schemes for (1) - (4) and (5) - (8) type problems are constructed in many works (see, for example, [2]-[4], [6]-[12] and references therein). Convergence of the first order finite-difference scheme for (1) - (4) problem is fixed in [11]. Using this type scheme some numerical experiments are made for establishing parabolic regularization fact for (1) - (4) problem in [9].

In the present note applying natural discretization for space derivatives the finite difference scheme, based on the semi-discrete scheme (9), is also constructed. Using this scheme several numerical experiments are carried out. Some results of them are given in figures below (Fig.1 and Fig.2).

The graphs in Fig.1 illustrate the exact and numerical solutions and the differences between them for problem (5) - (8) with suitable right parts and for $\gamma = 1$. The exact solution is taken as:

$$U(x,t) = x(1-x)(1+t), \quad W(x,t) = x(1-x)\left(1+t+t^2\right).$$

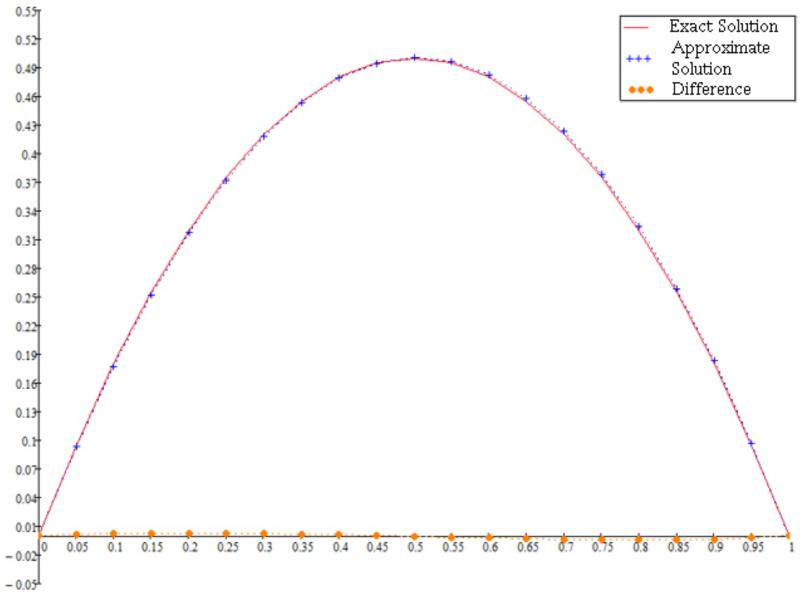

Fig.1. Exact and numerical solutions and the differences between them for the function U.

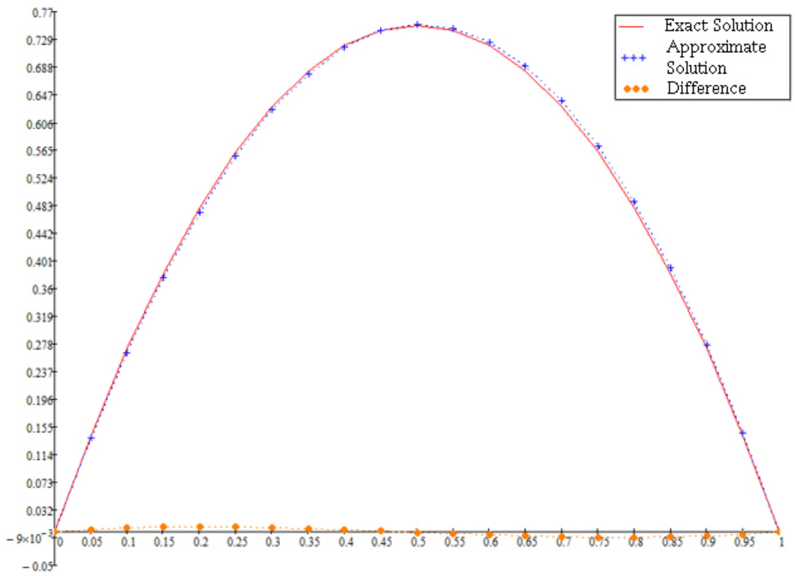

Fig.2. Exact and numerical solutions and the differences between them for the function W.

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